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ARSENIOS N. TSOKAS

Investigation of the Depth and the Broadness of the Dependence Among Behavioral Structures in Adult Education Class Networks

Supervisors: Antoniou Ioannis
Hasanagas Nikolaos
Mpekiari Alexandra

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ABSTRACT

A complete network analysis had been carried out in two Adult High Schools in Trikala and Ampelokipoi, Thessaloniki. The goal of the present research is to introduce new measures in statistical analysis, the certainty depth and the certainty broadness and apply these measures in social network analysis. The aim is also to study the concentrationism of the relation types involved in network analysis, as well as the pervasiveness of the structures involved and reach conclusions about each one of them. The diffused influence is also studied and, here, the notions of pure certainty depth and pure certainty broadness are introduced.

ΠΕΡΙΛΗΨΗ

Πλήρης δικτυακή ανάλυση είχε εφαρμοστεί σε εκπαιδευτικά ιδρύματα ενηλίκων στα Τρίκαλα και στους Αμπελοκήπους Θεσσαλονίκης. Στόχος της παρούσης έρευνας είναι η εισαγωγή νέων μέτρων στην στατιστική ανάλυση: η βεβαιότητα βάθους και η βεβαιότητα ευρύτητας, και η εφαμογή αυτών των μέτρων στην ανάλυση δικτύων των εκπαιδευτικών ιδρυμάτων. Στόχος επίσης είναι η μελέτη του συγκεντρωτισμού κάθε τύπου σχέσης η οποία εμπλέκεται στην δικτυακή ανάλυση καθώς και η διεισδυτικότητα κάθε εμπλεκόμενης δομής και η εξαγωγή συμπερασμάτων. Η διασπαρμένη επιρροή μελετάται επίσης και εδώ εισάγονται οι έννοιες της βεβαιότητας καθαρού βάθους και της βεβαιότητας καθαρής ευρύτητας.

CHAPTER 1

Introduction

The present study aims at introducing new measures, notions and ideas in network analysis and at applying these ideas in existing networks of classes of second-chance schools. The target is to delve deeper into network analysis and unveil connections between network variables via the proper interpretation of the results. The main contributions of the present study are:

- The introduction of the certainty depth and the certainty broadness in the interpretation of statistical results. How can one make the best use out of the result of a statistical or of the measure of an index?
- The introduction and study of the intrinsic structural concentrationism of a relation type, *i.e.*, of the concentration of different structures of power under one pattern within a given relation type.
- The introduction and study of the intersectoral pervasiveness of structure, *i.e.*, of the ability of a given structure to transcend different relation types.
- The study of the diffused influence, in the context of the analysis of structures of power in social networks, and the introduction and study of pure certainty depth and pure certainty broadness towards that direction.

CHAPTER 2

Literature Review

The algorithms of authority and pagerank are considered as the most suitable in evaluating the influence of an actor in a network (Brin & Page 1998, Garfield 1972, Kleinberg 1998, 1999). Applications of the pagerank algorithm include but are not limited to the influence of webpages and the influence of scientific papers published in journals. The pagerank algorithm has also been used as an index of the distribution of power in a social context (Sun *et al* 2012, Jøsang 2007, Wasserman & Faust 1994) Applications of the authority algorithm include finding the most "authoritative" member in a judiciary (Fowler & Jeon 2008). Katz's algorithm (Katz 1953), here referred to as status, has been applied in various psychology and sociology studies.

The choice of Adult School as the field of study of social research has been made in the past (Heidler 2014). The present study follows two such studies (Katsikas and Hasanagas 2013, Hasanagas *et al*, 2013), in which common methods of social network analysis were applied.

CHAPTER 3

Method and Theoretical Framework

Two identical surveys were conducted in two second-chance schools, one in Trikala and one in Ampelokipoi, Thessaloniki independently of one another. Such schools have been the subject of other studies. These are schools that offer high-school level education to individuals who cannot attend regular high school curriculum for various reasons, including but not limited to employment and parenthood. In each school, two classes took part in the survey: the first and second class of high school. A questionnaire was given to the students of the schools that is presented in table 3.1. The questions are meant to examine the relationship among the students and to trace the balance and flow of power within the microcosms of the classes.

Separate studies have examined the interaction between network features for the school in Trikala (Katsikas and Hasanagas, 2013) and for the school in Ampelokipoi (Hasanagas *et al*, 2013). The authors of these studies conducted the sampling. These studies also correlate network features with non-network ones, such as sex, income, age; thus attempting to explain network hierarchies using pre-determined and, at some point, unaltered characteristics. The present study focuses on network features and attempts to find similarities among different structures and among different relation types in the combined sample from both schools. We also examine the strength of the intrinsic structural concentrationism as well as of the intersectoral pervasiveness of structure. The former, an intrarelational, interstructural index, evaluates the dominance of a specific attribute throughout different types of hierarchy. The latter, an interrelational, intrastructural index, measures an algorithm's capability of penetrating in various contextual frames.

Although the present study does not target to the descriptive statistical representation of the classes, some basic characteristics of the sample are cited here. In total, there are 53 students in the classes surveyed. 26 are men and 27 are women, while 20 are married. The participants were born between 1951 and 1993, the average age being 36 years. The revenue of the participants varies between 0 and 1300€ with an average revenue of 559€.

In each question, the students were asked to point out those among their classmates that are most trusted or most wanted (e.g. for cooperation) with respect to specific disciplines or aspects of life. Based on the responses of the students, a network of power is formed indicating the flow of social power. The vertices correspond to the students of the class. An arc indicates that the student at the origin is susceptible to the student at the destination of the arc, in terms of the particular form of social power depicted in the current network.

It can be seen that the question try to take into account all three possible forms of power. The dominant one being trust, eight out of thirteen questions (GD, PD, TH, TN, TP, TA) are related more or less with the sense of trust. Provision of incentives is present in four elements (TH, TN). Irreplaceability plays a crucial role as well in tracking down power in school (IS, IO, GD, PD, AP). Finally there are two questions (DE, DI) that express negative forms of power, such as mistrust.

Table 3.1: Questionnaire and corresponding network

Question investigating network relation:	*Arising Network of:*	*Abbreviation:*
Please cite those among your classmates…		
…with whom you spend the most time during intervals	*Investing company time in school*	[IS]
…with whom you spend the most time after school	*Investing company time outside school*	[IO]
…with whom you would like specifically to cooperate in a group project at school	*Group work desire*	[GD]
…with whom you cooperate on a professional level or you would like to do so	*Professional cooperation desire*	[PD]
…whom you would consult if you had questions in language or socio-political and in general humanitarian courses	*Trust in humanitarian courses*	[TH]
… whom you would consult if you had questions in Mathematics, Physics, Chemistry, Biology	*Trust in natural sciences*	[TN]
…whom you would speak with about a serious professional issue that is concerning you	*Trust for professional advice*	[TP]
… whom you would speak with about a serious personal or family issue that is concerning you	*Trust for personal advice*	[TA]
…for whom you feel a specific appreciation	*Appreciation*	[AP]
… who, according to you, are evaluated by professors better than they deserve	*Derogatory characterization of educational attainment*	[DE]
…who, according to you, try to show off or	*Derogatory characterization*	[DI]

demonstrate something better than what they really are	*of public image*	

In all networks arcs start from each student and point to the student(s) suggested by the student at the origin in the respective question. It follows that arcs express the flow of power.

There is not a single way of evaluating the overall power exercised by an individual in a network. Many different approaches can be followed; each one of them measures the impact of a member on the rest of the network. These are usually called centralities in network analysis; they will be called algorithms in the present study. These are:

- Authority (Kleinberg, 1998): Characterizing someone as an authoritative source of information is based primarily on who asks them about the issue in question. An authoritative source is not necessarily asked (pointed) by many classmates (arcs) but is asked by those in the class that tend to seek and collect the best information (hubs). Intuitively, authoritative students are referenced by many hub students while hubs point to many authoritative students; in this way authoritative students and hubs reinforce one another. It is calculated as the leading eigenvector (the eigenvector corresponding to the largest eigenvalue, in terms of absolute value) of the matrix $A^T A$, where A is the adjacency matrix of the graph.
- In-degree: The in-degree of a vertex is calculated as the total of arcs pointing to that vertex normalized by the total number of vertices of the graph. The in-degree expresses how much immediate power a student exercises over his classmates. It is also called occasional dependence, as it takes into account only the first-step interplay between actors.
- Status (Katz, 1953): It is also referred to as Katz centrality and it is a natural generalization of the in-degree. Instead of considering only the adjacent vertices, we take into account all the vertices that can reach a specific vertex through a walk on the graph. In addition, we consider all possible paths that connect a pair of vertices but the longest the path, the less we take it into account through the implementation of an attenuation factor α. The status is calculated as $\mathbf{1}^T(\sum_{k=1}^{\infty}(\alpha A)^k)$, where **1** is the vector whose all elements are equal to 1.
- Pagerank (Page and Brin, 1998): This algorithm was introduced by Google in order to rank webpages, in terms of importance. Suppose that one starts from a given vertex on the graph and chooses some vertex to move to on each iteration. It is supposed that it is equally likely to pick any vertex pointed by the source to move to. One could think that this happens on every iteration, however this could lead to being trapped in sinks. Therefore we assume that the above happens with probability α. Alternatively, with probability $1-\alpha$, a random node v is picked with probability b_v. The pagerank score of all vertices is calculated as the leading eigenvector of the matrix $M = \alpha P + (1-\alpha)\boldsymbol{b}\mathbf{1}^T$, where **1** is the vector of ones, $\boldsymbol{b}$ is the vector of probabilities b_v and P is equal to A^Tcolumn-normalized (so that the sum of all elements in a column is equal to 1).

Authority, status, pagerank and in-degree were applied to all networks. Two different coefficients were calculated in the process of evaluating the correlation between any two centralities: Spearman's rho and mutual information.

Spearman's rank correlation coefficient - or Spearman's rho – is widely used in social sciences. The main advantage of Spearman's rho is that, being a nonparametric test, it does not require that the sample come from a population that follows some specific probability distribution as well as being not sensitive to outliers. In fact, graph theory assures that a sample consisting of the centralities of the nodes of a network does not follow the normal distribution, no matter what centrality is calculated; therefore Pearson's coefficient could not be used. Spearman's rho is an easily manipulated coefficient that provides an insight to whether there is a monotonic relation between two variables. In our case, Spearman's rho can trace such monotonic relationships between two variables corresponding to different algorithms and/or different relation types, therefore establishing whether there is a connection between these variables. The SPSS package and MATLAB was used for the calculation of Spearman's coefficient as well as of the p-value associated with it.

Mutual information $I(X,Y)$ is a general index of the dependence between two quantities, as it evaluates the joint probability distribution between two variables, X and Y, providing a measure of the heterogeneity of that distribution. There are many different ways of calculating mutual information – depending on the selection of the basis of the logarithm – and as many units of measurement, however they only differ by a multiplication with a constant. In the present study, mutual information is calculated using the natural logarithm and is measured in $nats$ as this is the most common method used in studies implementing the mutual information in continuous distributions. The mutual information has a natural intuitive meaning, although it is not obvious. Assume that on average n Yes/No questions are required in order to find X and on average m Yes/No questions are required to find X after observing Y. Then the mutual information is proportional to $n-m$. The mutual information takes values between 0 and $\ln N$, where N is the size of the sample, in our case $\ln 53 \cong 4$. The upper bound however is rarely achieved and one could find a better upper bound based on the properties of the sample. The upper bounds are of no concern in this study.

A permutation test is implemented, with 300 random permutations of the sample, in order to assign a p-value to the mutual information of each pair. Mutual information and the corresponding p-values were calculated with MATLAB.

What do Spearman's rho and mutual information suggest and what are the roles of the corresponding p-values? The first two are indexes of the interdependence between two different aspects of the hierarchy of our sample consisting of the students of the schools under examination, each one corresponding to a specific relation type. They provide an insight as to whether there is a strong connection between those aspects. The p-values reveal the probability that the observed score or quantity of information is due to pure luck, rather than a well-established and deep connection. In follows that 1-p expresses the certainty by which one

can generalize a result or, equivalently, the probability that the strength of connection suggested by Spearman's coefficient or mutual information is true more generally.

Based on the above, we define two new measures to evaluate the interdependence between two different hierarchies in the social network of the class. Let dv denote the dependence value for some specific test; it can be either Spearman's coefficient or mutual information. The certainty depth of that test is defined as:

Equation 3.1: $$\boldsymbol{CD = |dv|(|dv| + 1 - p)}$$

For the sake of clarity, we will denote the certainty depth with CD_S if it refers to a Spearman test and with CD_{MI} if it the dependence value is mutual information. The certainty broadness is defined as:

Equation 3.2: $$CB = (1 - p)(1 - p + |dv|)$$

Likewise, CB_S will be used for a certainty of broadness based on a Spearman test and CB_{MI} will be used for a certainty of broadness of the mutual information. We sometimes refer to certainty depth and certainty broadness as just depth and broadness respectively for simplicity.

Table 3.2: Conceptual framework

		Algorithm	
		Same (intrastructural)	Different (interstructural)
Relation type	Same (intrarelational)	Coefficient=max, p=0 (no research question)	*Intrinsic structural concentrationism (concentrating different patterns within the same relation)*
	Different (interrelational)	*Intersectoral pervasiveness of structure (talent to play the same role in various relational sectors)*	Diffused influence

These indexes were calculated for all possible pairs of hierarchy types: each element of the pair may refer to any algorithm and any relation type. However some cases are of specific interest. One of these is when the two hierarchy types correlated refer to the same relation type. What is under investigation, in this case, is whether a relation type can make two different patterns of power coincide and, hence, whether a relation type is capable of concentrating two different structures. We will refer to such cases as *intrinsic structural concentrationism*. Conversely, if the two hierarchy types that are correlated are using the same

algorithm but refer to different relation types, what is investigated is the ability of a specific structure to pierce through two different sectors of social power. Such cases will be referred to as *intersectoral pervasiveness of structure*. In the general case, where both the algorithm and the relation type are different, each case of strong correlation has to be examined independently, as the cause behind an interplay may be just incidental. However, such cases may trace the general influence exercised by some specific type of hierarchy over other types. These ideas are summarized in table 2.

These four indexes, CD_S, CD_{MI}, CB_S and CB_{MI}, are also used to evaluate the penetrability of each algorithm and of each relation type. Similar indexes correlating hierarchies of a specific algorithm but of all different relation types are averaged to provide a measure of the omnipresence of that specific algorithm in various relational contexts. Likewise, similar indexes correlating hierarchies of a specific relation type and of all different algorithms are averaged to yield a measure of the strength of that relation type across different patterns of imposition.

The method followed in order to study the extrinsic general influence is comparative. We study the depth and broadness of mutual information and Spearman's coefficient by applying a normalization. Let a measure (CD_S, CD_{MI}, CB_S or CB_{MI}) of the correlation of two algorithms referring to two different relation types have a great value. Such a measure having a great value may be important on its own but it might be a side-effect of the pervasiveness of an algorithm and of the concentrationism of a relation type.

To formalize ideas, let k_{aibj} be a measure of the correlation between algorithm a applied to relation i (variable 1) and algorithm b applied to relation j (variable 2). If the aforementioned measure is great, it might be just because k_{aiaj} and k_{ajbj} are both great. Alternatively, k_{aibj} may be great because k_{aibi} and k_{bibj} are both great. In order to counter the influence of the mediating measures, we propose the following formula:

Equation 3.3: $$purek_{aibj} = \sqrt[4]{\frac{k_{aibj}^4+1}{k_{aiaj}k_{ajbj}k_{aibi}k_{bibj}+1}}.$$

The above formula is applied to all measures (CD_S, CD_{MI}, CB_S or CB_{MI}), thus creating the corresponding pure measures ($pureCD_S$, $pureCD_{MI}$, $pureCB_S$ or $pureCB_{MI}$). In the above, we compare k_{aibj}^4 with the product $k_{aiaj}k_{ajbj}k_{aibi}k_{bibj}$. Intuitively, if the former is great independent of the latter, then there is a deeper connection between the two variables in question, a connection that does not depend on mediating measures. We choose to compare the ratio of $k_{aibj}$with the mediating measures $k_{aiaj}, k_{ajbj}, k_{aibi}$ and k_{bibj} as these measures are calculated from the results of a statistical test and the corresponding p-values, hence it is necessary that all of the mediating measures be great so that k_{aibj} be great. The ratio provides us with this property. In addition, we add 1 in both the numerator and the denominator in order to avoid dividing by zero or values close to zero.

Assuming that k_{aibj} takes values between 0 and M, then $purek_{aibj}$ takes value between $\sqrt[4]{\frac{1}{M^4+1}}$ and $\sqrt[4]{M^4+1}$. In the case of CD_{MI}, such limits are approximately 0.05 and 20, while in the case of CB_{MI}, the limits are 0.2 and 5. In the case of CD_{SP} and CB_{SP} the limits are 0.4925 and 2.0305.

CHAPTER 4

Results and Discussion

4.1 Study of Intrinsic Structural Concentrationism

4.1.1 Appreciation (AP)

All different algorithms applied to the networks share a strong connection via mutual information. In particular, the status and the authority are very deeply connected (Table 4.1 and 4.2), revealing that students receiving cumulative appreciation by their classmates are also sought after by those who try to give their appreciation only to those who are worth it (the hubs). Apart from the connection between deep-rooted hierarchies (authority, pagerank, status), the occasional dependence (in-degree) is related as well to those hierarchies, indicating that the apportionment of appreciation is in accordance with the overall high opinion one receives from the total of the class. The broadness of mutual information is also great in value, validating the significance of the above results.

The depth of Spearman's coefficient for the pairwise relationship between authority, pagerank and status is great as well, indicating that not only the apportionment but also the order of appreciation is capable of transcending through different hierarchies. However, the correlation between the structures of the occasional dependence and the other algorithms is even greater. This fact shows that the order of appreciation is well established and common among all students and thus determines all other hierarchies based on deeper structures.

What is it that causes the in-degree to have less mutual information with authority, pagerank and status respectively than these structures have between them pairwise? And what

causes, on the contrary, greater correlation between the in-degree and the aforementioned structures than the correlation between these hierarchies pairwise? The immediate appreciation, as expressed by the in-degree, can capture partially the overall appreciation one enjoys. More precisely, it is an adequate indicator of who is appreciated the most, who comes second, etc. as verified by Spearman's coefficient. However, if one attempts to quantify the appreciation received by a student, then it turns out that the immediate appreciation can provide a lot but not all of the necessary information about deep-rooted structures such as authority, pagerank and status.

Table 4.1 : Depth by mutual information

	Authority	Pagerank	Status
Indegree	4.4766	4.0148	5.9964
Authority		6.2952	7.9216
Pagerank			6.6422

Table 4.2 : Broadness by mutual information

	Authority	Pagerank	Status
Indegree	2.6594	2.5509	2.9832
Authority		3.0420	3.3410
Pagerank			3.1087

Table 4.3: Depth by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.3200	1.4141	1.3604
Authority		0.9777	0.4284
Pagerank			0.7281

Table 4.4: Broadness by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.7530	1.7900	1.769
Authority		1.6080	1.2877
Pagerank			1.4890

4.1.2 Derogatory characterization of public image (DI)

Surprisingly, the information obtained by observing the distributions of indegree, authority, pagerank and status of depreciate image pairwise is low (tables 4.5 and 4.6). However, the broadness is greater than the depth for all instances, indicating that this amount of information is significant.

Table 4.5: Depth by mutual information

	Authority	Pagerank	Status
Indegree	1.5584	1.6628	1.4666
Authority		1.7457	1.5584
Pagerank			1.4666

Table 4.6: Broadness by mutual information

	Authority	Pagerank	Status
Indegree	1.8335	1.8716	1.7990
Authority		1.9011	1.8335
Pagerank			1.7990

That being said, the great depth (table 4.7) of Spearman's coefficient and the even greater broadness (table 4.8) validate the connection between the different hierarchies. Presumably, there is a simple yet well-established structure in the classrooms with regard to who tries to show off. More precisely, there are few students that receive a derogatory characterization of their public image and no perplexed paths of derogatory characterizations exist, as indicated by the fact that the in-degree agrees with the authority, pagerank and status.

Table 4.7: Depth by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.7607	1.9641	1.9880
Authority		1.9701	1.7607
Pagerank			1.9523

Table 4.8: Broadness by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.9180	1.9880	1.9960
Authority		1.9900	1.9180
Pagerank			1.9840

4.1.3 Derogatory characterization of educational attainment (DE)

Similar to the case of derogatory characterization of public image, there is little information provided by the pairwise common distribution of different hierarchies including indegree, authority, pagerank and status. The broadness is greater than the depth, indicating that there are few and targeted characterizations among the students regarding the question of who receives better grades than he or she deserves.

Table 4.9: Depth by mutual information

	Authority	Pagerank	Status
Indegree	1.5584	1.6628	1.4666
Authority		1.7457	1.5584
Pagerank			1.4666

Table 4.10: Broadness by mutual information

	Authority	Pagerank	Status
Indegree	1.8335	1.8716	1.7990
Authority		1.9011	1.8335
Pagerank			1.7990

Spearman's coefficient verifies that the same hierarchy is deeply-rooted, regardless of the algorithm by which it is measured. The in-degree, pagerank and status are very strongly correlated while the authority is just strongly correlated to the other algorithms (tables 4.11 and 4.12), meaning there is a slight differentiation in the hierarchy as suggested by authority and that suggested by the other three algorithms. What could be the reason behind this deviation? There are some students that are believed to receive the favor of the professors by the total of the class. However there are some others that are believed to receive such treatment by the most observant of their classmates (the hubs); these are pointed out only by authority. These students receive –or are thought to receive- beneficial treatment by the teachers in a more latent, under-the-radar fashion which can only be discovered by those who carefully examine the way the teachers distribute the grades.

Table 4.11: Depth by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.4452	1.9671	1.9790
Authority		1.3401	1.3401
Pagerank			1.9345

Table 4.12: Broadness by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.8020	1.9890	1.9930
Authority		1.7610	1.7610
Pagerank			1.9780

Considering the derogatory characterizations of public image and educational attainment, the structures observed in the classes are relatively simple while few specific

students are characterized as either trying to show off or receiving beneficial treatment in grading. Students generally hesitate in pointing at someone in a negative manner, which may be natural: one tends to be really sure before forming a negative opinion about somebody else.

4.1.4 Group work desire (GD)

Table 4.13: Depth by mutual information

	Authority	Pagerank	Status
Indegree	2.4065	1.8499	2.1264
Authority		1.9556	2.3537
Pagerank			1.8030

Table 4.14: Broadness by mutual information

	Authority	Pagerank	Status
Indegree	2.1174	1.9374	2.0295
Authority		1.9733	2.1012
Pagerank			1.9212

It follows from the values of the certainty of depth and the certainty of broadness of the mutual information that the hierarchies suggested by these algorithms are involved and separate the students into multiple levels (the values are significant; this particular question was posed only to the students of the school at Trikala). These hierarchies agree with each other, as indicated by Spearman's coefficient. The general idea as of who is more worthy grouping together with is well-established in the classes and it is also very precise, as it can be detected by all the algorithms.

Table 4.15: Depth by Spearman's coefficient

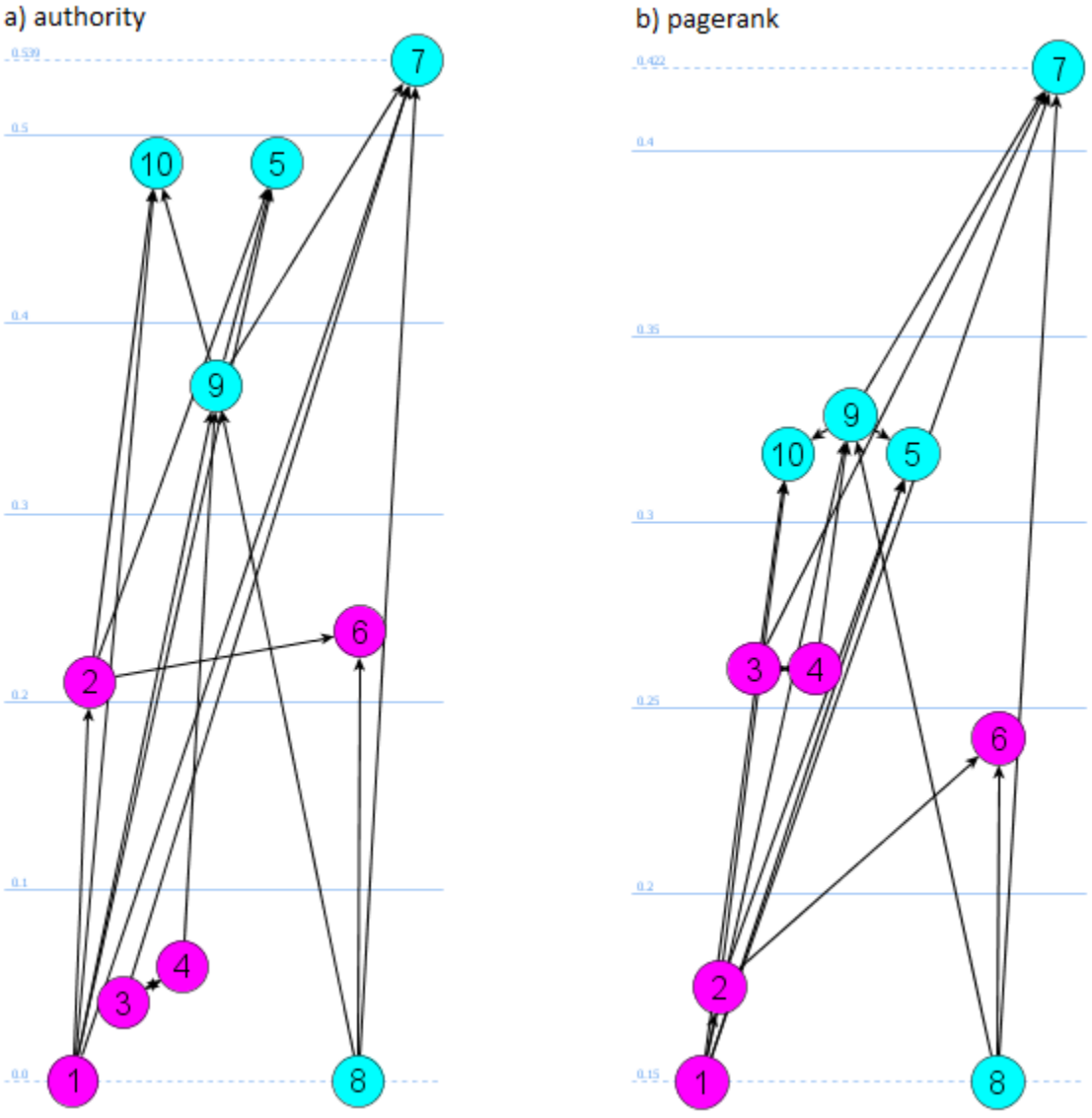
	Authority	Pagerank	Status
Indegree	1.8699	1.5887	1.7466
Authority		1.3351	1.8496
Pagerank			1.4950

Table 4.16: Broadness by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.9560	1.8560	1.9130
Authority		1.7590	1.9490
Pagerank			1.8210

In-degree, authority and status share a very strong relation, yet pagerank's correlation with these three algorithms is a bit weaker: the hierarchy suggested by pagerank is slightly different (figure 4.1). That is, those who are sought after as group mates by the majority (great in-degree) are also sought after by those who look for the best (the hubs - great authority) and are also those who are most wanted in an immediate, cumulative fashion (great status). However, things become different when one considers who is most likely to end up in a collaborating group (great pagerank). The probability of joining a work group does not necessarily agree with how much one is wanted as a partner –at least not in terms of order among the classmates.

Figure 4.1: The graph of GD of the second class of Trikala layered using authority (a) and pagerank (b)



4.1.5 Desire to invest company time in school (IS)

The mutual information between the different algorithms has great depth and great broadness (tables 4.17 and 4.18). More precisely, the fact that the mutual information takes great values indicates that the algorithms break down the networks into nodes, where almost each one of which assumes a different value (figures 4.2 and 4.3). The mutual information depth and broadness between the in-degree and any of the other algorithms is lower than the depth and broadness respectively between authority, pagerank and status. In other words, the occasional dependence fails to capture the breakdown of hierarchy as suggested by the other three algorithms, which delve deeper into the structures of the networks.

Figure 4.2: Network of IS in Ampelokipoi, 2nd grade, layered by pagerank.

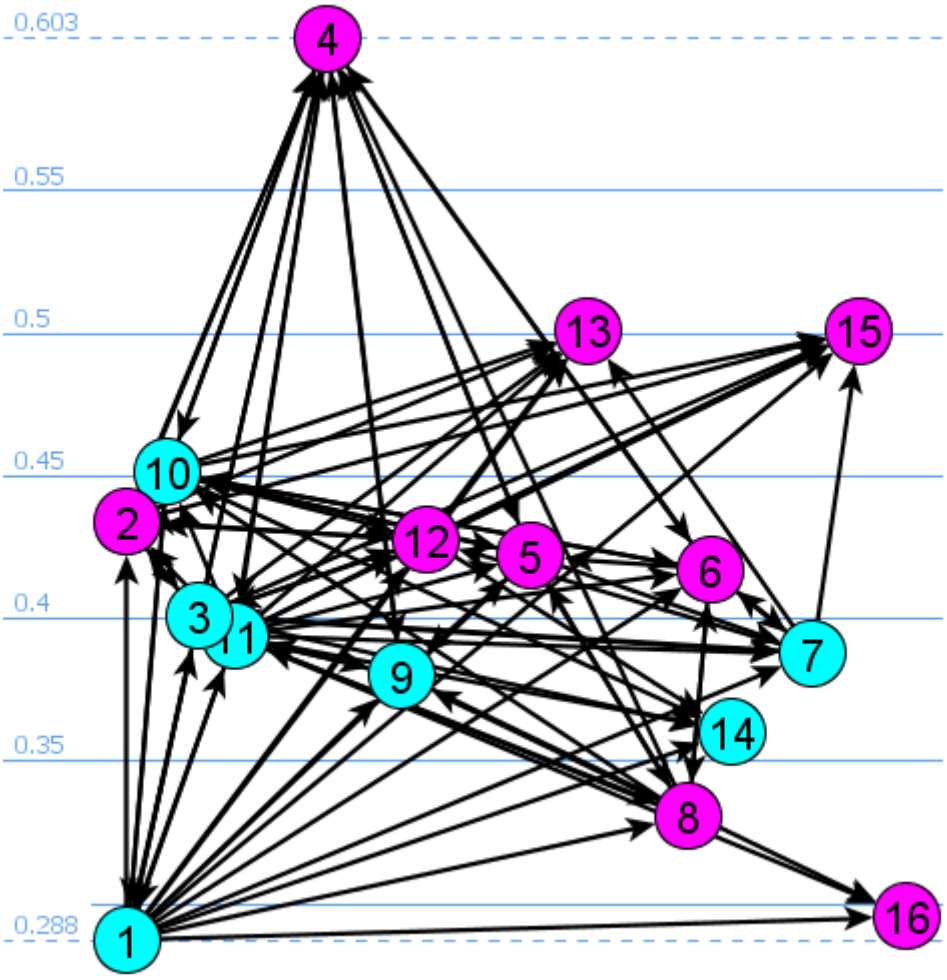


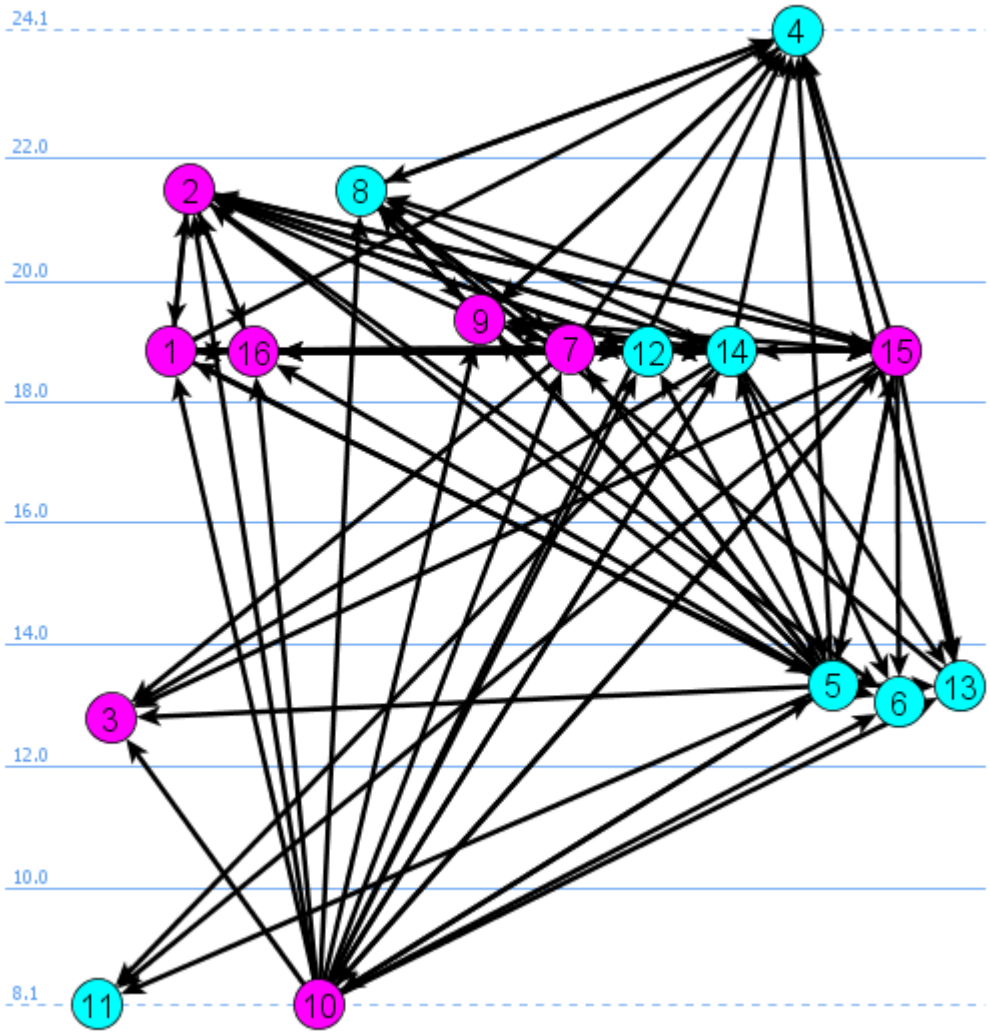
Table 4.17: Depth by mutual information

	Authority	Pagerank	Status
Indegree	5.4293	4.8232	5.7752
Authority		7.9459	6.9028
Pagerank			7.0621

Table 4.18: Broadness by mutual information

	Authority	Pagerank	Status
Indegree	2.8675	2.7373	2.9387
Authority		3.3452	3.1576
Pagerank			3.1871

Figure 4.3: Network of IS in Ampelokipoi, 1st grade, layered by status.



Both the depth and the broadness of Spearman's coefficient assume great values as well. However, the two pairs that stand out of the crowd are in-degree-authority and pagerank-status, while all other values are significantly lower. What could have led to this phenomenon? One is as much likely to be sought after for company at school by a classmate chosen at random as he or she is to be sought after by a classmate who looks for the best to spend his time with; or simpler: one's quality as company is evident to all. Meanwhile, the probability that one has some company in school at a specific time (pagerank) strongly agrees with the cumulative hierarchy (status).

Table 4.19: Depth by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.6682	1.2950	1.3175
Authority		0.8337	0.9056
Pagerank			1.6599

What causes the slight differentiation between in-degree and authority on the one hand and pagerank and status on the other hand? Generally, how good company one is determines the time you spend with others at school. However, there are some students that play their cards better (or worse) in terms of the network of companionships they form, so that they end up with more (or less respectively) company than they deserve.

Table 4.20: Broadness by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.8850	1.7430	1.7520
Authority		1.5410	1.5750
Pagerank			1.8820

4.1.6 Desire to invest company time outside of school (IO)

Mutual information between different algorithms pairwise assumes average-to-great values in depth (table 4.21) and great values in broadness (table 4.22). The latter indicates that the algorithms agree with each other on the breakdown of hierarchy in the network; the former implies that this hierarchy has many ties and the structure of the networks is rather simple (see for example figure 4.4). In particular, the status has a very strong connection with all other algorithms. In other words, the cumulative hierarchy manages to capture most of the information provided by the graphs of the classes. Apparently, it is if one is preferred as company by some who are also preferred as company by others and these in turn are also preferred as company that he/she tends to dominate the out-of-school fellowship. He/she not

only draws attention by the most and the best of his/her classmates, but is also more likely to have a company out of school.

Figure 4.4: Network of IO in Ampelokipoi, 2nd grade, layered by status.

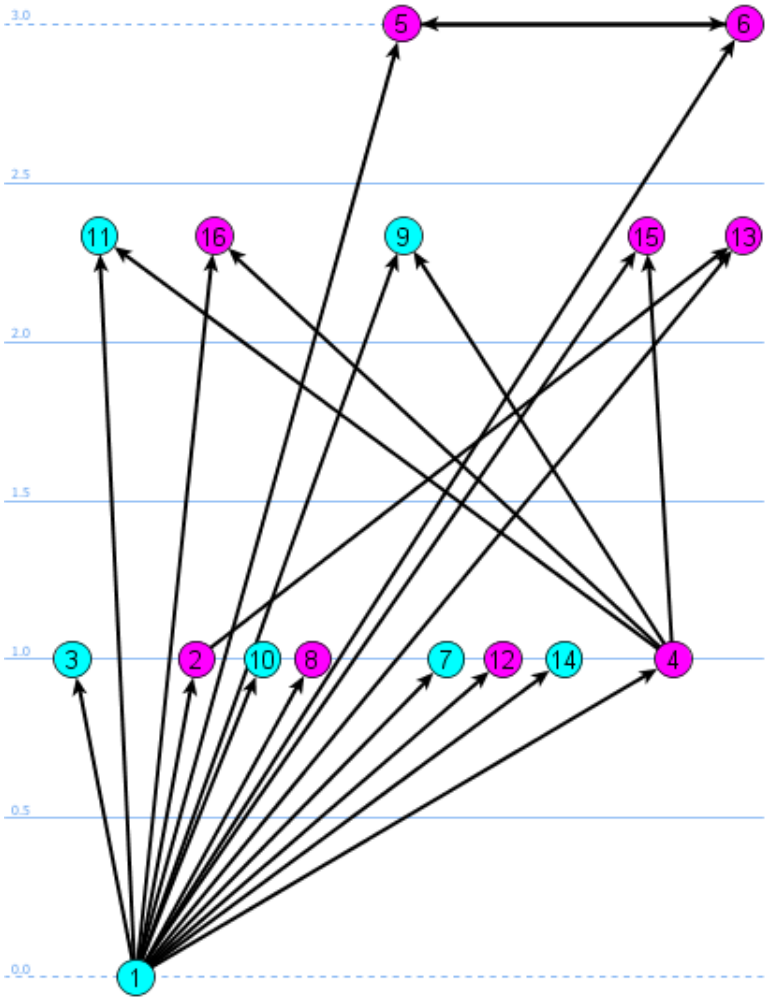


Table 4.21: Depth by mutual information

	Authority	Pagerank	Status
Indegree	3.7408	2.7968	4.5680
Authority		3.0103	4.6251
Pagerank			3.5312

Table 4.22: Broadness by mutual information

	Authority	Pagerank	Status
Indegree	2.4837	2.2326	2.6802
Authority		2.2925	2.6931
Pagerank			2.4308

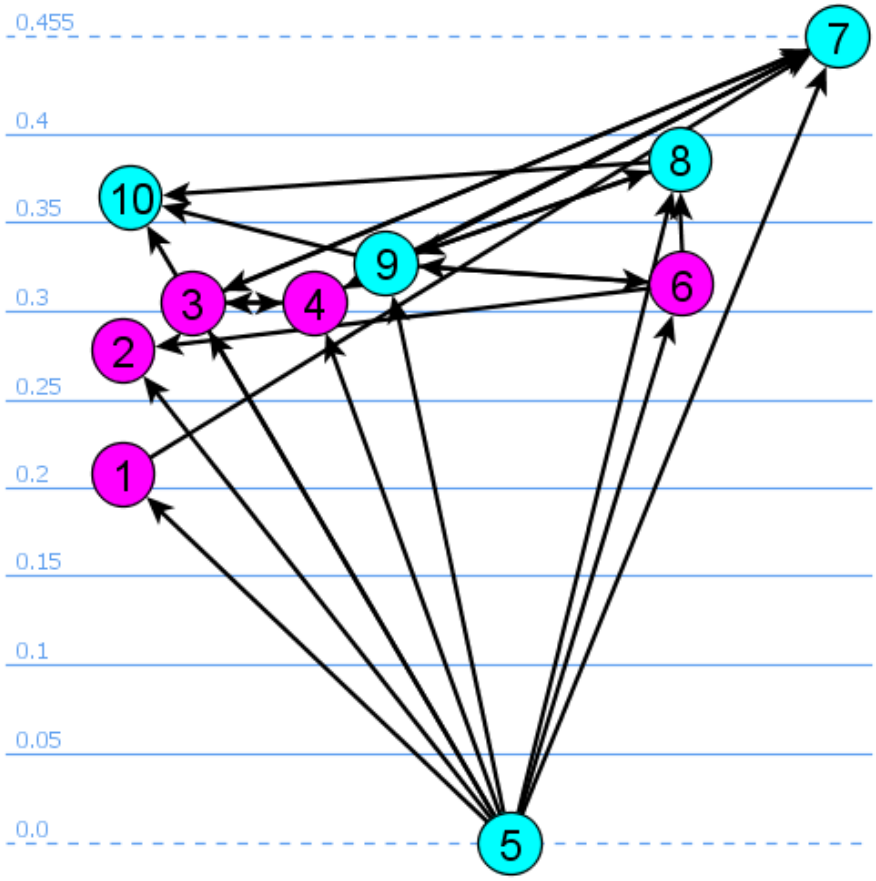
Table 4.23: Depth by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.6489	1.7978	1.8845
Authority		1.6599	1.7863
Pagerank			1.9404

Table 4.24: Broadness by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.8780	1.9310	1.9610
Authority		1.8820	1.9270
Pagerank			1.9800

Figure 4.5: Network of IO in Trikala, 2nd grade, layered by authority.



The above findings are validated and reinforced by Spearman's coefficient (tables 4.23 and 4.24). In addition, the authority presents a slightly weaker correlation with the other three algorithms than the in-degree, status and pagerank present between one another (while this is not the case with the mutual information). The amount of attention one gets as possible out-of-

school company does not necessarily coincide with how really nice it is to hang out with him/her (the hubs' choice). What could be the reason behind this discrepancy? A person worth spending time with is not always recognizable, let alone in the environment of the classroom. The opposite may also hold: one may be considered a good company in the classroom although in fact he/she does not have this quality.

4.1.7 Professional Cooperation desire (PD)

Table 4.25: Depth by mutual information

	Authority	Pagerank	Status
Indegree	3.1246	1.5579	2.7174
Authority		2.6619	3.1692
Pagerank			1.7245

Table 4.26: Broadness by mutual information

	Authority	Pagerank	Status
Indegree	2.3237	1.8333	2.2098
Authority		2.1937	2.3358
Pagerank			1.8936

The algorithms applied to the networks of professional cooperation desire are characterized by low values of depth of mutual information (table 4.25) and by average values of broadness of mutual information (table 4.26), revealing that the hierarchy suggested by the algorithms has many ties –many people are equally likely to be considered good potential partners (figure 4.6).

In addition, the greatest values are observed when authority is involved, while the lowest ones are observed when pagerank is involved. The prevalence of authority is based on the fact that the students tend to be very careful in their professional life and selective as to whom they will cooperate with. As a result, students that are equally likely to be preferred by those who look for the best (the hubs) are also equally likely to be chosen generally. On the other hand, the slightly lower mutual information when pagerank is involved indicates that one possibly has more (or less) chances of ending up in a professional cooperation than his/her actual evaluation from his classmates would suggest.

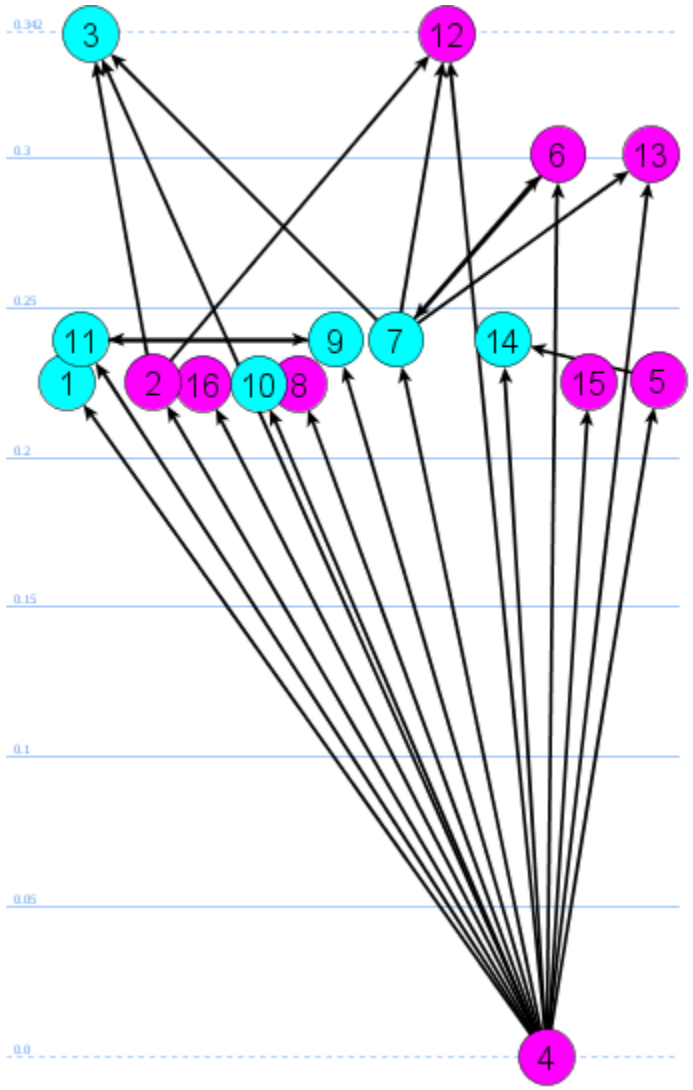
Table 4.27: Depth by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.5510	1.8265	1.9315
Authority		1.6793	1.6627
Pagerank			1.9227

Table 4.28: Broadness by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.8420	1.9410	1.9770
Authority		1.8890	1.8830
Pagerank			1.9740

Figure 4.6: Network of PD in Ampelokipoi, 2nd grade, layered by authority.



Spearman's coefficient assumes great values both in depth and in broadness (tables 4.27 and 4.28), reinforcing the strong connection between the different algorithms. However, the authority presents a slightly weaker correlation with the other algorithms than the status, pagerank and in-degree demonstrate between them pairwise. What is the reason behind this turning of the table, as compared to mutual information, behind the authority suggesting a slightly different order of hierarchy? One that is chosen as a potential professional partner by

those who seek for the best professional partners (the hubs) may not be sought by the majority and may not be likely to eventually cooperate with someone; in fact authoritative nodes are revealed and appreciated mostly by the hubs while the majority will classify them together on a different level of hierarchy.

4.1.8 Trust for humanitarian courses (TH)

The mutual information assumes great depth and broadness values (tables 4.29 and 4.30), indicating that the hierarchies suggested by the algorithms are complicated and also agree with one another. The greatest values appear when the status is involved, demonstrating the dominance of status over the other algorithms. In other words, one's position in the order of preference regarding questions about humanitarian courses is determined by the cumulative hierarchy; intuitively, if many questions –where each question is asked successively to different persons, the one asked becoming the questioner- end up to someone, then he is much trusted and his/her opinion affects the others. The mouth-to-mouth character of the class networks can be attributed to the subjectivity of the issues addressed by humanitarian courses, which enhances the spread of rumors.

Table 4.29: Depth by mutual information

	Authority	Pagerank	Status
Indegree	5.2597	4.7598	6.2676
Authority		5.6981	6.4193
Pagerank			5.8723

Table 4.30: Broadness by mutual information

	Authority	Pagerank	Status
Indegree	2.8318	2.7233	3.0366
Authority		2.9230	3.0660
Pagerank			2.9583

The strong connection among the algorithms is verified by the great depth and broadness of Spearman's coefficient (tables 4.31 and 4.32), reinforcing the fact that the hierarchies suggested by each of the algorithms present little differences. In particular, the in-degree demonstrates a surprisingly great correlation with the other algorithms, especially the authority and the status. Apparently, one's position as determined by the authority and the status relies heavily on his/her occasional dependence, the in-degree: it is necessary to be asked by the majority to end up dominating the network. Again the subjectivity of the humanitarian sciences plays its role: whoever is asked is much likely to influence others, while the more elaborate algorithms do not offer much of extra information.

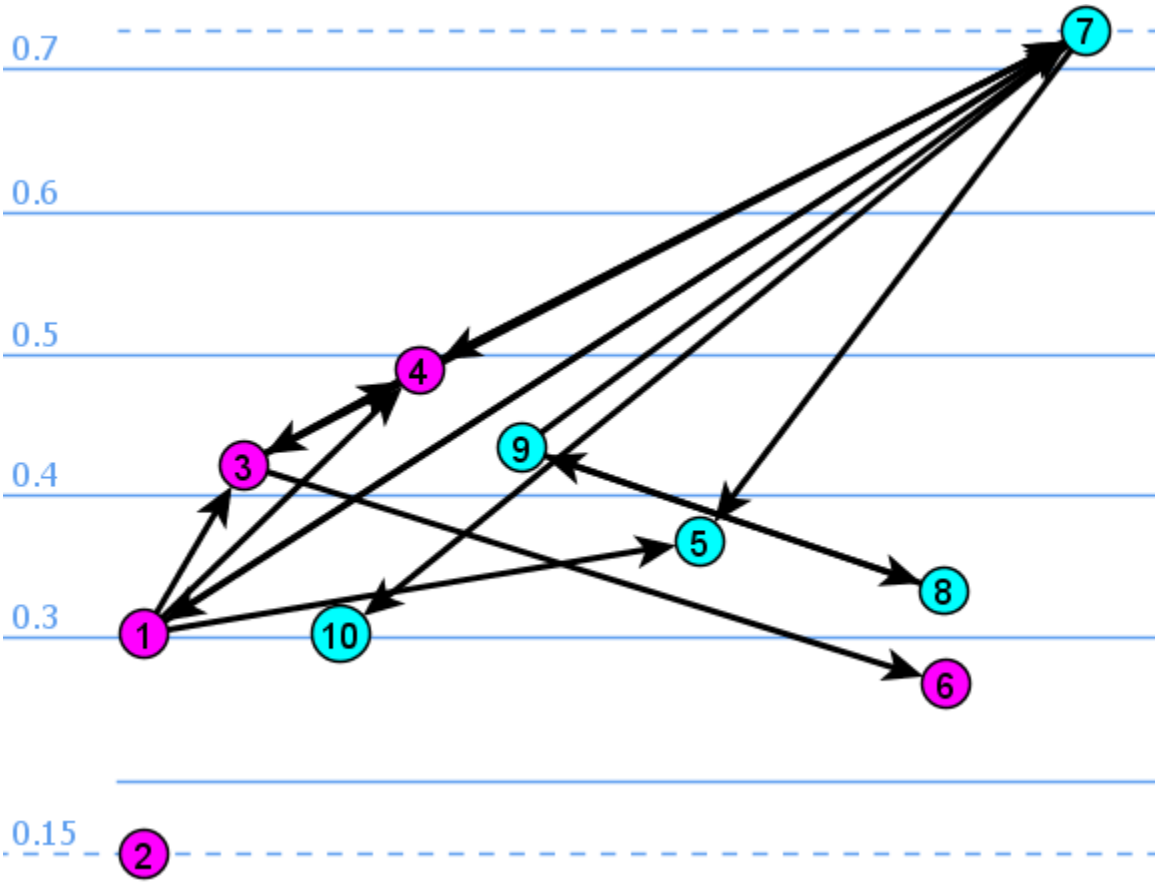
Table 4.31: Depth by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.7325	1.2851	1.6324
Authority		0.7762	1.2580
Pagerank			1.3654

Table 4.32: Broadness by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.9080	1.7390	1.8720
Authority		1.5130	1.7280
Pagerank			1.7710

Figure 4.7: Network of TH in Trikala, 2nd grade, layered by pagerank.



4.1.9 Trust for natural sciences (TN)

Table 4.33: Depth by mutual information

	Authority	Pagerank	Status
Indegree	4.7037	4.6948	4.9060
Authority		5.8699	6.2465
Pagerank			7.0790

Table 4.34: Broadness by mutual information

	Authority	Pagerank	Status
Indegree	2.7108	2.7088	2.7556
Authority		2.9579	3.0325
Pagerank			3.1902

The mutual information assumes great values for all pairs involved (tables 4.33 and 4.34) in depth, revealing a much complicated hierarchy, and in broadness, proving the hierarchy to be well-established. Both the depth and broadness are particularly high when the status is involved, due to the status being more capable of categorizing the students in a network having complex form. The complexity appears as each student poses a question to lots of his classmates prior to establishing an answer, an optimal strategy in exact sciences, where the correct is known to be specific.

Table 4.35: Depth by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.7085	1.4373	1.4164
Authority		0.9967	1.0665
Pagerank			1.1535

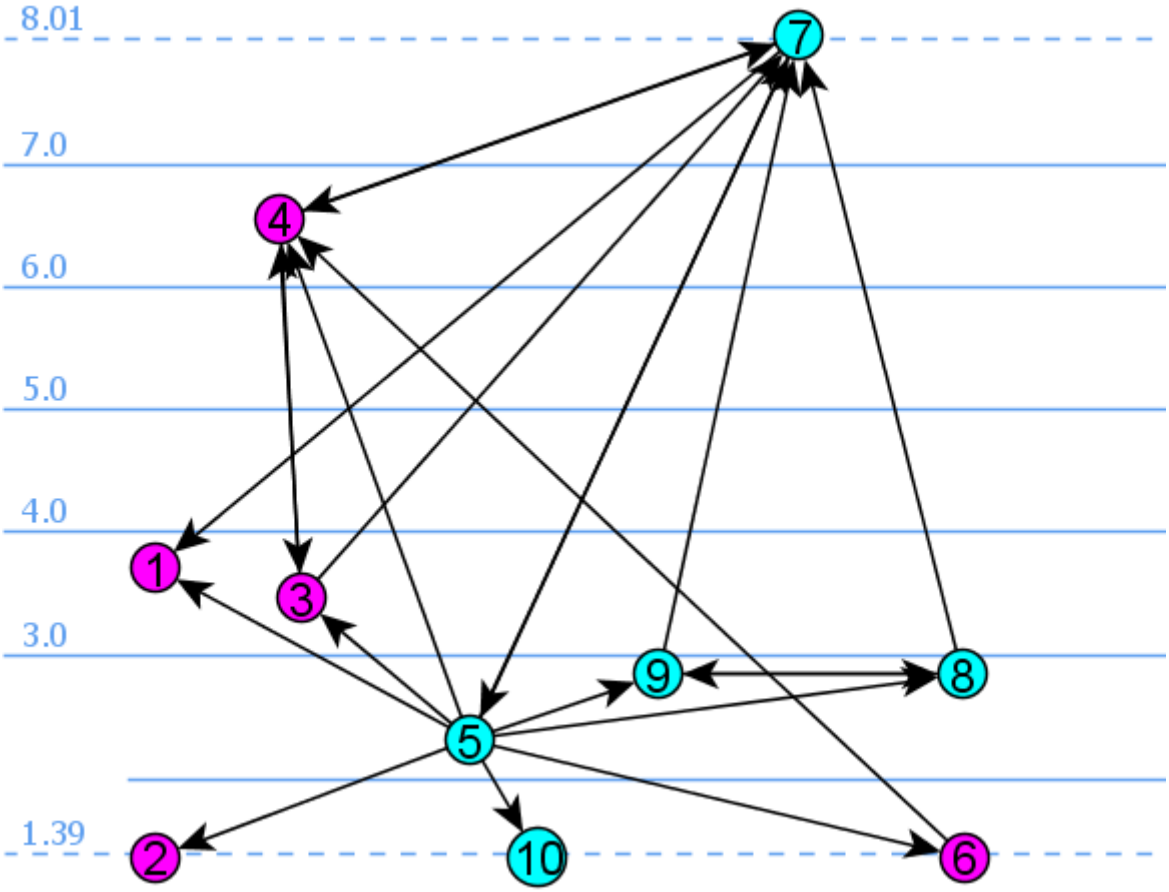
Table 4.36: Broadness by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.8995	1.7990	1.7909
Authority		1.6166	1.6474
Pagerank			1.6847

Spearman's coefficient also takes great values in both depth and broadness (tables 4.35 and 4.36). However, the values are high when the in-degree is involved while they are average-high when it is not. Apparently, the authority, pagerank and status, the more deep-rooted hierarchies, differ slightly from one another; meanwhile the in-degree suggests a more simplistic hierarchy that tries to compromise the other three. Namely, it turns out that if one is likely to be asked by those who seek for the best answers, he/she may not be as much likely to

have many questions ending up at them while someone who is much likely to be asked a question might neither be asked by the hubs nor have many questions ending up at them. Despite the exact nature of these courses, it turns out that some authorities elude the attention of the others while some end up being asked more frequently that they deserve.

Figure 4.8: Network of TN in Trikala, 2nd grade, layered by status.



4.1.10 Trust for personal advice (TA)

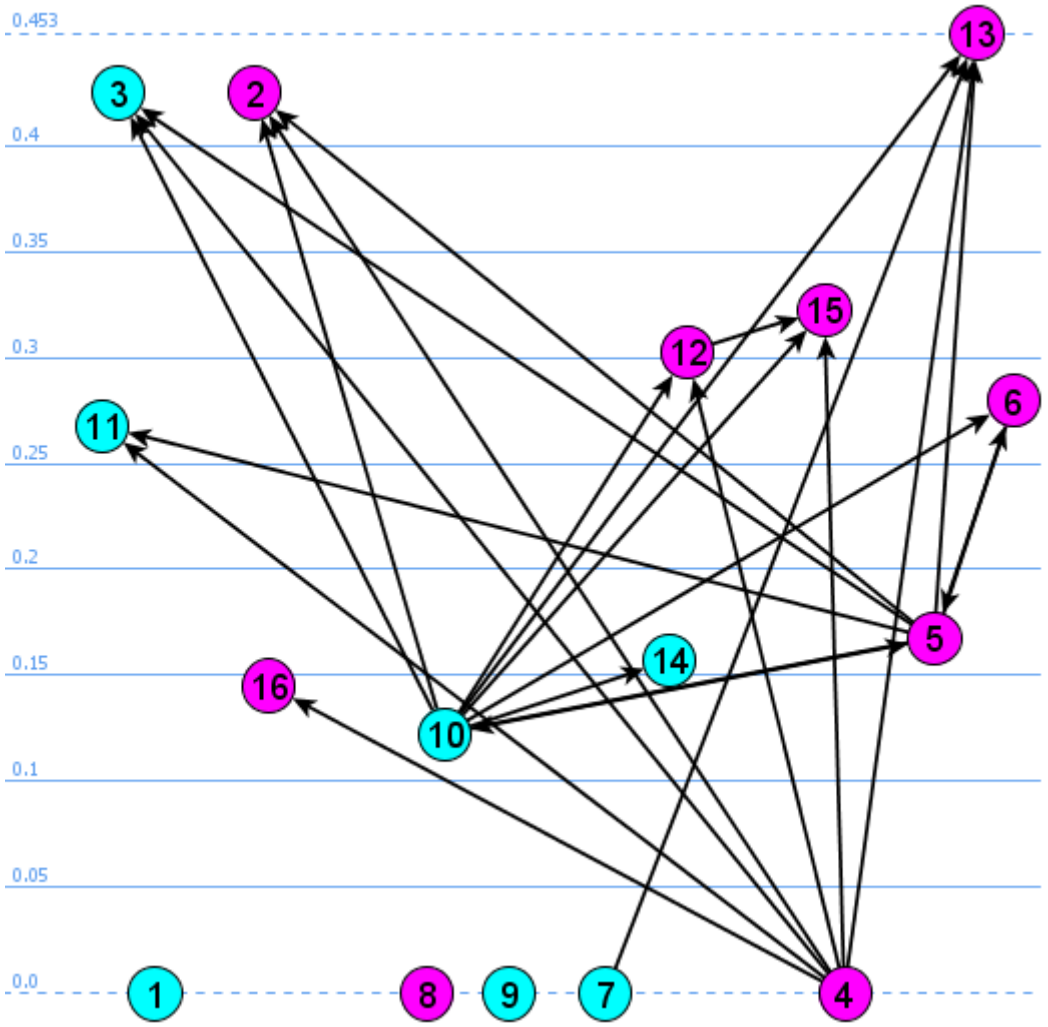
Table 4.37: Depth by mutual information

	Authority	Pagerank	Status
Indegree	3.6426	1.8978	3.6101
Authority		2.5552	4.2250
Pagerank			2.1523

Table 4.38: Broadness by mutual information

	Authority	Pagerank	Status
Indegree	2.4591	1.9538	2.4509
Authority		2.1623	2.6010
Pagerank			2.0378

Figure 4.9: Network of TA in Ampelokipoi, 2nd grade, layered by authority.



The depth and broadness by mutual information have low values (tables 4.37 and 4.38), revealing the simplicity of structure of the network of trust for personal advice (in most cases the graphs are very sparse). More precisely, the values of depth and broadness are lower when the pagerank algorithm is involved; that is, one's classification based on the in-degree, authority or status may –in some cases- not be indicative of the amount of personal discussions in which he/she actually participates.

Table 4.39: Depth by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.8092	1.8845	1.9404
Authority		1.6379	1.5806
Pagerank			1.8816

Table 4.40: Broadness by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.9350	1.9610	1.9800
Authority		1.8740	1.8530
Pagerank			1.9600

Spearman's coefficient has great scores in both depth and broadness, meaning that the algorithms demonstrate high coincidence in classifying the students as of their quality in giving personal advice. The coefficient is lower when the authority is involved, suggesting that some few authorities may escape the attention of the majority; these can give good advice yet only the hubs trust them. Whenever pagerank is involved, the coefficient is as great as when pagerank is not involved, unlike mutual information, showing that while algorithms agree on the order of the hierarchy, the pagerank may give the same score to students having different status or authority scores. This phenomenon -one being asked about personal issues slightly more or less than they deserve- unveils that the students decide solemnly on who they should trust based on who is good at advice, however there are some other criteria that affect at a low percentage their choice. Personal issues usually are very delicate, thus one tends to decide on who to trust based primarily on reason, yet he/she allows emotion to add something to the final decision.

4.1.11 Trust for professional advice (TP)

Table 4.41: Depth by mutual information

	Authority	Pagerank	Status
Indegree	4.3696	2.9676	4.6160
Authority		3.1226	4.5166
Pagerank			3.7822

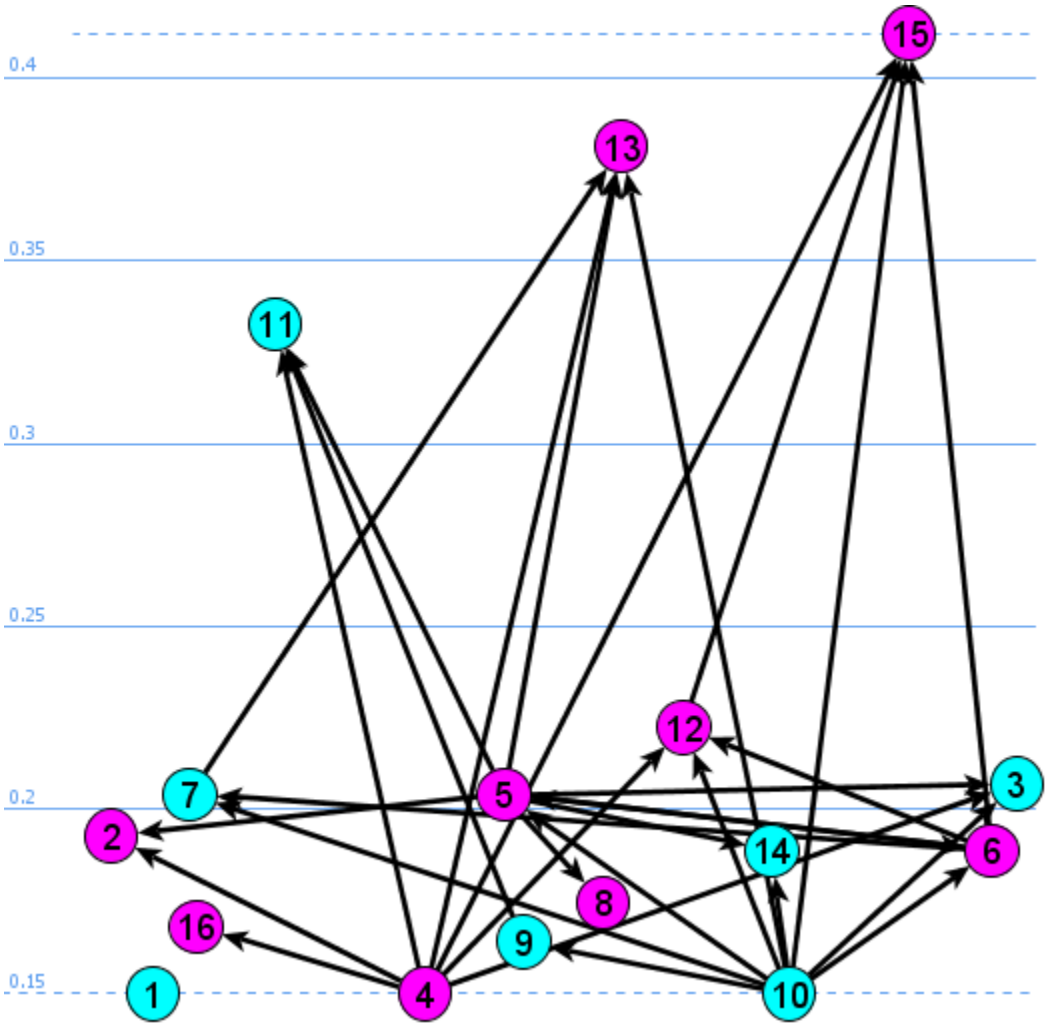
Regarding the trust for professional advice, the mutual information is of average depth (table 4.41) and of average-to-high broadness (table 4.42), all pairs having approximate values. The former fact reveals that the networks of the classes have relatively simple structure –in fact

this is true for the two networks corresponding to 1st grade. No algorithm is more important than any other, more precisely all four algorithms moderately agree with one another on how to distribute hierarchy in the network. This phenomenon –of agreement- is due not only to the simplicity of the networks but also to the fact that the students decide on who to consult on professional issues based on rational criteria.

Table 4.42: Broadness by mutual information

	Authority	Pagerank	Status
Indegree	2.6347	2.2807	2.6911
Authority		2.3232	2.6685
Pagerank			2.4940

Figure 4.10: Network of TP in Ampelokipoi, 2nd grade, layered by pagerank.



This rationality can be also observed in Spearman's coefficient (tables 4.43 and 4.44), as all values of both depth and broadness are great yet they do not reach the maximum value they

can obtain. The algorithms generally agree on the hierarchy of professional trust in the network, however there are some details that differentiate one from another. The lowest value is that between the authority and the pagerank. Considering that for someone to have a great authority score means that those who search for the best partners think highly of him/her, then this person may not have a great pagerank as well, meaning that he is not as much likely to find themselves cooperating with a classmate. This slight discrepancy is indicative of the harsh nature of the professional world, where one may attain more (or less) than they actually deserve.

Table 4.43: Depth by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.4296	1.4141	1.7128
Authority		1.0089	1.2580
Pagerank			1.7920

Table 4.44: Broadness by Spearman's coefficient

	Authority	Pagerank	Status
Indegree	1.7960	1.7900	1.9010
Authority		1.6220	1.7280
Pagerank			1.9290

4.2 Study of Intersectoral Pervasiveness of Structure

4.2.1 Authority

Table 4.45: Depth by mutual information

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	0.7193	0.3706	2.2777	5.5731	2.6895	2.2430	5.1728	4.2957	3.4632	4.3994
DI		0.3321	0	0.9400	0.9471	1.2471	1.2886	1.2817	1.2780	0.9903
DE			0	0.4279	0.2516	0.4033	0.3989	0.2817	0.2738	0.8429
GD				1.7052	1.9149	0.7554	1.5276	1.8680	0.7856	0.8456
IS					3.3565	1.8261	2.8861	3.4997	2.2450	3.6647
IO						2.3172	2.8637	2.7581	2.5873	3.3133
PD							2.4784	2.3820	2.2588	2.1937
TH								4.8656	3.5804	3.4022
TN									3.2773	3.3312
TA										3.0573

Table 4.46: Broadness by mutual information

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	0.3665	0.1053	1.8119	1.6600	1.5764	1.4549	2.8133	2.6175	2.1747	2.6416
DI		0.3888	0.9911	0.9978	1.3187	1.7127	1.6973	1.7106	1.7252	1.1842
DE			0.9911	0.2188	0.0356	0.4467	0.1882	0.0580	0.0292	1.3733
GD				1.1423	1.9596	1.4927	0.8743	1.6411	1.5076	1.7159
IS					2.3256	0.6834	0.0567	1.4812	0.3302	1.8556
IO						2.0899	2.2516	2.2215	2.1718	2.3742
PD							2.1392	2.1099	2.0717	2.0511
TH								2.7467	2.4434	2.3975
TN									2.3646	2.3789
TA										2.3054

The mutual information (tables 4.45 and 4.46) between the distributions of authority on the different relation types assumes low values both in depth (mean: 1.7539) and in broadness (mean: 1.3972), in spite of the fact that there are some great values. It appears that the apportionment of authority follows different patterns in different types of relation: in general persons holding the same amount of authority in one relation type are much unlikely to have equal authority in another relation type. Spearman's coefficient (tables 4.47 and 4.48) has low depth (mean: 0.4247) but average-to-great broadness (mean: 0.9470). There is a weak but well-established relationship between the authorities in different relation types, that is, someone that is sought after by those who look for the best in one sort of quality is, more or less, likely to be considered an authority in another form of relation.

The appreciation (AP) is primarily connected to the trust in issues related to humanitarian courses (TH), showing that one will be considered an authority in humanitarian courses if he/she has gained the appreciation of those who look for the most qualitative persons in the class and, conversely, if one is asked by those who look to find the best answers in questions related to humanitarian courses then he/she will also be deeply appreciated. Apparently, the humanitarian courses, touching various aspects of social life, are quite indicative of one's overall quality as a person.

Table 4.47: Depth by Spearman's coefficient

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	0.0846	0.1953	0.6341	0.4812	0.4251	0.0024	1.1900	0.7560	0.4173	0.7741
DI		0.6760	-	0.0934	0.8037	1.1780	0.4454	0.5277	0.2687	0.4799
DE			-	0.1419	0.6837	0.4494	0.5199	0.7178	0.1318	0.3174
GD				0.2088	0.4286	0.0468	0.3584	0.2936	0.0934	0.2917
IS					0.6774	0.4134	0.3362	0.4653	0.5360	0.7843
IO						1.0817	0.1122	0.3744	0.1794	1.4582
PD							0.1494	0.0138	0.0355	1.0066
TH								1.3452	0.7138	0.6108
TN									0.4461	0.7026
TA										0.7575

Table 4.48: Broadness by Spearman's coefficient

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	0.3392	0.7034	1.3512	1.3359	1.2858	0.0118	1.7000	1.5030	1.2416	1.5120
DI		1.4468	-	0.3717	1.5220	1.6950	1.2213	1.3219	0.8956	1.2678
DE			-	0.5387	1.4508	1.2277	1.3134	1.4726	0.5066	1.0051
GD				0.6229	1.0794	0.1006	0.9526	0.8216	0.1982	0.8171
IS					1.4630	1.2352	1.1693	1.3223	1.3628	1.5170
IO						1.6540	0.5488	1.2258	0.7308	1.8070
PD							0.6321	0.0686	0.1715	1.6210
TH								1.7630	1.4795	1.4256
TN									1.2753	1.4760
TA										1.5015

In addition, there is great depth by mutual information between the appreciation and the invested company time in school (IS) and the trust for natural courses (TN) and professional advice (TP), meaning that the apportionment of authority between general appreciation and these relation types follows the same patterns. They also partially agree on the order of the hierarchy, as indicated by the great broadness by Spearman's coefficient. Appreciation in the microcosm of the classroom is not detached from the forms of interaction one encounters in

there, namely courses, the professional trust and the time spent together in school, on the contrary it is through these forms that one establishes himself as an authority much appreciated.

The depth and broadness by Spearman's coefficient between the derogatory characterization of public image and the professional cooperation desire is particularly great while the mutual information between the two is low, showing that the two relation types agree on the order of authority in the classes, despite the fact that the hierarchies are simple with many ties. The quality of being a good professional partner –and acknowledged as such by those looking for the best partners- coincides with creating and maintaining a public image that does not reflect reality –instead it gives the impression of a better person than one truly is. That coincidence is present in the classes, maybe subconsciously.

There is also great depth and broadness by Spearman's coefficient between the professional cooperation desire, the invested company time outside of school and the trust for professional advice pairwise, indicating a strong relation between the authorities of these three relations. The corresponding depth and broadness by mutual information is average, revealing the simplicity of the hierarchies involved. It appears that the ones who have established themselves as notable (potential) professional partners or advisers are also sought after for company in extracurricular hours. The others try to keep the authorities close and develop a relationship beyond the walls of the classroom with them; this kind of relationship can prove to be particularly helpful in the future.

In addition, the depth by Spearman's coefficient between the trust for natural sciences and the trust for humanitarian sciences assumes great values. The best in natural sciences who are also recognized as such by the experts are recognized as experts in humanitarian sciences as well. The mutual information, having great depth, verifies that the two hierarchies are very elaborate in the apportionment of power in the networks and that they are accurate with respect to one another. Apparently, students are really good (or bad) to the point that those who seek for the best advice will always come to them at both natural and humanitarian courses simultaneously.

The mutual information between the four forms of trust (TH, TN, TA and TP) pairwise is particularly great, indicating that there are deep relations between these types of relation. Developing trust for somebody else is a demanding task within the classroom; a particular form of trust may influence another form, even at a lower level, especially since the authority is measured here, meaning that trust is developed after careful consideration. The hierarchies involved are very elaborate, showing that students are very exhaustive when attributing authoritative trust.

4.2.2 In-degree

Like in the case of authority, the mutual information between the distributions of in-degree on the different relation types assumes low values in depth (mean: 1.296) and average-to-low values in broadness (mean: 1.613). This indicates that people sharing the same amount of power in one type of relation are not likely to have equal power in another type, *i.e.*, their immediate influence is probably different from relation to relation. As for Spearman's coefficient, the in-degree scores a low depth (mean: 0.404) but an average-to-great broadness (mean: 0.946), showing that there is a weak relationship among hierarchies in different relations: someone's immediate influence in a particular context little does it affect their influence in another context.

Table 4.49: Depth by mutual information

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	0.5063	1.3241	1.0435	2.4259	1.5497	1.0122	2.3367	2.1104	1.4846	2.0738
DI		0.7625	0.5605	0.3945	0.6576	0.9141	0.5276	0.6300	0.1086	0.4115
DE			0.5702	1.4003	1.7068	0.9119	1.3807	1.3958	0.9423	1.1608
GD				1.5608	1.4974	1.0311	1.4589	1.5852	0.4796	1.6264
IS					2.0107	1.4031	2.7046	2.2831	1.4309	2.3746
IO						1.5426	1.9157	1.9895	1.3197	2.2836
PD							1.5291	1.4847	0.6498	1.2392
TH								3.2244	1.6235	1.9866
TN									1.4449	1.9829
TA										1.8592

Table 4.50: Broadness by mutual information

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	1.2212	1.7437	0.5334	2.1049	1.8135	1.4027	2.0960	2.0243	1.8058	2.0124
DI		1.4962	1.2503	0.8863	1.4428	1.5686	1.2601	1.4283	0.2097	1.1676
DE			1.3959	1.7736	1.8874	1.5676	1.7659	1.7718	1.5816	1.6770
GD				1.4818	1.7776	1.1590	1.3888	1.7110	0.0432	1.7914
IS					1.9916	1.7583	2.2061	2.0793	1.7854	2.1077
IO						1.8276	1.9598	1.9846	1.7419	2.0795
PD							1.8226	1.8059	1.3101	1.7095
TH								2.3506	1.8574	1.9836
TN									1.7907	1.9824
TA										1.9406

The appreciation has strong depth and broadness of mutual information with three forms of trust: trust for natural sciences, trust for humanitarian sciences and trust for

professional advice as well as with the will to spend time together in school. The corresponding broadness by Spearman's coefficient is also high for these pairs. Apparently, the immediate appreciation one receives is a product of the trust they enjoy in terms of courses and professional activities. This forms of dependence is carried over in more complex forms of power (authority, status, pagerank), showing that the trust in courses and professional matters is a primordial factor in shaping appreciation towards a classmate. The aforementioned forms of trust and the desire to invest company time in school also have great depth and broadness of mutual information between one another. This shows that people trusted the same in courses are much likely to be targeted as school time companions at the same level as well. However, the desire to spend time in school has a low depth and a great broadness of Spearman's coefficient, that is, although the apportionment of hierarchy is the same, the order is different, showing that trust and company time in school intertwine in a complicated fashion.

Table 4.51: Depth by Spearman's coefficient

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	0.2726	0.0317	0.9488	0.1916	0.0170	0.0535	1.0610	0.4582	0.6238	0.3587
DI		1.2408	0.4441	0.6521	0.0228	0.5791	0.6108	0.7924	0.0390	0.0492
DE			0.3342	1.0702	0.6445	0.1819	0.5099	1.0840	0.3466	0.1576
GD				0.0902	0.1112	0.0000	0.4148	0.0711	0.5782	0.4824
IS					0.6597	0.0272	0.4057	0.5388	0.0032	0.3482
IO						0.7863	0.0002	0.2469	0.0081	1.1756
PD							0.2606	0.0721	0.2928	0.8483
TH								1.3528	0.1563	0.2502
TN									0.0502	0.2622
TA										0.6967

Table 4.52: Broadness by Spearman's coefficient

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	1.0500	0.1735	0.5856	0.8366	0.0945	0.2852	1.6450	1.3160	1.4326	1.2037
DI		1.7210	1.1034	1.4476	0.1262	1.4062	1.4256	1.5210	0.2110	0.2633
DE			0.9068	1.6490	1.4436	0.8061	1.3588	1.6550	1.1860	0.7223
GD				0.2917	0.3540	0.0000	1.0565	0.2325	1.2904	1.1637
IS					1.4515	0.1483	1.2636	1.3795	0.0176	1.1870
IO						1.5180	0.0015	0.9897	0.0453	1.6940
PD							1.0231	0.3741	1.0902	1.5480
TH								1.7660	0.7198	0.9975
TN									0.2679	1.0260
TA										1.4730

The derogatory characterization of educational attainment has great depth and broadness of Spearman's coefficient with the derogatory characterization of public image, the will to invest company time in school and the trust in natural sciences. Although one may be trusted in natural science courses and sought after as company in school, they are believed to receive better grade than they actually deserve. It appears that the immediate influence presents a weird behavior: the fact that one receives better grade than they deserve serves as a motive to befriend them and trust their opinion in natural sciences.

There is also great depth and broadness between the trust in professional advice and the time invested outside of school. Evaluating someone as a good professional partner leads naturally to the necessity of maintaining a strong relation with them that surpasses the walls of the classroom.

Also, the group work desire has a great broadness of Spearman's coefficient with the trust in personal advice while the corresponding depth is low. This indicates a weak relation between the two relation types: one's confidence in personal matters is a criterion when choosing someone to work together. It is natural that the immediate influence in group work desire is based on personal attributes and the personal connection with one another.

4.2.3 Pagerank

Table 4.53: Depth by mutual information

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	0.9229	0.4037	1.8499	4.0198	2.3736	1.2561	4.1473	3.5794	1.3997	2.1369
DI		1.1583	0.5501	0.5292	0.4829	0.4726	0.9190	1.2290	0.1183	0.2344
DE			0.5586	0.2716	0.7457	0.5576	1.3609	2.1082	0.3684	0.8688
GD				0.8358	0.3963	0.5610	1.7264	1.3119	0.9645	0.7744
IS					2.0830	0.9496	3.6120	3.4242	0.7714	2.3019
IO						1.2170	1.8416	2.4599	1.1796	1.8140
PD							0.8104	1.3354	0.5330	1.2122
TH								4.2336	1.3875	2.0696
TN									1.7287	2.6022
TA										1.7459

Moving towards more elaborate forms of hierarchy, the pagerank scores low in both depth (mean: 1.339) and broadness (mean: 1.291) of mutual information, showing the large discrepancy among the apportionment of power in different relation types. Spearman's coefficient has low depth (mean: 0.349) but average-to-great broadness (mean: 0.910) showing –like in the cases of authority and in-degree- a weak but well-established relation among the various forms of hierarchy.

Table 4.54: Broadness by mutual information

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	1.4086	0.0042	1.9374	0.8978	2.0525	1.4678	2.5826	1.9320	1.5813	1.7039
DI		1.6760	1.3848	0.2047	0.9431	1.2161	1.4655	1.7053	0.2435	0.1959
DE			1.3895	1.0917	1.3287	1.3320	1.7582	2.0236	0.7098	1.5023
GD				0.0827	1.7394	0.6905	1.8772	1.1037	1.2423	0.5025
IS					1.1014	0.8445	1.9200	0.7009	0.1325	1.2358
IO						1.7004	1.5848	2.1337	1.6849	1.9250
PD							0.7019	1.6045	1.0784	1.6984
TH								2.6030	1.6875	1.8192
TN									1.8951	2.1762
TA										1.9012

Table 4.55: Depth by Spearman's coefficient

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	0.1625	0.3518	0.8277	0.4320	0.2418	0.0258	1.0933	0.7341	0.1702	0.4850
DI		1.0246	0.4422	0.1704	0.0030	0.8462	0.2672	0.3052	0.0082	0.0342
DE			0.1181	0.3536	0.6108	0.3744	0.2742	0.4389	0.2536	0.0196
GD				0.0671	0.8930	0.0006	0.5351	0.3608	0.5312	0.8470
IS					0.0684	0.0871	0.3655	0.0289	0.1754	0.1866
IO						0.4707	0.0163	0.2451	0.0968	0.6928
PD							0.0362	0.0024	0.2047	0.5134
TH								0.9799	0.2688	0.3344
TN									0.1595	0.3001
TA										1.0473

Table 4.56: Broadness by Spearman's coefficient

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	0.7401	1.1933	1.5132	1.2920	0.9770	0.1423	1.6590	1.4920	0.7649	1.3402
DI		1.6290	1.1004	0.7667	0.0169	1.5470	1.0369	1.1157	0.0461	0.1857
DE			0.3761	1.1964	1.4256	1.2258	1.0529	1.2982	1.0053	0.1078
GD				0.2212	1.5531	0.0024	1.2365	0.9594	1.2324	1.5246
IS					0.3554	0.4400	1.2120	0.1569	0.7850	0.8195
IO						1.3276	0.0900	0.9848	0.4839	1.4710
PD							0.1971	0.0132	0.8748	1.3608
TH								1.6090	1.0399	1.1662
TN									0.7316	1.1065
TA										1.6390

Some patterns already observed in authority and/or in-degree appear here as well: the appreciation is closely related to the various forms of trust, indicating that this form of hierarchy has deeper roots. The broadness of Spearman's coefficient between the appreciation and the group work desire is particularly great, as is that among the appreciation, the trust in humanitarian courses and the trust in natural sciences. These forms of power share a deep connection: if one is deeply appreciated, this appreciation should be credited to their ability to work efficiently in the environment of a group.

There is also great broadness of Spearman's coefficient between the derogatory characterization of educational attainment, the desire to invest company time outside of school and the professional cooperation desire. The connection between the latter two may be obvious: one wants to keep close to those whom they acknowledge as possible professional partners. However, the connection between the two and the derogatory characterization of educational attainment may seem odd. It is revealing, however, to the fact that being an achiever is evaluated as an advantage in a professional/general setting, leading to the positive acknowledgement as a companion beyond the walls of the classroom and as a potential professional partner.

4.2.4 Status

Table 4.57: Depth by mutual information

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	0.8807	2.2536	1.6150	4.8889	4.0056	1.9468	6.0091	6.0476	3.6640	3.6870
DI		0.6214	0.5826	0.7254	0.5992	0.5457	0.4501	0.7079	0.2850	0.4145
DE			0.5881	1.6789	0.8018	0.7357	1.8748	1.7771	1.2047	1.3001
GD				1.1644	1.8993	0.8805	1.1501	1.8065	0.5701	0.7975
IS					3.3871	1.8060	4.0351	4.2498	1.5420	2.5193
IO						1.5790	3.6555	3.4298	2.3535	2.6576
PD							1.6559	1.5585	0.9933	1.1523
TH								5.0771	2.6778	2.6069
TN									2.9085	2.9510
TA										2.5480

The status scores low in depth (mean: 1.798) and broadness (mean: 1.660) of mutual information. It also scores low in depth (mean: 0.378) but average-to-great in broadness (mean: 0.918) of Spearman's coefficient. The pervasiveness of the status algorithm is low, showing that it is highly improbable that someone play the same leading role in different contexts. Both the apportionment and the order of power differ considerably from one relation type to another.

Table 4.58: Broadness by mutual information

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	1.5379	2.0700	1.5301	1.6069	2.5487	1.9703	2.9857	2.9934	2.4645	2.4703
DI		1.4236	1.3739	1.4335	1.4117	1.3823	0.9503	1.4687	0.8031	1.1694
DE			1.4057	1.8774	1.9208	1.4828	1.9459	1.9122	1.6953	1.7341
GD				0.3793	1.9543	0.8115	0.4637	1.5742	0.0309	0.2438
IS					2.3935	1.9222	2.2050	2.3122	0.4286	1.6204
IO						1.8411	2.4624	2.4047	2.1012	2.1924
PD							1.8691	1.8168	1.6046	1.6577
TH								2.7928	2.1983	2.1588
TN									2.2642	2.2760
TA										2.1601

Table 4.59: Depth by Spearman's coefficient

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	0.0072	0.3536	0.9488	0.4479	0.4179	0.1283	1.1166	0.9316	0.6832	0.2049
DI		1.3150	0.4406	0.3049	0.0146	0.7782	0.5714	0.6616	0.1436	0.0333
DE			0.1720	0.4196	0.5882	0.4582	0.2812	0.4459	0.4512	0.0144
GD				0.1766	0.1272	0.2682	0.6697	0.3605	0.2847	0.2863
IS					0.0315	0.5370	0.5423	0.4425	0.0191	0.0004
IO						0.5696	0.1674	0.0784	0.0243	0.8006
PD							0.3692	0.4372	0.3952	0.7924
TH								1.6627	0.1136	0.0011
TN									0.1286	0.0004
TA										1.0179

Table 4.60: Broadness by Spearman's coefficient

	DI	DE	GD	IS	IO	PD	TH	TN	TA	TP
AP	0.0407	1.1964	1.5856	1.3078	1.2773	0.6130	1.6690	1.5870	1.4660	0.8767
DI		1.7510	1.0995	1.1136	0.0821	1.5140	1.3998	1.4525	0.6718	0.1802
DE			0.5252	1.2782	1.4112	1.3160	1.0690	1.3045	1.3097	0.0798
GD				0.5374	0.4013	0.7640	1.3867	0.9574	0.8031	0.8057
IS					0.1710	1.3785	1.3815	1.3025	0.1055	0.0025
IO						1.3988	0.7581	0.4032	0.1341	1.5250
PD							1.2184	1.2972	1.2509	1.5210
TH								1.8830	0.5539	0.0063
TN									0.6162	0.0026
TA										1.6260

Most of the forms of dependence that were observed in pagerank appear also in status. Both the depth and the broadness of mutual information between the appreciation and the trust in both natural and humanitarian courses is great. These two algorithms depict deeper forms of hierarchy –in fact the status takes into account all possible paths of power. Therefore, it is the trust one enjoys in courses that defines the appreciation they receive from the class. The corresponding depth and broadness by Spearman's coefficient are average, meaning that the relation between appreciation and trust in courses is not monotonous.

A similar phenomenon happens between the appreciation, the will to invest company time in school and the will to invest company time outside of school. The three relation types have pairwise great mutual information but low Spearman's coefficient. This means that one's cumulative appreciation plays a crucial role in the time others spend with them, although the relation is, again, not monotonous.

Finally, the two derogatory forms of power have a great Spearman's coefficient yet a low mutual information. That is, those who strive for a better public image are also believed to receive better grades than they deserve. The hierarchies involved are rather simple, as suggested by the low mutual information. At the end of the day, the ones who achieve better than they deserve are recognized and categorized together, regardless of the context that that characteristic is first spotted.

4.3 Study of Diffused Influence

For every measure mentioned the 10 pairs with the greatest score are reported. The number was chosen arbitrarily. For simplicity, $pure^{4}_{aibj}$ is reported in the following without harm.

Table 4.61: Greatest values of $pureCD_{MI}$

Variable1	Variable2	$pureCD^{4}_{MI}$
TA authority	DE pagegank	7.97954224071182
IO authority	DE pagegank	5.20717534647513
AP authority	DE pagegank	4.87343672433552
TH authority	DE pagegank	4.57649541876712
TN authority	DE in-degree	4.35398417331188
PD authority	DE pagegank	4.27977407719711
TA authority	DE in-degree	3.70412612622999
TN authority	DE pagegank	3.66086350379653
TH authority	DE in-degree	3.64587954565231
PD authority	DE status	3.40829898103572

Table 4.62: Greatest values of $pureCB_{MI}$

Variable1	Variable2	$pureCB^{4}_{MI}$
TH authority	IS in-degree	23.5705918489245
TH authority	IS status	19.152500977442
TA authority	DE pagerank	13.6779715340826
AP authority	DE pagerank	13.6196658657673
TA authority	DE in-degree	11.6162359785348
TA authority	DE status	11.5322289919312
AP status	DE pagerank	11.1177812243194
IO authority	DE pagerank	10.3711156513395
IO authority	DE status	9.11179208053838
TN authority	DE pagerank	8.73564975181517

The greatest values of $pureCD_{MI}$ (Tables 4.61 and 4.62) occur when the pagerank (primarily) or the in-degree and the status (secondarily) of the derogatory characterization of educational attainment are present. A similar phenomenon occurs for $pureCD_{MI}$. This relation type appears to influence authority in a series of other relation types, such as the trust for personal advice, the desire to spend time outside of school, the appreciation, the trust for both natural and humanitarian courses and the professional cooperation desire. This reveals an underlying connection: whether one is sought after as an authority in various relational sectors depends on whether they prove themselves capable of gaining more than they deserve in

terms of school grades. However, the relation between the two is not monotonous, as the corresponding values of $pureCD_{SP}$ and $pureCB_{SP}$ is not high. In addition, the pagerank, status and in-degree of the other relation types (appreciation, trust for natural sciences, etc.) is not affected by the great $pureCD_{MI}$ and $pureCB_{MI}$. The experts in each domain are -possibly subconsciously- recognizing the ones who gain better grades than they deserve and attribute to them the same amount of desirability.

Table 4.63 : Greatest values of $pureCD_{SP}$

Variable1	Variable2	$pureCD_{SP}^{4}$
DE authority	TN status	2.27533624031856
DE authority	TH status	1.59712148871203
DI authority	IO pagerank	1.41048106912477
IS authority	IO status	1.3439014703037
DI authority	IO status	1.31891977437627
TN authority	DE pagerank	1.29578779305345
IS authority	DE pagerank	1.23397637472695
DI authority	IO in-degree	1.2312821802161
IS authority	DE status	1.23085926578619
IS authority	IO pagerank	1.22774975205785

Table 4.64: Greatest values of $pureCB_{SP}$

Variable1	Variable2	$pureCB_{SP}^{4}$
DI authority	IO pagerank	5.818338486505
DI authority	IO status	4.21660580058641
AP status	IO in-degree	3.83377381725137
IS authority	IO status	3.60257801092182
DI authority	IO in-degree	3.46550531662054
IS authority	TP status	3.34915128156891
AP authority	DI status	3.27185292213126
IO status	IS in-degree	3.09444865747146
AP authority	DE in-degree	3.02706410581323
TP authority	DE status	3.00051371034158

The greatest values of $pureCD_{SP}$ occur between the authority of the derogatory characterization of educational attainment and the status of the trust in natural and humanitarian sciences. Here, a slightly different phenomenon than the ones discussed above emerges. The ones who actually establish themselves at the top of the hierarchy of the class in terms of trustfulness in matters related with courses are also recognized as experts in getting

better grades than they deserve (by those who look deeper in order to recognize those who attain better grades than they deserve).

Let us assume that the algorithm of authority reflects better the truth, *i.e.*, the hubs (the experts) can truly discover the ones who are dominant in each relation type. Then, the ones who have gained the trust of the class in matters related with courses (TN, TH) are actually receiving better grades than they deserve but only the most experienced eyes can identify that. Meanwhile, the ones who are truly authorities in a series of relational sectors are believed, cumulatively and occasionally (status, pagerank, in-degree), by the class to receive better grades than they deserve. This discrepancy shows that the opinion of the experts (the hubs) is significantly different than that of the total of the class. The latter is rather incapable of recognizing the ones who truly are good in a discipline or, more generally, in a relational sector.

The $pureCD_{SP}$ and $pureCB_{SP}$ assume great value between the authority of the derogatory characterization of the public image and the pagerank, status and in-degree of the desire to invest company time outside of school. The majority of the class prefers to spend time outside of school with those who are considered authorities in striving for a better image. It seems that the class appreciates this characteristic as a social skill. However, this can be seen from a different point of view: the ones considered authorities in achieving a good public image also try to spend a lot of time with others (outside of school). These others, *i.e.*, the majority of the class apparently enjoy the company of the former.

CHAPTER 5

Conclusions

In this study we introduce the use of mutual information as a measure of the correlation between network variables. The mutual information reveals an aspect of the distribution of power in social networks, namely the apportionment that has not been taken into account in past studies. The latter, using only Spearman's coefficient ρ as a measure of correlation, capture the order of power but tend to neglect the apportionment of power. When studied parallel, as in the present study, the Spearman's ρ and the mutual information provide a deeper understanding of the patterns of power in the network.

We also define and introduce two new notions in network analysis, which can be used in any statistical analysis: the notion of certainty depth and the notion of certainty broadness combining the value of a correlation measure with the p-value of the test of significance on that correlation measure. These two notions prove to be particularly helpful in the case of mutual information, where no standard method of testing the significance of mutual information exists. The certainty depth and the certainty broadness are also used on Spearman's ρ providing a helpful insight, while they can be applied even if the significance is measured with a parametric test. The certainty depth can tell whether a connection between two variables has deeper roots or is superficial. The certainty broadness can tell us whether this connection is well-established and can be generalized.

In the present study we follow a systematic method of isolating each relation type and studying its intrinsic structural concentrationism. We then isolate each algorithm of network

analysis and study the intersectoral pervasiveness of the particular structure proposed by the algorithm. Most existing studies of such extent reach their conclusions by isolating pairs of variables of specific interest, hence possibly missing the general image. We examine every relation type and every algorithm separately, thus finding the attributes of each one of them.

We introduce also the notion of pure certainty depth and pure certainty broadness in the study of the extrinsic general influence in order to counterbalance the effects of the intrinsic structural concentrationism and of the intersectoral pervasiveness of structure. The pure certainty depth and the pure certainty broadness prove to be particularly helpful in discovering deeper relations between network variables examining different forms of power in different relational sectors. In particular, in the microcosm of the classes, the majority tends to trust -in many aspects, personal, professional or related to courses- students whom the experienced eyes consider as attaining greater grades than they deserve. Meanwhile, the experienced eye trusts –personally, professionally, or in terms of school courses- students for whom the majority of the class believes they receive better grades than they deserve.

The present study does not take into account non-network variables such as age, gender, revenue, etc. These variables and their influence is the subject of other studies. The study is limited by the small size of the sample (the classes) and the fact that some students may be reluctant to answer some questions. In addition, the questionnaires given to the school of Ampelokipoi and the school of Trikala differ by one question. Moreover, the samples of all schools were treated as one, in order to find connections that can transcend through different classes; however, some characteristics of each individual class might have escaped our attention after the union of the sample.

Last but not least, we assume that, in the context of a network, actors can increase their role (measured by a network algorithm) if they wish to. However, there is no guarantee that the network will respond to such a desire by an actor. This indicates that an actor may have ended up in a position of power despite their will. The study of the actors' motives in conjunction with what they have attained should be studied further in the future.

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