RANDOM WALK MODELS IN EARTH SCIENCES

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ΠΕΡΙΛΗΨΗ

Τα Μοντέλα τυχαίας διαδρομής είναι απλές τυχαίες ανελίξεις. Θεωρούμε κάποιον που στέκεται σε ένα σταθερό σημείο ενός δρόμου και αρχίζει και βηματίζει. Κάθε βήμα του είναι του ίδιου μήκους και ανεξάρτητο από το προηγούμενο. Κινείται προς τα δεξιά με πιθανότητα p και προς τα αριστερά με πιθανότητα q. Μοντέλα σαν το παραπάνω μπορούν να χρησιμοποιηθούν στις Γεωεπιστήμες με κατάλληλη τροποποίηση. Σε αυτή την εργασία αναφέρονται μερικές εφαρμογές στη Γεωμορφολογία και στη Γεωλογία. Ο σκοπός αυτού του άρθρου είναι να δείξει πως εφαρμόζονται στις γεωεπιστήμες μερικά πορίσματα της θεωρίας των πιθανοτήτων.

ABSTRACT

One of the simple stochastic processes is the random walk. We imagin a man starting at a fixed point on a road and in each time unit making a step to the left or right. Each step of him has equal length and is independent of the preceeding one. The man moves to the right or to the left with probability p and q respectively. Such models can be used to represent different processes in earth sciences especialy if one extends these ideas. In this paper we investigate some uses of this model in geomorphology and in geology. The concern of this paper is to illustrate the application of some advanced results of the probability theory in geosciences.

INTRODUCTION

The nature of some phenomena has not a unique but a probability distribution of solutions. This kind of problems are known as stochastic ones. It is clear that to solve these problems probability theory must be used. One of the most illustrative example of the above theory is the so-called «random-walk problem» [1].

We suppose a man (or a particle) standing initially at a given point (x=0) and starts making a step to the left or to the right on a road (x-axis). Each step on him has a unique distance and is independent of the preceeding one. The probability to move to the right or to the left is p and q respectively. The question of interesting is: after n

steps, what is the probability of being at a certain position. The set of all steps, is considered to be a collection of random variables. Using the binomial distribution, we can easily yield that at time.

$$t=n*T$$

our man will be at the position $t(x)=R!-(n-R)!=2R!-n!$, where n the number of trials, R the right steps and $n-R=l$ the left steps. The probability of this position is:

For n very large the binomial distribution is approaching the Gaussian one, by which we can easily solve the above problems. The above simple concepts can be extended and generalized to more dimensions.

$$P\{t(x) = 2R! - n!\} = \frac{n!}{R!(n-l)!}\, p^R\, q^{n-R}$$

APPLICATION OF RANDOM WALK MODELS TO EARTH SCIENCES

The alluvial fan formation

We concider a gravel in a plane at the position zero (Fig. 1) and moving to the left or to the right with equal probabilities (each step we suppose having a unit lenght) while at the same time is suffers a forward steady displacement. The resulting triangle (Fig. 1) shows the most probable position to arrive the particle. It is clear that the formation

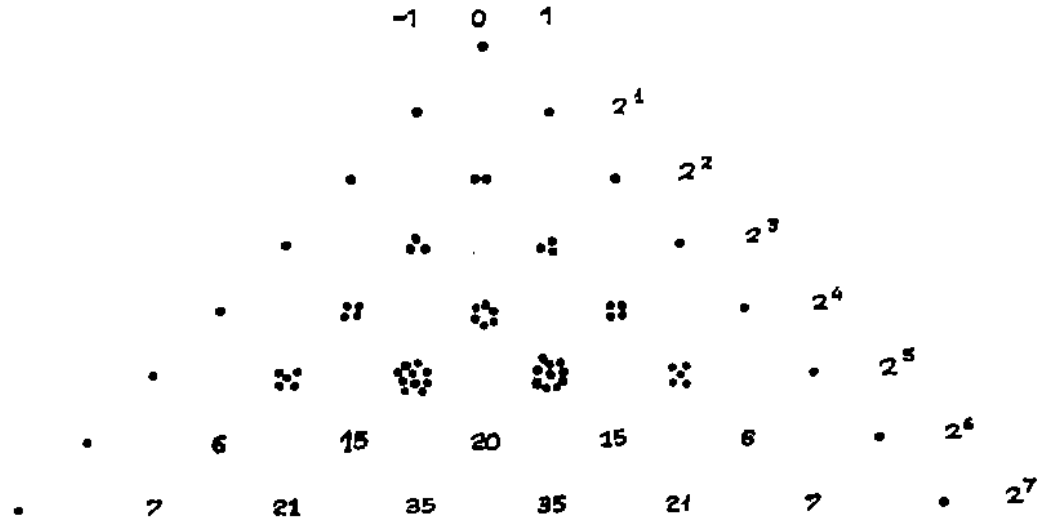

Fig. 1

of alluvial fans can be described by a random walk mechanism [3]. The mouth of the stream in the mountain front corresponds to zero point of the Figure 1.

Some volcanic forms

If we concider a particle starting at the point (0,0) of xy place, and moving to the

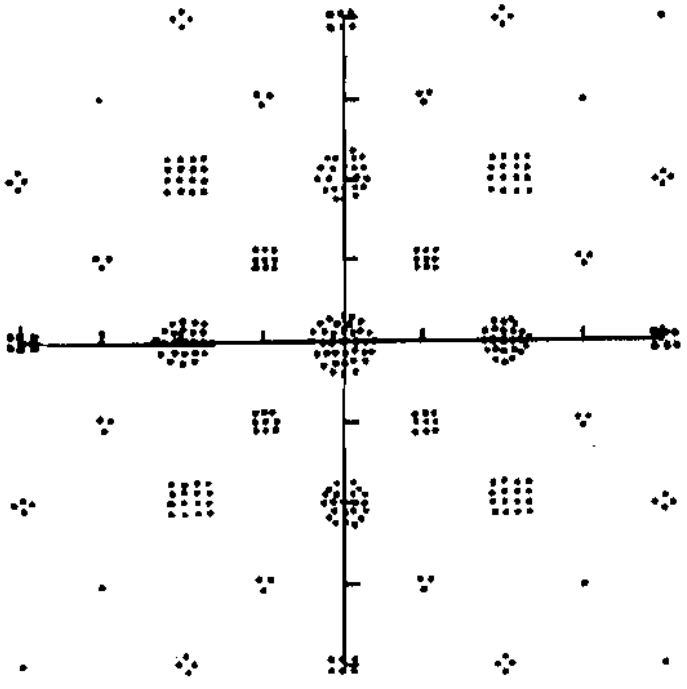

Fig. 2

four diagonal directions with equal probabilities. The most probable positions are shown in Figure 2. The process of exodus of some volcanic products and their form can be assimilated with the above two dimensional random walk model.

It must be noted that, since in both random walk examples we accepted an equal probability model we arrive to symmetrical spatial distribution (symmetrical geoforms). However, it is easy to change the model and to produce a variety of geoforms.

The processes of Sedimentation and Erosion

The random walk model can be used to study the Sedimentation - Erosion processes. [2,4]. We consider a sedimentary basin with p=0,75, the probability of deposition of a sediment and q=0,25, the probability of its erosion. The next step is to simulate this process in computer, and have a great number of random realizations.

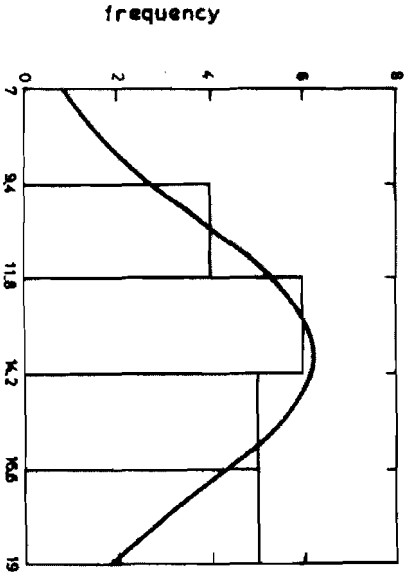

Fig. 4

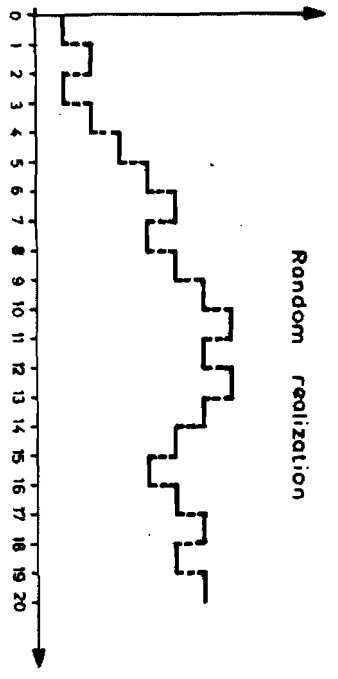

Fig. 3

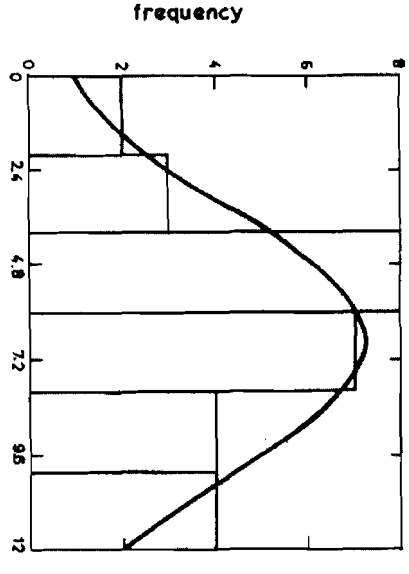

Fig. 5

Figure 3 shows that the upward movement means the deposition of the material, and the downward movement the erosion (axis x represents the base level). Using this model we can calculate the expected time to have a certain thickness of the sedimentery column (Fig. 4) and the expected thickness of the deposits after a certain time (Fig. 5).

CONCLUSIONS

The random walk model can be used in two ways to earth sciences, firstly to explain some characteristic geoforms by the spatial probability distribution and secondly to simulate some processes and to calculate different variables. It is certain that to investigate complex natural systems, the probabilistic approach is becoming necessary. The uses of random walk models in various braches of earth sciences are providing a new approach to understand the different geomorphological processes.

REFERENCES

Cox, D.R. and Miller, H.D., 1968, The Theory of Stochastic Processes: Wiley, New York, 398 p.

Kolmogorov, A. N., 1951, Solution of a Problem in Probability Theory Connected with the Problem of the Mechanism of Stratification: Am. Math. Soc. Transl., v. 53, p. 171-177.

Price, W.E. jr. (1974), Simulation of alluvial fan deposition by a random walk model, Water Res. Resch., 10, pp. 263-274.

Schwarzacher, Walther, 1975, Sedimentation Models and Quantitave Stratigraphy: Elsevier, Amsterdam, 382 p.