Study of the Mean Monthly Air Temperatures during the Successive Sunspot Cycles

by

JOHN XANTHAKIS

Study of the Mean Monthly Air Temperatures during the Successive Sunspot Cycles

by

JOHN XANTHAKIS

SUMMARY

In the first part of this paper (1-3) we study the mean monthly air temperatures during the successive sunspot cycles in 6 stations of Central and Northwest Europe.

Based upon the observation data we study the variations from one 11-year period of solar activity to the next of the following:

a) The difference $T_h - T_c$ between the mean temperatures of the 4 warmer (May-August) and the 4 colder (November-February) months of the year.

b) The differences $R = T_7 - T_1$ and $R' = T_6 - T_{12}$ between the mean temperatures of the months July-January and June-December.

c) The difference $T'_{eq} - T_{eq}$ between the mean temperatures of the months of the equinoxes, $T'_{eq} = \frac{1}{2}(T_9 + T_{10})$, $T_{eq} = \frac{1}{2}(T_3 + T_4)$.

d) The quantities $R - R'$ and $X = (T'_{eq} - T_{eq}) - \frac{1}{2}(R - R')$.

All the above differences are expressed in terms of the 6 constants A, C, V, P, e, W, each of which has a special physical signification. By the same constants are also expressed the corresponding mean monthly air temperatures T_i , $i = 1, 2, \ldots 12$.

In the second part (4-7) we show that only the constants e, P and their functions $e_o = APe$ and $Q_o = A(P-1)$ present a correlation with the mean annual sunspot numbers N, if the time scale corresponding to e_o and Q_o is shifted a whole period to the left. Then we show that:

1) There exists a close correlation between the mean annual sunspot number N and the average $\overline{X}$ of the values of X corresponding to 13 stations of Europe and Near East, if the time scale corresponding to $\overline{X}$ is shifted a whole period to the left.

2) There exists a close correlation between N and the average $\overline{X}$ of the values of X corresponding to 13 stations of the United States.

3) The values of $\overline{X}$ and $\overline{T'_{eq} - T_{eq}}$ for Europe and the United States show a phase difference amounting to one 11-year period.

ΠΕΡΙΛΗΨΙΣ

Εἰς τὸ πρῶτον μέρος τῆς παρούσης (1 - 3) μελετῶνται αἱ μέσαι μηνιαῖαι θερμοκρασίαι 6 σταθμῶν τῆς Κεντρικῆς καὶ Βορειοδυτικῆς Εὐρώπης αἱ ἀντιστοιχοῦσαι εἰς διαδοχικὰς περιόδους τῆς ἡλιακῆς δράσεως.

Ἰδιαιτέρως μελετῶνται αἱ μεταβολαὶ ἀπὸ περιόδου εἰς περίοδον τῆς ἡλιακῆς δράσεως τῶν κάτωθι μεγεθῶν:

α) Τῆς διαφορᾶς $T_h - T_c$ τῶν μέσων θερμοκρασιῶν τῶν 4 θερμοτέρων (Μάϊος-Αὔγουστος) καὶ τῶν 4 ψυχροτέρων (Νοέμβριος - Φεβρουάριος) μηνῶν τοῦ ἔτους.

β) Τῶν διαφορῶν $R = T_7 - T_1$ καὶ $R' = T_6 - T_{12}$ τῶν θερμοκρασιῶν τῶν μηνῶν Ἰουλίου - Ἰανουαρίου καὶ Ἰουνίου - Δεκεμβρίου.

γ) Τῆς διαφορᾶς $T'_{eq} - T_{eq}$ τῶν μέσων θερμοκρασιῶν τῶν μηνῶν τῶν ἰσημεριῶν, $T'_{eq} = \frac{1}{2}(T_9 + T_{10})$, $T_{eq} = \frac{1}{2}(T_3 + T_4)$.

δ) Τῶν ποσοτήτων $R - R'$ καὶ $X = (T'_{eq} - T_{eq}) - \frac{1}{2}(R - R')$.

Ὅλα τὰ μεγέθη ταῦτα ἐκφράζονται συναρτήσει 6 σταθερῶν A, C, V, P, e, W, ἑκάστη τῶν ὁποίων ἔχει ἰδίαν φυσικὴν σημασίαν. Συναρτήσει τῶν αὐτῶν σταθερῶν ἐκφράζονται ἐπίσης καὶ αἱ μέσαι μηνιαῖαι θερμοκρασίαι T_i, $i = 1, 2, .. 12$.

Εἰς τὸ δεύτερον μέρος (4 - 7) ἀποδεικνύεται κατ' ἀρχὴν ὅτι ἐκ τῶν ἀνωτέρω 6 σταθερῶν μόνον αἱ σταθεραὶ e καὶ P καὶ αἱ συναρτήσεις αὐτῶν $e_o = APe$ καὶ $Q_o = A(P - 1)$ συσχετίζονται μὲ τὰς ἀντιστοίχους μέσας ἐτησίας τιμὰς N τοῦ σχετικοῦ ἀριθμοῦ τῶν κηλίδων καὶ δὴ μόνον ἐφόσον ἡ κλῖμαξ τοῦ χρόνου εἰς ἣν ἀναφέρονται τὰ e_o καὶ Q_o μετατεθῇ κατὰ μίαν περίοδον πρὸς τὰ ἀριστερὰ ἐν σχέσει πρὸς τὴν κλίμακα τοῦ χρόνου τὴν ἀντιστοιχοῦσαν εἰς τὰ N. Κατόπιν τούτου δεικνύεται ὅτι:

1) Ὑπάρχει στενὴ συσχέτισις μεταξὺ τοῦ μέσου ἐτησίου σχετικοῦ ἀριθμοῦ N τῶν κηλίδων καὶ τοῦ μέσου ὅρου $\overline{X}$ τῶν τιμῶν τοῦ X τῶν ἀντιστοιχουσῶν εἰς 13 σταθμοὺς τῆς Εὐρώπης καὶ τῆς Ἐγγὺς Ἀνατολῆς, ἐφόσον ἡ κλῖμαξ τοῦ χρόνου εἰς ἣν ἀναφέρονται τὰ $\overline{X}$ μετατεθῇ κατὰ μίαν περίοδον πρὸς τὰ ἀριστερὰ ἐν σχέσει πρὸς τὴν κλίμακα τοῦ χρόνου τὴν ἀντιστοιχοῦσαν εἰς τὰ N.

2) Ὑπάρχει ἐπίσης στενὴ συσχέτισις μεταξὺ τοῦ N καὶ τοῦ μέσου ὅρου $\overline{\overline{X}}$ τῶν τιμῶν τοῦ X τῶν ἀντιστοιχουσῶν εἰς 13 σταθμοὺς τῶν Ἡνωμένων Πολιτειῶν.

3) Αἱ ἀντίστοιχοι τιμαὶ τῶν $\overline{X}$ καὶ $\overline{T'_{eq} - T_{eq}}$ διὰ τὴν Εὐρώπην καὶ τὰς Ἡνωμένας Πολιτείας παρουσιάζουν μίαν διαφορὰν φάσεως ἀνερχομένην εἰς μίαν περίοδον τῆς ἡλιακῆς δράσεως.

RÉSUMÉ

Dans la première partie de l'ouvrage (1 - 3) sont étudiées les températures moyennes mensuelles de l'air durant les cycles successifs de l'activité solaire en 6 stations de l'Europe Centrale et du Nord - Ouest.

Sont étudiées notamment les variations d'une période à l'autre de l'acti vité solaire des quantités suivantes:

a) De la différence $T_h - T_c$ des températures moyennes des 4 mois les plus chauds de l'année (Mai à Août) et des 4 mois les plus froids (Novembre à Février).

b) Des différences $R = T_7 - T_1$ et $R' = T_6 - T_{12}$ des températures moyennes mensuelles des mois Juillet - Janvier et Juin - Décembre.

c) De la différence $T'_{eq} - T_{eq}$ des températures moyennes des mois d'équinoxes, $T'_{eq} = \frac{1}{2}(T_9 + T_{10})$, $T_{eq} = \frac{1}{2}(T_3 + T_4)$.

d) Des quantités $R - R'$ et $X = (T'_{eq} - T_{eq}) - \frac{1}{2}(R - R')$.

Toutes ces quantités sont exprimées en fonction des 6 constantes A, C, V, P, e, W, qui ont chacune une signification physique spéciale. En fonction des mêmes constantes s'expriment également les températures moyennes mensuelles correspondantes T_i, $i = 1, 2, \ldots 12$.

Dans la seconde partie (4 - 7) de l'ouvrage constatation est faite que seules les constantes e, P ainsi que leurs fonctions $e_0 = APe$ et $Q_0 = A(P-1)$ présentent une corrélation avec le nombre moyen annuel N des taches solaires, lorsque l'échelle du temps pour e_0 et Q_0 est déplacée d'une période vers la gauche par rapport à celle des N. D'où, l'on peut tirer les conséquences suivantes vérifiées par les observations:

1) Il existe une haute corrélation entre N et $\overline{X}$, où $\overline{X}$ est la valeur moyenne de la quantité X qui correspond à 13 stations de l'Europe et du Proche - Orient. L'échelle du temps pour $\overline{X}$ est déplacée aussi d'une période vers la gauche par rapport à celle des N.

2) Il existe aussi une haute corrélation entre N et $\overline{X}$, où $\overline{X}$ est la valeur moyenne de X qui correspond à 13 stations des États - Unis.

3) Les valeurs des quantités $\overline{X}$ et $\overline{T'_{eq} - T_{eq}}$ qui correspondent aux stations en question de l'Europe et des États - Unis présentent une différence de phase s'élevant à une période entière.

ZUSAMMENFASSUNG

In dem ersten Abschnitt dieser Arbeit (1 - 3) werden die den aufeinander folgenden Sonnenfleckenzyklen entsprechenden Monatsmittel der Lufttemperatur in 6 Stationen Zentral- und Nordwesteuropas untersucht.

Im besonderen werden die Änderungen von einem Sonnenfleckenzyklus zu dem anderen der folgenden Grössen behandelt:

a) Der Differenz $T_h - T_c$ der Mitteltemperaturen der 4 wärmeren (Mai-August) und 4 kälteren (November - Februar) Monate des Jahres.

b) Der Differenzen $R = T_7 - T_1$ und $R' = T_6 - T_{12}$ der Mitteltemperaturen der Monate Juli - Januar und Juni - Dezember.

c) Der Differenz $T'_{eq} - T_{eq}$ der Mitteltemperaturen der Monate der Äquinoktien, $T'_{eq} = \frac{1}{2}(T_9 + T_{10})$, $T_{eq} = \frac{1}{2}(T_3 + T_4)$.

d) Der Quantitäten $R - R'$ und $X = (T'_{eq} - T_{eq}) - \frac{1}{2}(R - R')$.

Alle diese Grössen werden als Funktionen von 6 Konstanten A, C, V, P, e, W, deren jede eine eigene physikalische Bedeutung hat, ausgedrückt. Als Funktionen derselben Konstanten werden auch die entsprechenden Monatsmitteltemperaturen T_i, $i = 1, 2, \ldots 12$, ausgedrückt.

In dem zweiten Abschnitt (4 - 7) wird erstens gezeigt, dass von den obigen 6 Konstanten nur die Konstanten e und P und ihre Funktionen $e_0 = APe$

und $Q_o = A(P-1)$ mit den entsprechenden Jahresmitteln N der Sonnenfleekenrelativzahlen korrelieren und zwar nur, wenn der Masstab der Zeit für e_o, Q_o um eine ganze 11 - Jährige Sonnenfleekenperiode naeh links versehoben wird.

Zweitens werden die folgenden gezeigt:

1) Es gibt eine enge Korrelation zwischen den den aufeinander folgenden Sonnenfleckenzyklen entspreehenden Jahresmitteln N der Sonnenfleekenrelativzahlen und den entsprechenden Mittelwerten $\overline{X}$ der Quantität X für 13 Stationen von Europa und dem Nahen Osten, wenn der Masstab der Zeit für $\overline{X}$ um eine ganze 11 - jährige Periode nach links verschoben wird.

2) Es gibt eine enge Korrelation zwischen N und den Mittelwerten $\overline{X}$ der Quantität X für 13 Stationen der Vercinigten Staaten Amerikas.

3) Die entsprechenden Werte der Quantitäten $\overline{X}$ und $\overline{T'_{eq} - T_{eq}}$ Europas und der Vereinigten Staaten, zeigen eine Phasendifferenz von einer ganzen 11 - jährigen Sonnenfleckenperiode.

1. INTRODUCTION

In a previous paper (1953) we found the following relations:

[1] $$\frac{1}{2}(T_i + T_{13-i}) = A + C \sin(L_i - V)$$

[2] $$\frac{T_{13-i}}{T_i} = \frac{P}{1 - e\cos(L_i - W)}$$

$$i = 1, 2, \ldots 6$$

where T_i and T_{13-i}, $i = 1, 2, \ldots 6$, represent the mean monthly air temperatures in absolute degrees, during the successive sunspot cycles *, L_i the longitude of the Sun corresponding to the middle of each of the months January, February,.... June, and A,C,P, e,V,W, six constants determined by the observation data.

A theoretical justification of relations [1] and [2] was given (Xanthakis, 1952) for the case in which T_i and T_{13-i} represent the meau monthly temperatures of the Earth's surface, the Earth being considered without atmosphere.

---

* Each cycle begins at the year following that in which the minimum of the annual sunspot number occurs, and ends at the year of the next minimum.

From the above two relations we can easily get the following expressions of the mean monthly air temperatures (Xanthakis, 1953):

$$[3] \qquad T_i = \frac{2P}{P+1}\left[A + C\sin(L_i - V)\right]\cdot\left[\frac{1}{P} - \frac{e}{P+1}\cos(L_i - W) - \frac{e^2}{(P+1)^2}\cos^2(L_i - W)\right]$$

$$[4] \qquad T_{13-i} = \frac{2P}{P+1}\left[A + C\sin(L_i - V)\right]\cdot\left[1 + \frac{e}{P+1}\cos(L_i - W) + \frac{e^2}{(P+1)^2}\cos^2(L_i - W)\right]$$

$$i = 1, 2, \ldots 6$$

As was shown by Mavridis (1954a, b) relations [3] and [4] represent the observed mean monthly air temperatures corresponding to successive sunspot cycles, with the same approximation as that given by the corresponding development of harmonic analysis.

2. DETERMINATION OF THE CONSTANTS AND THEIR PHYSICAL MEANING

Constants P, e, W of the ellipse [2] can be determined from the system:

$$\frac{T_i}{T_{13-i}} = \frac{1}{P} - \frac{e}{P}\cos(L_i - W) \qquad i = 1, 2, \ldots 6$$

by the method of least squares. To facilitate calculations we put:

$$L_i = L_1 + y_i \qquad i = 1, 2, \ldots 6$$

The values of L_1 and y_i are given in Table I for each period of solar activity (Mavridis, 1954 b).

Constants A, C, V can also be determined from system [1] by the method of least squares (Mavridis, 1954 b).

However, we can also determine these constants in the following way, which practically leads us to the same values.

Subtracting each equation from the next in the system [1] we obtain:

$$(T_2 - T_1) + (T_{11} - T_{12}) = 4\ C \cos\left(\frac{L_1 + L_2}{2} - V\right) \sin\frac{1}{2}(L_2 - L_1)$$

$$(T_3 - T_2) + (T_{10} - T_{11}) = 4\ C \cos\left(\frac{L_2 + L_3}{2} - V\right) \sin\frac{1}{2}(L_3 - L_2)$$

$$(T_4 - T_3) + (T_9 - T_{10}) = 4\ C \cos\left(\frac{L_3 + L_4}{2} - V\right) \sin\frac{1}{2}(L_4 - L_3)$$

$$(T_5 - T_4) + (T_8 - T_9) = 4\ C \cos\left(\frac{L_4 + L_5}{2} - V\right) \sin\frac{1}{2}(L_5 - L_4)$$

$$(T_6 - T_5) + (T_7 - T_8) = 4\ C \cos\left(\frac{L_5 + L_6}{2} - V\right) \sin\frac{1}{2}(L_6 - L_5)$$

or, considering Table I, we have approximately:

[5]

$$(T_2 - T_1) + (T_{11} - T_{12}) = 4\ C \sin 15^\circ \cos[(V - 11^\circ) + 60^\circ]$$

$$(T_3 - T_2) + (T_{10} - T_{11}) = 4\ C \sin 15^\circ \cos[(V - 11^\circ) + 30^\circ]$$

$$(T_4 - T_3) + (T_9 - T_{10}) = 4\ C \sin 15^\circ \cos[(V - 11^\circ)]$$

$$(T_5 - T_4) + (T_8 - T_9) = 4\ C \sin 15^\circ \cos[(V - 11^\circ) - 30^\circ]$$

$$(T_6 - T_5) + (T_7 - T_8) = 4\ C \sin 15^\circ \cos[(V - 11^\circ) - 60^\circ]$$

by putting:

$$f_1 = (T_{10} - T_{11}) + (T_{11} - T_{12}) + (T_2 - T_1) + (T_3 - T_2)$$

$$f_2 = (T_5 - T_4) + (T_6 - T_5) + (T_7 - T_8) + (T_8 - T_9)$$

$$g = (T_4 - T_3) + (T_9 - T_{10})$$

and taking into account equations [5] we get:

[6]

$$f_2 - f_1 = 4\ (1 + \sqrt{3})\ C \sin 15^\circ \sin(V - 11^\circ)$$

$$g = 4\ C \sin 15^\circ \cos(V - 11^\circ)$$

[7]

$$\operatorname{tg}(V - 11^\circ) = \frac{1}{1 + \sqrt{3}} \ \frac{f_2 - f_1}{g}$$

Relations [6] and [7] permit the calculation of the angle V in terms of the differences of the mean monthly air temperatures of the successive months.

From system [1] we can easily obtain the values of C and A

TABLE I

Periods		L_1	y_1	y_2	y_3	y_4	y_5	y_6
I	1776 - 1784	296° 25′	0°	30° 3′	59° 42′	89° 44′	119° 17′	148° 30′
II	1785 - 1798	» 31	»	» »	» »	» »	» »	» »
III	1799 - 1810	295 37	»	» »	» 44	» 48	» 23	» 36
IV	1811 - 1823	» 41	»	» »	» »	» »	» »	» »
V	1824 - 1833	» 47	»	» »	» »	» »	» »	» »
VI	1834 - 1843	» 50	»	» »	» »	» »	» »	» »
VII	1844 - 1856	» 57	»	» »	» »	» »	» »	» »
VIII	1857 - 1867	296 3	»	» »	» »	» »	» »	» »
IX	1868 - 1878	» 8	»	» »	» »	» »	» »	» »
X	1879 - 1889	» 12	»	» »	» »	» »	» »	» »
XI	1890 - 1901	» 17	»	» »	» »	» »	» »	» »
XII	1902 - 1913	295 21	»	» 4	» 47	» 53	» 29	» 43
XIII	1914 - 1923	» 27	»	» »	» »	» »	» »	» »
XIV	1924 - 1933	» 32	»	» »	» »	» »	» »	» »
XV	1934 - 1944	» 36	»	» »	» »	» »	» »	» »

as well:

[8] $$C = \frac{0{,}5}{\sqrt{2}+\sqrt{6}} \left[\sum_{\text{Apr.}}^{\text{Sept.}} T - \sum_{\text{Oct.}}^{\text{Mar.}} T \right] \sec\,(V - 11^\circ)$$

[9] $$A = \frac{1}{12} \sum_{1}^{12} T_i + \frac{\sqrt{2}+\sqrt{6}}{6}\, C \sin(V - 11^\circ) = T_m + 0{,}644\, C \sin(V - 11^\circ)$$

where $\sum_{\text{Apr.}}^{\text{Sept.}} T$ is the sum of the mean monthly air temperatures of the months April ..... September, $\sum_{\text{Oct.}}^{\text{Mar.}} T$ the sum of the mean monthly air temperatures of the months October .... March and T_m the mean annual temperature corresponding to each of the time intervals considered.

By putting :

$$\frac{1}{2}(T_6 + T_7) = T_{sol} \quad , \qquad \frac{1}{2}(T_{12} + T_1) = T'_{sol}$$

$$\frac{1}{2}(T_3 + T_4) = T_{eq} \quad , \qquad \frac{1}{2}(T_9 + T_{10}) = T'_{eq}$$

where T_{sol} , T'_{sol} represent respectively the mean temperatures at the viciuity of the solstices (June, July - December, January) and T_{eq} , T'_{eq} the mean temperatures at the vicinity of the equinoxes (March, April - September, October) we get :

[10]
$$\frac{1}{2}(f_2 - f_1) = (T_{sol} + T'_{sol}) - (T_{eq} + T'_{eq})$$
$$\frac{1}{2}\ g = T'_{eq} - T_{eq}$$

Thus, relation [7] becomes :

[11]
$$tg(V - 11^0) = \left[(T_{sol} + T'_{sol}) - (T_{eq} + T'_{eq})\right] : (T'_{eq} - T_{eq})$$

Table II gives the values of the constants A,C,P,e,V and W for each sunspot cycle for the following stations :

1) Vienna	1776 - 1944	4) Copenhagen	1799 1944
2) Prague	1776 - 1944	5) Oslo	1824 - 1944
3) Berlin	1776 - 1933	6) Bergen	1824 - 1944

The values of the mean monthly air temperatures for Prague were taken from Hlavàč (1940) and Schindler (1948); those for the other stations from "World Weather Records" (Clayton, 1934, 1944, 1947).

From relations [8], [9] and [11] we can see immediately the physical meaning of the constauts A, C and V. Thus, when the phase angle V takes values near the value $V = 11°$, the constant A differs only slightly from the mean annual temperature corresponding to the same sunspot cycle. In Central Europe and especially in Vienna, where mostly $V < 11°$ (see Table II) the constant A takes values slightly smaller than those of the mean annual temperature T_m. In Northwest Europe ou the contrary (Copenhagen, Oslo, Bergen), where $V > 11°$ in general, constant A takes values greater than the mean annual temperature. However, owing to the fact that the temperature is expressed in absolute degrees, the relative variation of this constant from period to period is very small, not exceeding the 0.7 per cent of its mean value.

Constant C, as is shown by relation [8], depends on the difference of the mean temperatures of the six warmer (April to September) and the six colder (October to March) months of the year. We must notice here the continuous decrease of this constant in Central Europe (Vienna, Prague, Berlin) and especially its sudden decrease during the period 1902 - 13 (see Table II).

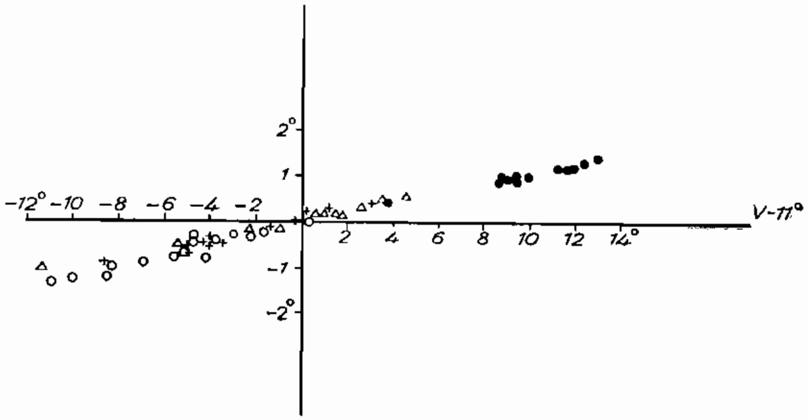

Fig. 1. Values of the differences $(T_{sol} + T'_{sol}) - (T_{eq} + T'_{eq})$ *and* V - 11° *for Vienna (open circles), Prague (crosses), Berlin (triangles) and Copenhagen (points).*

Finally the phase angle V plays a special role in the distribution of the mean temperatures at the vicinity of the solstices and the equinoxes. In fact, as in all the six considered stations we have $T'_{eq} - T_{eq} > O$, it follows from relation [11] that :

$$\text{if} \quad V = 11° \ , \quad T_{sol} + T'_{sol} = T_{eq} + T'_{eq}$$
$$» \quad V < 11° \ , \quad T_{sol} + T'_{sol} < T_{eq} + T'_{eq}$$
$$» \quad V > 11° \ , \quad T_{sol} + T'_{sol} > T_{eq} + T'_{eq}$$

Thus, in the three stations of Northwest Europe (Copenhagen, Oslo, Bergen) where V> 11° we have for all the periods considered :

$$T_{sol} + T'_{sol} > T_{eq} + T'_{eq}$$

whereas in Central Europe and specially in Vienna and Prague we generally have the reverse (see Fig. 1).

TABLE II

Values of the constants A, C, V, P, e and W

Periods		VIENNA A	VIENNA C	VIENNA V	VIENNA P	VIENNA e	VIENNA W	PRAGUE A	PRAGUE C	PRAGUE V	PRAGUE P	PRAGUE e	PRAGUE W	BERLIN A	BERLIN C	BERLIN V	BERLIN P	BERLIN e	BERLIN W
I	1776 - 1784	282.°34	11.°20	3.°9	1.00518	0.0135	7.°8	281.°81	10.°47	6.°0	1.00562	0.0138	11.°3	282.°08	10.°47	9.°8	1.00457	0.0154	9.°1
II	1785 1798	3.16	10.81	8.8	0.99753	229	16.0	3.14	10.22	10.9	0.99735	237	13.3	2.24	10.08	12.0	0.99520	262	17.7
III	1799 - 1810	2.75	11.02	6.2	1.00177	218	10.1	2.40	10.62	6.9	1.00193	228	9.4	0.68	10.17	5.5	1.00340	209	12.0
IV	1811 - 1823	2.06	10.96	0.1	1.00065	173	7.8	1.89	10.34	2.5	1.00015	187	7.0	0.53	10.03	1.6	1.00006	191	7.6
V	1824 - 1833	2.24	10.86	5.7	1.00955	86	—11.3	2.41	10.54	7.5	1.00906	123	—10.9	1.46	10.05	6.8	1.00645	133	—11.2
VI	1834 - 1843	2.46	10.70	9.4	0.99915	238	11.7	2.19	10.31	10.5	0.99946	250	8.4	1.93	9.59	11.4	1.00021	218	9.2
VII	1844 - 1856	1.82	10.56	7.2	0.99627	256	10.9	1.63	10.10	9.6	0.99635	265	10.5	1.77	9.65	12.4	0.99676	263	10.8
VIII	1857 - 1867	1.25	10.63	2.4	0.99611	259	15.7	1.62	10.20	5.9	0.99797	239	15.2	2.19	9.46	10.8	0.99573	268	13.8
IX	1868 - 1878	1.70	10.39	5.5	1.00071	197	18.1	2.30	10.35	10.6	0.99942	223	20.8	2.84	9.33	13.8	0.99828	230	21.3
X	1879 1889	1.45	10.32	6.8	1.00160	185	12.2	2.00	10.03	11.0	0.99982	211	11 4	2 41	9.43	15.5	0.99934	224	9.9
XI	1890 - 1901	0.99	10.30	1.0	1.00264	172	12.5	1.65	10 03	7.1	1.00124	197	11.1	1.99	9.55	8.8	1.00080	192	13.0
XII	1902 - 1913	1.98	9.35	8.0	1.00243	145	11.2	2.53	9.42	12.2	1.00007	169	13.6	2.87	8.94	14.6	0.99770	200	12.6
XIII	1914 - 1923	2.48	9.21	11.3	1.00122	149	18.6	2.88	9.40	14 4	1.00022	167	17.9	2.73	8.98	13.5	1.00033	156	20.5
XIV	1924 - 1933	1.84	9.90	6.3	0.99860	245	12.6	2.41	9.81	10.6	0.99824	250	13.1	2.46	9.18	12.0	0.99950	227	15.7
XV	1934 - 1944	1.47	10.00	2.6	1.00356	168	6.9	2.19	10.06	6.8	1.00089	211	11.0	—	—	—	—	—	—

TABLE II (Continued)

Values of the constants A, C, V, P, e and W

Periods		Copenhagen A	Copenhagen C	Copenhagen V	Copenhagen P	Copenhagen e	Copenhagen W	Oslo A	Oslo C	Oslo V	Oslo P	Oslo e	Oslo W	Bergen A	Bergen C	Bergen V	Bergen P	Bergen e	Bergen W
III	1799 - 1810	280.°96	8.°68	13.°8	0.99904	0.0335	11.°1												
IV	1811 - 1823	1.08	8.21	13.8	0.99787	297	11.5												
V	1824 - 1833	1.86	8.92	19.9	0.99986	273	4.2	279.°50	9.°62	19.°5	0.99951	0.0256	8.°8	281.°35	6.°78	19.°4	1.00075	0.0167	14.°7
VI	1834 - 1843	0.84	8.54	19.5	0.99908	273	11.0	278.67	10.16	15.2	0.99895	252	14.2	0.84	6.22	24.0	1.00228	165	13.8
VII	1844 - 1856	1.07	8.62	20.4	0.99699	322	10.6	278.75	10.90	17.4	0.99894	281	11.0	0.99	6.74	23.4	0.99857	217	17.0
VIII	1857 - 1867	1.37	8.28	20.4	0.99477	334	12.3	279.40	10.41	19.5	0.99565	317	13.6	1.11	6.59	22.4	0.99589	268	12.5
IX	1868 - 1878	1.61	8.49	22.2	0.99621	288	19.4	279.46	10.87	19.2	0.99483	310	17.8	0.80	6.72	21.5	0.99481	254	25.4
X	1879 - 1889	1.62	8.65	24.0	0.99670	291	11.4	279.82	10.75	21.0	0.99392	313	12.7	0.86	6.76	22.4	0.99621	247	16.3
XI	1890 - 1901	1.78	8.57	20.9	1.00044	245	8.9	279.98	10.85	19.4	1.00110	221	9.7	1.08	6.57	23.4	1.00094	178	10.6
XII	1902 - 1913	2.06	8.07	23.4	0.99790	252	10.7	280.25	9.88	20.9	0.99798	233	13.6	1.27	5.99	25.5	0.99777	197	10.9
XIII	1914 - 1923	2.00	8.18	22.9	0.99951	215	15.8	279.71	10.28	18.1	1.00222	178	16.9	0.49	6.13	19.1	1.00252	136	26.4
XIV	1924 - 1933	2.27	7.96	22.7	0.99937	271	15.1	280.40	9.82	22.8	0.99917	243	20.6	1.67	6.17	27.1	1.00066	179	23.9
XV	1934 - 1944	2.28	8.47	19.8	1.00220	248	10.2	280.10	10.31	17.1	1.00573	182	6.1	1.63	6.18	19.7	1.00345	171	15.8

Of special interest are the variations of P, e and W on which the differences between the seasonal temperatures depend. From these three constants the two former represent the parameter and the eccentricity of the ellipse [2], whereas W plays a role similar to that of V. In fact, if we call:

$$T_{win} = \frac{1}{3}(T_1 + T_2 + T_3) \quad , \quad T_{spr} = \frac{1}{3}(T_4 + T_5 + T_6)$$

$$T_{sum} = \frac{1}{3}(T_7 + T_8 + T_9) \quad , \quad T_{aut} = \frac{1}{3}(T_{10} + T_{11} + T_{12})$$

we can easily show that:

if $W = 11^\circ$, $T_{sum} + T_{win} = T_{aut} + T_{spr}$ or $T_{sum} - T_{spr} = T_{aut} - T_{win}$

» $W < 11^\circ$, $T_{sum} + T_{win} < T_{aut} + T_{spr}$ » $T_{sum} - T_{spr} < T_{aut} - T_{win}$

» $W > 11^\circ$, $T_{sum} + T_{win} > T_{aut} + T_{spr}$ » $T_{sum} - T_{spr} > T_{aut} - T_{win}$

where,

T_{win} represents the mean temperature of winter

T_{spr} » » » » » spring

T_{sum} » » » » » summer

T_{aut} » » » » » autumn

Thus, during the period 1824 - 1833 we have:

	Vienna	Prague	Berlin	Copenh.	Oslo	Bergen
W—11°	−22°.3	−21°.9	−22°.2	−7°.2	−2.°2	+3°.7
$(T_{sum}+T_{win})$ - $(T_{aut}+T_{spr})$	−1°.04	−1°.33	−1°.72	−0°.96	−0°.03	+0°.66C

Whereas during the period 1914 - 1923 is:

	Vienna	Prague	Berlin	Copenh.	Oslo	Bergen
W—11°	+7°.6	+6°.9	+9°.5	+4°.8	+5°.9	+15°.4
$(T_{sum}+T_{win})$ - $(T_{aut}+T_{spr})$	+1°.17	+1°.20	+1°.28	+0°.93	+0°.83	+1°.73C

3. VARIATIONS OF CERTAIN CHARACTERISTIC TEMPERATURE DIFFERENCES FROM PERIOD TO PERIOD OF SOLAR ACTIVITY

3.1. *Difference between the mean temperatures of the 4 warmer and the 4 colder months of the year.* Let T_h be the average of the mean monthly air temperatures of the months May, June, July, August and T_c that of the months November, December, January and February i.e.:

$$[12] \qquad \begin{aligned} T_h &= \tfrac{1}{4}(T_5 + T_6 + T_7 + T_8) \\ T_c &= \tfrac{1}{4}(T_{11} + T_{12} + T_1 + T_2) \end{aligned}$$

From relation [1] we can easily find that:

$$[13] \qquad T_h - T_c = 1.67\ C \cos(V - 11°)$$

Since the factor $\cos(V - 11°)$ varies slightly from cycle to cycle, it follows from [13] that the difference $T_h - T_c$ varies proportionally to the constant C.

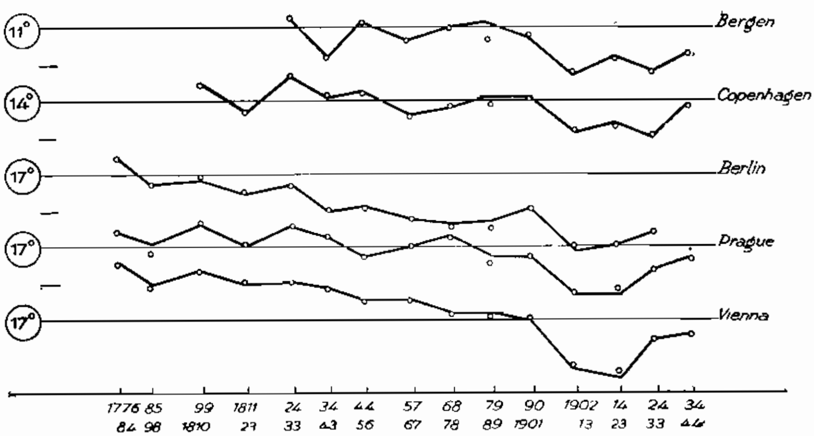

Fig. 2. Values of the difference $T_h - T_c$ for the successive sunspot cycles. The open circles represent the values of this difference given by the observations, and the continuous line the calculated ones.

In Fig. 2 the open circles represent the values of the difference $T_h - T_c$ given by the observations, whereas the continuous lines represent the values of this difference given by relation [13]. A noticeable decrease of this difference in Central Europe is clearly seen, a fact already noticed by other investigators too.

Fig. 3 represents the values of the difference $T_h - T_c$ for Central England. The values of the temperatures for this region were taken from the Table of Prof. G. Manley (1953).

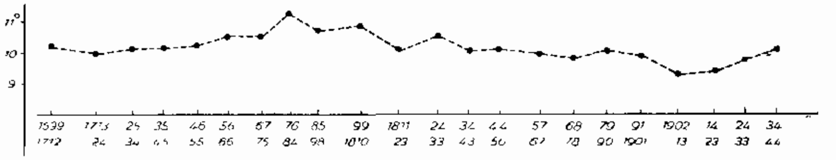

Fig. 3. Observed values of the difference $T_h - T_c$ for Central England (the values of the mean monthly air temperatures were taken from the Table of Prof. G. Manley).

The general march of the difference $T_h - T_c$ in Central Europe, as well as in Central England, leads one to suspect that probably this difference has a long period variation, whose last maximum took place during the period 1776 - 1784 and the last minimnm during the period 1902 - 1913.

3.2. *Difference between the mean temperatures at the vicinity of the equinoxes.* As the parameter P of the ellipse [2] takes values differing slightly from the unit (see Table II) we can approximately put $P + 1 \simeq 2$ in relations [3] and [4] and omit at the same time the square of the eccentricity e. Thus we have:

$$T_i = A - \tfrac{1}{2} A P e \cos (L_i - W) + C \sin (L_i - V) - \\ - \tfrac{1}{2} C P e \sin (L_i - V) \cos (L_i - W)$$

$$T_{13-i} = A P + \tfrac{1}{2} A P e \cos (L_i - W) + C P \sin (L_i - V) + \\ + \tfrac{1}{2} C P e \sin (L_i - V) \cos (L_i - W)$$

$$i = 1, 2, \ldots 6$$

If we take the principal terms* only, we have finally:

$$[14] \qquad T_i = A - \tfrac{1}{2} A P e \cos (L_i - W) + C \sin (L_i - V)$$

$$[15] \qquad T_{13-i} = A P + \tfrac{1}{2} A P e \cos (L_i - W) + C P \sin (L_i - V)$$

$$i = 1, 2, \ldots 6$$

* The coefficient $\frac{1}{2}$ C P e is of the order 0°.1 C.

From these relations we get, for i=3 and i=4:

$$T_3 + T_4 = 2\,A - \tfrac{1}{2}\,A\,P\,e\,[\cos(L_3 - W) + \cos(L_4 - W)] + \\ + \;C\;[\sin(L_3 - V) + \sin(L_4 - V)]$$

$$T_9 + T_{10} = 2\,A\,P + \tfrac{1}{2}\,A\,P\,e\,[\cos(L_3 - W) + \cos(L_4 - W)] + \\ + C\,P\;[\sin(L_3 - V) + \sin(L_4 - V)]$$

But:

$$\cos(L_3 - W) + \cos(L_4 - W) \simeq 2\cos 15^\circ \cos(W - 11^\circ) = \\ = 1.932\cos(W - 11^\circ)$$

$$\sin(L_3 - V) + \sin(L_4 - V) \simeq -2\cos 15^\circ \sin(V - 11^\circ) = \\ = -1.932\sin(V - 11^\circ)$$

Consequently:

$$\tfrac{1}{2}(T_3 + T_4) \equiv T_{eq} = A - 0.483\,A\,P\,e\cos(W - 11^\circ) - \\ - 0.483\;C\sin(V - 11^\circ)$$

$$\tfrac{1}{2}(T_9 + T_{10}) \equiv T'_{eq} = A\,P + 0.483\,A\,P\,e\cos(W - 11^\circ) - \\ - 0.483\,C\,P\sin(V - 11^\circ)$$

Therefore:

$$T'_{eq} - T_{eq} = A\,(P - 1) + 0.966\;A\;P\;e\cos(W - 11^\circ) - \\ - 0.483\;C\;(P - 1)\sin(V - 11^\circ)$$

or, neglecting the last term **

[16] $$T'_{eq} - T_{eq} = Q_o + 0.966\;e_o\cos(W - 11^\circ)$$

where

[17] $$Q_o = A\,(P - 1)$$

[18] $$e_o = A\,P\,e$$

** The coefficient $0.483\,C\,(P - 1)$ is of the order $0^o.02$ C.

3.3. *Difference between the mean temperatures of the months July-January and June - December.* From relations [14] and [15] we have for $i = 1$ and $i = 6$:

$$T_7 - T_1 = A\ (P - 1) + \tfrac{1}{2} A\ P\ e\ [\cos (L_6 - W) + \cos (L_1 - W)] + + C\ [P \sin (L_6 - V) - \sin (L_1 - V)]$$

$$T_6 - T_{12} = A\ (1 - P) - \tfrac{1}{2} A\ P\ e\ [\cos (L_6 - W) + \cos (L_1 - W)] + + C\ [\sin (L_6 - V) - P \sin (L_1 - V)]$$

but:

$$\cos (L_6 - W) + \cos (L_1 - W) \simeq 2 \sin 15° \cos (W - 11°) = = 0.259 \cos (W - 11°)$$

$$\sin (L_6 - V) - \sin (L_1 - V) \simeq 2 \cos 15° \cos (V - 11°) = = 1.932 \cos (V - 11°)$$

hence, taking into account relations [17] and [18] we get:

[19] $R = T_7 - T_1 = + Q_o + 1.932 C \cos (V - 11°) + 0.259\ e_o \cos (W - 11°)$

[20] $R' = T_6 - T_{12} = - Q_o + 1.932 C \cos (V - 11°) - 0.259\ e_o \cos (W - 11°)$

Relation [19] shows that the difference between the mean temperatures of the months July - January, which usually represents the annual range R of the temperature, depends on Q_o, C and e_o. However, owing to the fact that the coefficient of e_o is small, the variations of the annual range must have, on the whole, the characteristics of the variations of the constant C and the parameter P [because $Q_o = A\ (P - 1)$]. Thus in Vienna, Prague and Berlin, where constant C shows a continuous decrease, the annual range R will also show a systematic decrease, with sudden rise, however, during the periods 1799-1810, 1824-1833 and 1890-1901, when the parameter P assumes high values (see Table 1I), and the quantity Q_o its greatest positive values.

This is clearly shown in Fig. 4 where the open circles represent the observed values of the mean annual range R for each period of solar activity, and the continuous line those calculated by relation [19].

Analogous results we have for the difference between the mean temperatures of the months June - December, $R' = T_6 - T_{12}$, which represents the "theoretical annual range" of the temperature; that is, in this case

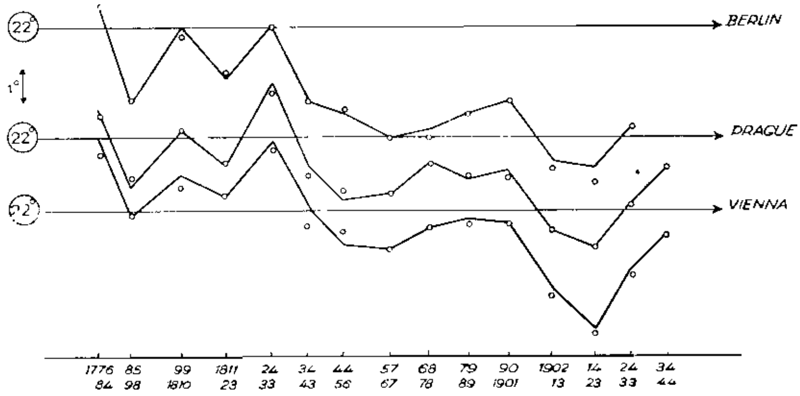

Fig. 4. Values of the difference $R = T_7 - T_1$ *for the successive sunspot cycles. The open circles represent the values of R given by the observations and the continuous line those calculated by relation* [19].

too, we shall have a systematic decrease of the difference $T_6 - T_{12}$ until the period 1914 - 1923 for the three stations of Central Europe. But here the influence of Q_0 will be opposite to that in the previous case of $T_7 - T_1$.

From [19] and [20] we get:

[21] $$R - R' = 2\,Q_0 + 0.518\,e_0 \cos(W - 11°)$$

From this relation we deduce that the variations of this difference from one period of solar activity to the next will mainly show the characteristics of the variations of Q_0, that is of the parameter P.

3.4. *The range X.* Combiniug relations [16] and [21] we get:

[22] $$X = (T'_{eq} - T_{eq}) - \tfrac{1}{2}(R - R') = 0.71\,e_0 \cos(W - 11°)$$

But,

$$(T'_{eq} - T_{eq}) - \tfrac{1}{2}(R - R') = \tfrac{1}{2}[(T_9 - T_4) + (T_{10} - T_3)] - \tfrac{1}{2}[(T_7 - T_6) + (T_{12} - T_1)]$$

heuce, quantity X represents the range of the variation of the difference $T_{13-i} - T_i$, $i = 1, 2, \ldots 6$ (see Fig. 5).

From relation [22] we deduce that, the range X depends mainly on e_0, becanse cos (W – 11°) changes slightly from period to period.

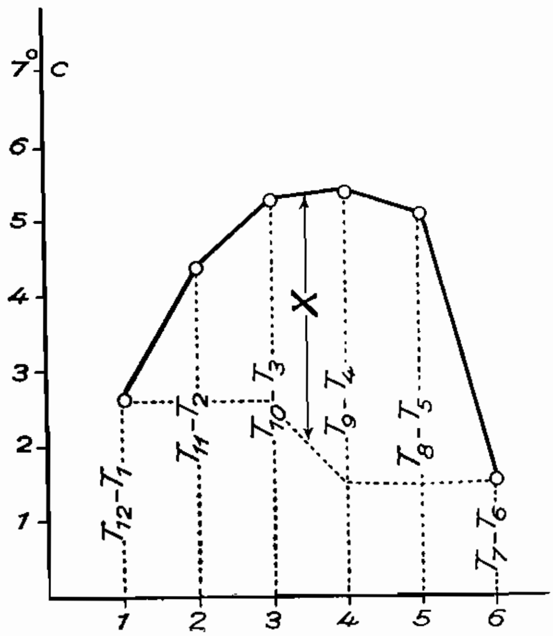

Fig. 5. Values of the difference $T_{13-i} - T_i$, $i = 1, 2, \ldots 6$ for Vienna (1934 - 44).

4. THE SUNSPOT NUMBER AND THE ELEMENTS OF THE TEMPERATURE ELLIPSE [2]

In a previous paper (1953) we pointed out that the eccentricity e and the parameter P of the ellipse [2] are correlated with the annual sunspot number. Thus, during the 4 periods with high mean annual sunspot number, the values of the eccentricity e corresponding to Vienna, Rome, Copenhagen and Bergen are great, and those of the parameter P are small. The opposite takes place during the following 3 periods with low mean annual sunspot number. An analogous correlation is observed for the quantities e_0 and Q_0 which depend on e and P.

Now a more detailed investigation shows that, the variations of e_0 and especially those of the average of its values corresponding to numerous stations of Central and Northwest Europe, follow in an impressive way the variations of the mean annual sunspot number N corresponding to the successive sunspot cycles.

The strangeness of this phenomenon lies in the fact that for the six considered stations *the origin of the time scale corresponding to* e_o *is shifted a whole period to the left of the origin of the time scale corresponding to the mean annual sunspot number N.* Thus, the values of e_o for the period 1914-1923 are correlated with that of N corresponding to the period 1902-1913 immediately before.

As regards Q_o, its variations from period to period are opposite to the variations of the mean annual sunspot number N. In this case also, the above mentioned shift of the origin of the time scale holds.

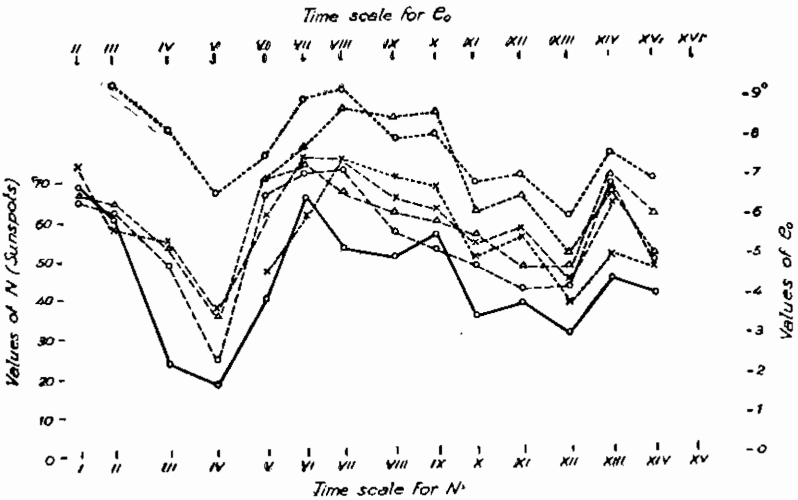

Fig. 6. Variations of N and e_o *from period to period.*

N = continuous line

o — — — o *Values of* e_o *for Vienna* o . . . o *Values of* e_o *for Copenhagen*

Δ — — — Δ » » » » *Prague* Δ . . . Δ » » » « *Oslo*

X — — — X » » » » *Berlin* X . . . X » » » » *Bergen*

(The origin of the time scale for e_o *is shifted a whole period to the left).*

These results are shown in Fig. 6 where the continuous line represents the values of the mean annual sunspot number N for each period (Stetson, 1947) (see Table III) and the broken lines represent the values of e_o in each of the six stations of Central and Northwest Europe respectively. The origin of the time scale of e_o is shifted a whole period to the left as regards the origin of the time scale of N.

TABLE III

Values of the mean annual sunspot number for the successive periods of solar activity

Periods		N	Periods		N	Periods		N
I	1776 - 1784	68.6	VI	1834 - 1843	65.4	XI	1890 - 1901	38.5
II	1785 - 1798	60.0	VII	1844 - 1856	53.2	XII	1902 - 1913	31.0
III	1799 - 1810	23.5	VIII	1857 - 1867	49.9	XIII	1914 - 1923	44.6
IV	1811 - 1823	18.2	IX	1868 - 1878	56.2	XIV	1924 - 1933	41.0
V	1824 - 1833	39.5	X	1879 - 1889	34.8	XV	1934 - 1944	55.4

5. THE ANNUAL SUNSPOT NUMBER AND RANGE X

If the above correlation between e_o, Q_o and N is real, then one might expect a corresponding relationship between N and the mean monthly air temperatures, not only in the six considered stations, but also in many others situated in various regions of the temperate zones.

However, as the mean monthly air temperatures themselves do not depend solely on e and P, but also on other constants, one must not expect an obvious relationship between the variations of the mean monthly air temperatures and those of solar activity expressed by the mean annual sunspot number, because the influence on the mean monthly temperatures of the variations of e and P from one period to the next, is screened out by the variations of the other constants, which do not show any correlation with N. The same holds for the difference $T_h - T_c$ between the mean temperatures of the 4 warmer and the 4 colder months of the year, which is independent of e_o and Q_o (see rel. 13), as well as for the annual range $R = T_7 - T_1$. Only for the quantities $\Delta R = R - R'$ and X, which depend respectively on Q_o and e_o (see rel. 21 and 22) one might expect to find an analogous relationship.

For this purpose we considered at first 13 stations in Europe and the Near East which have relatively long series of thermometrical observations and from their data we calculated the values of ΔR and X, by using relations:

$$\Delta R = R - R' = (T_7 - T_1) - (T_6 - T_{12}) = (T_7 - T_6) + (T_{12} - T_1)$$

$$X = \frac{1}{2}\left[(T_9 + T_{10}) - (T_3 + T_4)\right] - \frac{1}{2}\Delta R$$

where T_i, $i = 1, 2, \ldots 12$, represent the mean monthly air temperatures for each period of solar activity.

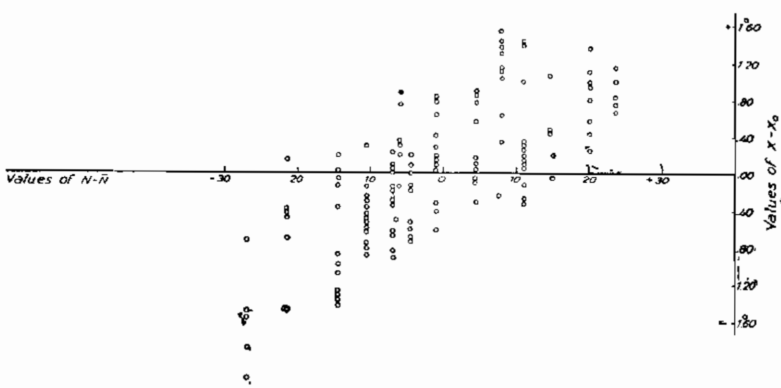

Fig. 7. Correlation between $N - \overline{N}$ *and* $X - X_0$ *for 13 stations in Europe and the Near East.* $\overline{N}$ *is the average of the values of* N *for the periods* I *to* XV, *and* X_0 *is the average of the values of* X *in each station. (The origin of the time scale for the difference* $X - X_0$ *is shifted a whole period to the left).*

The numerical results of these calculations are given in Tables IV and V. In the last column of these Tables are given for each period the averages $\overline{X}$ and $\overline{\Delta R}$ of the values of X and ΔR corresponding to the 13 stations considered, whereas their last line gives for each station the averages X_0 and ΔR_0 of the values of X and ΔR corresponding to all the periods considered here.

A comparison of the values of X and ΔR with those of N given by Table III shows, in fact, a more or less high correlation between these quantities (see Table VI and Fig. 7 and 8), *when the origin of the time scale for* X *and* ΔR *is shifted a whole period to the left of the origin of the time scale of the sunspots; that is if we correlate the values of* N *corresponding to the period* I (1776 - 84) *with those of* X *and* ΔR *corresponding to the period* II (1785 - 98) *etc.*

TABLE IV

Values of X, X_o and $\overline{X}$ for 13 stations in Europe and the Near East

Periods		VIENNA	PRAGUE	BERLIN	TRONDHEIM	VILNA	COPENHAGEN	ROME	BERGEN	OSLO	GIBRALTAR	ALEXANDRIA	BEIRUT	BUSHIRE	$\overline{X}$	Number of stations
I	1776 - 1784	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
II	1785 - 1798	4.°74	4.°96	5.°35	4.°80	6.°70	—	—	—	—	—	—	—	—	5.°31	5
III	1799 - 1810	4.38	4.73	4.27	4.26	5.20	6.°72	—	—	—	—	—	—	—	4.94	6
IV	1811 - 1823	3.51	3.85	3.86	3.37	4.07	5.95	4.°83	—	—	—	—	—	—	4.20	7
V	1824 - 1833	1.65	2.37	2.63	4.39	4.80	5.54	3.29	3.°40	5.°21	—	—	—	—	3.50	9
VI	1834 - 1843	4.79	5.04	4.50	3.54	5.88	5.47	5.01	3.46	5.08	—	—	—	—	4.75	9
VII	1844 - 1856	5.25	5.38	5.32	3.92	6.10	6.42	5.34	4.17	5.41	—	—	—	—	5.26	9
VIII	1857 - 1867	5.22	4.83	5.48	5.53	6.58	6.80	5.01	5.47	6.37	3.°82	—	—	—	5.51	10
IX	1868 - 1878	3.83	4.33	4.40	4.83	6.07	5.53	4.81	4.78	5.83	4.17	4.°86	—	—	4.86	11
X	1879 - 1889	3.76	4.36	4.64	5.45	5.65	5.90	4.30	4.91	6.22	3.72	5.25	6.°01	5.°46	5.05	13
XI	1890 - 1901	3.43	3.90	3.83	3.69	4.64	4.81	4.99	3.45	4.30	3.97	4.95	5.72	4.83	4.34	13
XII	1902 - 1913	2.95	3.43	4.10	3.77	4.95	5.09	4.30	3.98	4.67	3.91	5.43	6.02	5.31	4.45	13
XIII	1914 - 1923	2.91	3.38	3.21	2.70	—	4.36	4.29	2.65	3.56	4.29	5.07	5.89	5.34	3.97	12
XIV	1924 - 1933	4.74	4.91	4.46	2.82	5.56	5.30	5.25	3.40	4.58	4.50	5.31	6.01	5.61	4.80	13
XV	1934 - 1944	3.29	4.14	—	3.42	—	4.91	4.48	3.40	3.52	4.29	5.29	5.93	—	4.28	10
	X_o	3.°82	4.°26	4.°31	4.°04	5.°52	5.°63	4.°66	3.°92	4.°98	4.°08	5.°18	5.°93	5.°31	$\overline{X}_m$ 4.°66	

TABLE V

Values of ΔR, ΔR_o and $\overline{\Delta R}$ for 13 stations in Europe and the Near East

Periods		VIENNA	PRAGUE	BERLIN	TRONDHEIM	VILNA	COPENHAGEN	ROME	BERGEN	OSLO	GIBRALTAR	ALEXANDRIA	BEIRUT	BUSHIRE	$\overline{\Delta R}$	Number of stations
IV	1811 - 1823	3.°12	3.°20	3.°03	3.°98	4.°52	3.°44	3.°02	—	—	—	—	—	—	3.°47	7
V	1824 - 1833	6.58	6.88	5.76	1.83	4.91	4.39	4.79	3.°26	4.°00	—	—	—	—	4.93	9
VI	1834 - 1843	3.24	3.60	3.75	5.69	3.68	3.76	3.68	4.27	3.52	—	—	—	—	3.90	9
VII	1844 - 1856	2.18	2.30	2.44	3.67	4.32	3.30	2.87	2.40	3.50	—	—	—	—	3.00	9
VIII	1857 - 1867	1.93	2.70	1.94	2.90	1.91	2.46	2.80	2.14	2.68	2.°71	—	—	—	3.42	10
IX	1868 - 1878	3.34	3.06	2.38	1.35	1.46	2.05	4.52	0.79	1.46	2.34	4.°13	—	—	2.44	11
X	1879 - 1889	3.97	3.55	3.49	1.79	4.04	2.93	4.32	1.80	1.49	2.77	3.40	3.°40	4.°62	3.24	13
XI	1890 - 1901	4.08	3.60	3.37	3.31	4.41	3.88	4.13	3.17	3.92	3.04	4.38	5.28	4.47	3.93	13
XII	1902 - 1913	3.62	2.76	2.07	2.50	2.69	2.91	4.68	1.96	2.60	2.95	3.88	4.44	4.19	3.16	13
XIII	1914 - 1923	2.97	2.96	3.08	3.83	—	3.42	3.97	3.70	4.30	3.15	3.65	3.90	3.04	3.50	12
XIV	1924 - 1933	2.77	2.87	3.30	4.47	4.75	3.78	3.60	3.07	3.10	2.65	3.57	4.60	2.98	3.50	13
XV	1934 - 1944	4.27	3.44	—	5.72	—	4.91	4.59	4.66	5.75	2.64	3.80	4.18	—	4.40	10
	ΔR_o	3.°51	3.°41	3.°15	3.°42	3.°67	3.°44	3.°91	2.°84	3.°30	2.°78	3.°83	4.°50	3.°74		

This correlation becomes closer if, instead of taking X in each station separately, we consider the average $\overline{X}$ which corresponds to as many stations of a large area as possible.

TABLE VI

Correlation coefficient between X and N

Stations	Periods	Cor. Coef. $r_{X,N}$	Stations	Periods	Cor. Coef. $r_{X,N}$
Vienna	II - XV	0.76 ± 0.04	Oslo	V - XV	0.75 ± 0.05
Prague	II - XV	0.81 ± 0.04	Bergen	V - XV	0.67 ± 0.06
Berlin	II - XIV	0.81 ± 0.04	Trondheim	II - XV	0.48 ± 0.09
Vilna	II - XIV	0.78 ± 0.04	Rome	IV - XV	0.56 ± 0.08
Copenhagen	III - XV	0.58 ± 0.08			

In Fig. 9 the continuous line represents the values of $N - \overline{N}$ where $\overline{N}$ is the average of the values of N for the periods I to XV, and the open circles the values of $\overline{X} - \overline{X}_m$ for 13 stations in Europe and

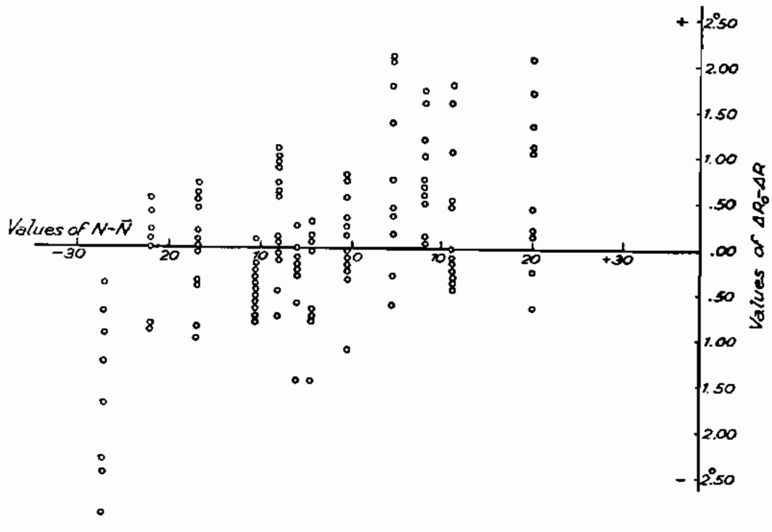

Fig. 8. Correlation between $N - \overline{N}$ *and* $\Delta R_0 - \Delta R$ *(open circles) for 13 stations in Europe and the Near East.* $\overline{N}$ *is the average of* N *for the periods* I *to* XV *and* ΔR_0 *is the average of* ΔR *in each station. The origin of the time scale for* $\Delta R_0 - \Delta R$ *is shifted a whole period to the left.*

the Near East. $\overline{X}_m$ is the average of the values of $\overline{X}$ given in the last column of the Table IV. As in Fig. 6, thus here again the origin of the time scale for $\overline{X} - \overline{X}_m$ is shifted a whole period to the left. Over the open circles is written the number of stations to which the average $\overline{X}$

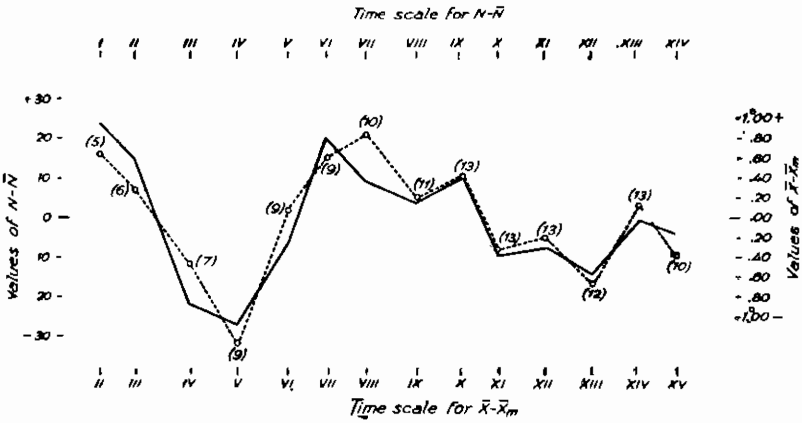

Fig. 9. Values of $N - \overline{N}$ *(continuous line) and* $\overline{X} - \overline{X}_m$ *(open circles) for 13 stations in Europe and the Near East, where* $\overline{X}_m$ *is the average of* $\overline{X}$ *for the periods* II *to* XV. *Over the open circles is written the number of stations to which the average* $\overline{X}$ *corresponds. The origin of the time scale for* $\overline{X} - \overline{X}_m$ *is shifted a whole period to the left*

corresponds. With the exception of the period VIII/VII during which we have a disagreement mainly in the stations of Northwest Europe, for all the other periods there exists a close correlation between $\overline{X}$ and N, the correlation coefficient being $r_{\overline{X},N} = 0.91 \pm 0.02$.

6. EXTENSION OF THE ABOVE INVESTIGATIONS TO 13 STATIONS IN THE UNITED STATES AND 9 STATIONS IN THE SOUTHERN HEMISPHERE

As the above stated relationship between $\overline{X}$ and N is obviously of special interest, we have tried to extend this research to other regions of the northern and southern temperate zone. For this purpose we considered 9 stations in the southern hemisphere having the relatively long series of thermometrical observations contained in "World Weather Records" (Clayton, 1934, 1944, 1947).

TABLE VII

Values of X, X_o and $\overline{X}$ for 13 stations of U. S. A.

Periods		NEW HAVEN	NEW YORK	ALBANY	CHARLESTON	ST. LOUIS	SAN DIEGO	SANTA FE	SAN FRANCISCO	DENVER	SALT LAKE CITY	EL PASO	PORTLAND	HELENA	$\overline{X}$	Number of stations
V	1824 - 1833	5.°72	6.°23	5.°16	4.°70	—	—	—	—	—	—	—	—	—	5.°45	4
VI	1834 - 1843	5.50	6.76	6.15	5.65	—	—	—	—	—	—	—	—	—	6.02	4
VII	1844 - 1856	6.10	7.00	6.19	3.81	5.°32	—	—	—	—	—	—	—	—	5.68	5
VIII	1857 - 1867	6.61	6.99	6.30	4.43	4.82	3.°02	5.°95	2.°84	—	—	—	—	—	5.12	8
IX	1868 - 1878	7.81	8.17	7.79	4.68	5.67	3.72	5.85	3.09	—	—	—	—	—	5.85	8
X	1879 - 1889	7.16	7.74	7.05	4.91	4.21	2.55	4.04	2.07	3.°82	—	—	—	—	4.84	9
XI	1890 - 1901	7.27	7.92	7.70	5.36	7.10	2.71	5.83	3.24	6.31	4.°70	4.°70	2.°86	3.°32	5.31	13
XII	1902 - 1913	6.36	6.65	6.48	4.88	5.46	3.14	5.76	2.95	5.78	4.58	5.09	2.61	3.77	4.89	13
XIII	1914 - 1923	7.23	7.50	6.93	5.57	6.04	2.64	6.42	2.91	6.27	4.88	5.70	3.75	4.59	5.42	13
XIV	1924 - 1933	7.47	7.82	7.42	5.66	6.16	1.95	6.56	2.36	6.13	4.21	6.37	2.44	2.72	5.17	13
XV	1934 - 1944	6.89	7.19	6.59	5.89	6.53	2.43	5.14	2.60	5.06	3.35	4.59	2.77	2.48	4.73	13
XVI	1945 - 1953 *	6.76	7.41	6.40	5.66	6.69	3.19	7.23	3.27	5.85	4.56	5.86	2.89	3.32	5.31	13
X_o		6.°75	7.°28	6.°68	5.°09	5.°80	2.°81	5.°86	2.°81	5.°60	4.°33	5.°11	2.°73	3.°27	$\overline{X}_m$ 5.°32	

* Incomplete period.

Apart from the above European stations we considered also 13 stations in the northern hemisphere distributed all over the United States and in various altitudes. Tables VII and VIII give for each period the values of X and ΔR of these stations and also the averages $\overline{X}$ and $\overline{\Delta R}$ corresponding to all the stations considered. In Table IX are given the values of X and $\overline{X}$ corresponding to the 9 stations in the southern hemisphere.

The observation data for these areas show that:

a) The values of $\overline{X}$ corresponding to the 13 stations in the United States show a close correlation with the values of N, for all the periods from V to XIV, the correlation coefficient being $r_{\overline{X},N} = 0.89 \pm 0.02$.

b) In contrast to the European region *no phase difference exists between $\overline{X}$ and N in the United States.*

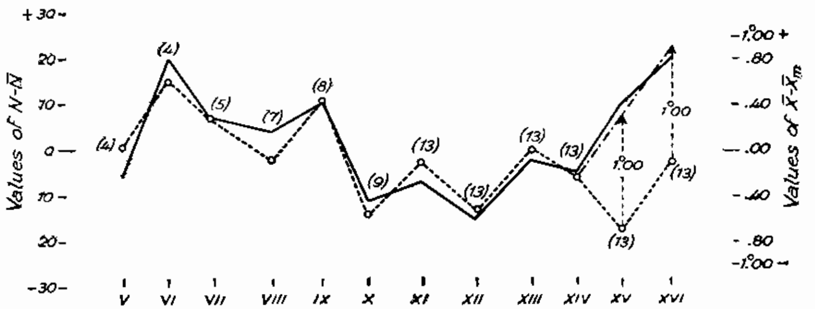

Fig. 10. Values of $N - \overline{N}$ *(continuous line) and* $\overline{X} - X_m$ *(open circles) for 13 stations in the United States, where* $\overline{X}_m$ *is the average of* $\overline{X}$ *for the periods* V *to* XIV. *Over the open circles is written the number of stations to which the average* $\overline{X}$ *refers.*

These results seem to hold for the South America too (Buenos Aires and Santiago) and South Africa (Cape Town, Durban). On the contrary, in Australia and New Zealand, very probably, there exists a phase difference between $\overline{X}$ and N, like that for the European stations. These results, however, referring to the latter three regions of the southern hemisphere, must be accepted with circumspection, because they refer to few stations only and, on the other hand, the periods for which we have thermometrical observations are relatively few.

In Fig. 10 the open circles represent the values of $\overline{X} - \overline{X}_m$ for the United States (13 stations) and the continuous line represents, here

TABLE VIII

Values of $\Delta R = R - R'$ and $\overline{\Delta R}$ for 13 stations of U.S.A.

Periods		NEW HAVEN	NEW YORK	ALBANY	CHARLESTON	ST. LOUIS	SAN DIEGO	SANTA FE	SAN FRANCISCO	DENVER	SALT LAKE CITY	EL PASO	PORTLAND	HELENA	$\overline{\Delta R}$	Number of stations
V	1824 - 1833	5.°18	5.°54	4.°59	1.°49	—	—	—	—	—	—	—	—	—	4.°20	4
VI	1834 - 1843	4.21	4.92	3.96	1.12	—	—	—	—	—	—	—	—	—	3.55	4
VII	1844 - 1856	4.95	4.95	4.95	3.13	3.°04	—	—	—	—	—	—	—	—	4.20	5
VIII	1857 - 1867	4.24	5.34	5.16	2.98	5.16	2.°61	2.°69	0.°52	—	—	—	—	—	3.59	8
IX	1868 - 1878	4.38	4.16	4.26	2.29	3.08	1.94	2.39	0.09	—	—	—	—	—	2.83	8
X	1879 - 1889	5.81	5.56	6.16	3.24	6.82	2.91	4.41	1.38	5.°93	—	—	—	—	4.69	9
XI	1890 - 1901	4.48	4.37	4.44	1.81	2.93	3.22	1.97	0.44	3.74	5.°82	0.°78	4.°56	8.°37	3.61	13
XII	1902 - 1913	5.12	4.97	5.28	1.67	4.44	2.38	0.61	0.97	1.97	4.32	−0.75	4.32	6.12	3.19	13
XIII	1914 - 1923	4.65	4.47	5.58	1.39	3.51	2.74	2.28	0.81	3.12	5.66	0.43	2.81	4.42	3.22	13
XIV	1924 - 1933	3.66	3.79	3.91	2.44	4.22	2.36	1.93	0.41	3.47	6.28	0.26	4.21	6.35	3.33	13
XV	1934 - 1944	4.70	4.60	5.21	0.49	4.58	3.40	3.57	1.18	5.63	8.35	1.87	4.76	9.82	4.47	13
XVI	1945 - 1953 *	4.70	4.39	4.41	−0.43	3.29	2.99	1.87	0.91	5.06	7.22	0.98	5.27	7.72	3.72	13

* Incomplete period.

too, the values of $N - \overline{N}$, where $\overline{X}_m$ and $\overline{N}$ are respectively the averages of $\overline{X}$ and N for the periods V to XIV. Above the open circles is written the number of the stations of the United States to which the values of $\overline{X}$ refer.

In this Figure we see that, while the values of $\overline{X} - \overline{X}_m$ and $N - \overline{N}$ are in a very satisfactory agreement at all periods from V to XIV, during the two last periods (XV and XVI) on the contrary, there appears a noticeable decrease of the difference $\overline{X} - \overline{X}_m$. This is mostly due to a considerable decrease in the values of X during the last two periods, as compared to the values of X during the previous period XIV, at New Haven, New York and Albany on the one hand and Denver, El Paso, Santa Fe and Salt Lake City on the other.

As we can easily see from Table X, the values of X at the above mentioned stations present, in fact, a decrease of about 1° C during the period XV, as compared with those during the previous one XIV; whereas an increase of the values of X of about 0°.3 C appears at the other stations. Similarly, during the period XVI and at the above stations — except Santa Fe and Salt Lake City — we have a decrease of about 0°.6 C in the values of X, as compared with those corresponding to the period XIV, while at the other stations we have an increase of about 0°.6 C.

Namely, we have a disagreement here between $\overline{X}$ and N, as was the case in Europe during the period VIII/VII, where the value of $\overline{X}$ instead of showing a decrease, if compared with that of the previous period, showed an increase contrary to the general relationship between $\overline{X}$ and N.

A disagreement between the valnes of $\overline{X}$ and N also appears in Central England* (see Fig. 11), where, during the period XIII/XII we have a maximum of $\overline{X}$ instead of the expected minimum.

If, however, we consider the average $\overline{X}_{Gen}$ of the values of $\overline{X}$ corresponding to Europe, Central England and the United States— always observing the phase difference between Europe and Central England on the one hand and the United States on the other—it is remarkable that these disagreements mostly disappear. In fact, in

* The values of $\overline{X}$ for Central England were calculated from the Table of the mean monthly air temperatures given by Prof. G. Manley (1953). For this area, within the period X is included the year 1890, i. e. X: 1879 - 1890.

TABLE IX

Valnes of X and $\overline{X}$ for 9 stations in the Southern Hemisphere

Periods		New Zealand, Australia						South Africa			South America		
		AUCKLANT	WELLINGTON	DUNEDIN	SYDNEY	ADELAIDE	$\overline{X}$	BUENOS-AIRES	SANTIAGO	$\overline{X}$	CAPE TOWN	DURBAN	$\overline{X}$
IX	1868 - 1878	3.°03	2.°27	1.°80	2.°52	2.°91	2.°51	—	—	—	—	—	—
X	1879 - 1889	2.72	2.17	1.45	2.47	2.87	2.34	3.°04	1.°71	2.°38	—	—	—
XI	1890 - 1901	2.59	1.75	0.98	1.55	2.97	1.97	3.90	2.40	3.15	2.°92	2.°42	2.°67
XII	1902 - 1913	2.83	2.44	1.52	2.36	2.41	2.30	3.62	1.82	2.72	2.78	1.75	2.29
XIII	1914 - 1923	2.12	1.88	1.09	2.08	2.89	2.01	3.68	3.10	3.39	3.22	1.91	2.57
XIV	1924 - 1933	3.09	2.59	1.83	2.08	3.08	2.53	3.82	2.16	2.99	2.93	1.83	2.38

TABLE X

Values of $X_{xv} - X_{xiv}$ and $X_{xvi} - X_{xiv}$ for 13 stations of U. S. A.

Periods	NEW HAVEN	NEW YORK	ALBANY	DENVER	EL PASO	SANTA FE	SALT LAKE CITY	HELENA	CHARLESTON	ST. LOUIS	SAN DIEGO	SAN FRANCISCO	PORTLAND
XV —XIV	—0.°58	—0.°63	—0.°90	—1.°07	—1.°78	—1.°42	—0.°86	—0.°24	+0.°23	+0.°37	+0.°48	+0.°24	+0.°33
XVI—XIV	— 0.71	— 0.41	— 1.09	— 0.28	— 0.51	+ 0.67	+ 0.35	+ 0.60	0.00	+ 0.53	+ 1.24	+ 0.91	+ 0.45

Fig. 12 the open circles represent the values of $(\overline{X} - \overline{X}_m)_{Gen}$ given by the formula:

$$(\overline{X} - \overline{X}_m)_{Gen} = \frac{1}{9}\left[4\,(\overline{X} - \overline{X}_m)_{Eur.} + 4\,(\overline{X} - \overline{X}_m)_{U.S.A.} + (\overline{X} - \overline{X}_m)_{C.\,Engl.}\right]$$

and the continuous line the values of N — N.

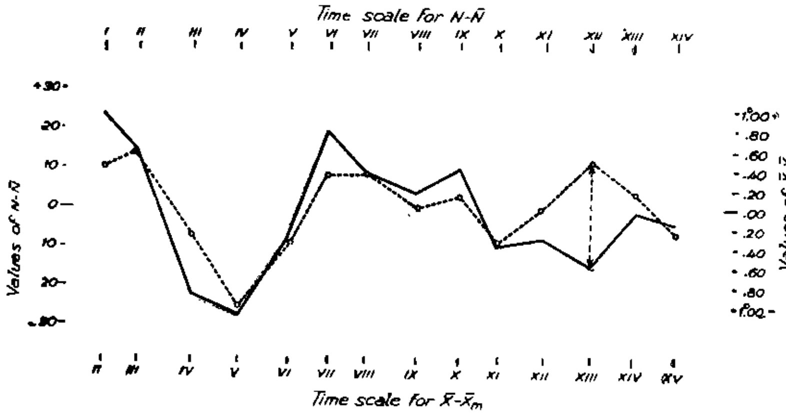

Fig. 11. Values of $N - \overline{N}$ *(continuous line) and* $\overline{X} - \overline{X}_m$ *(open circles) for Central England, where* $\overline{X}_m$ *is the average of* $\overline{X}$ *for the periods* I *to* XV. *The origin of the time scale for* $\overline{X} - \overline{X}_m$ *is shifted a whole period to the left.*

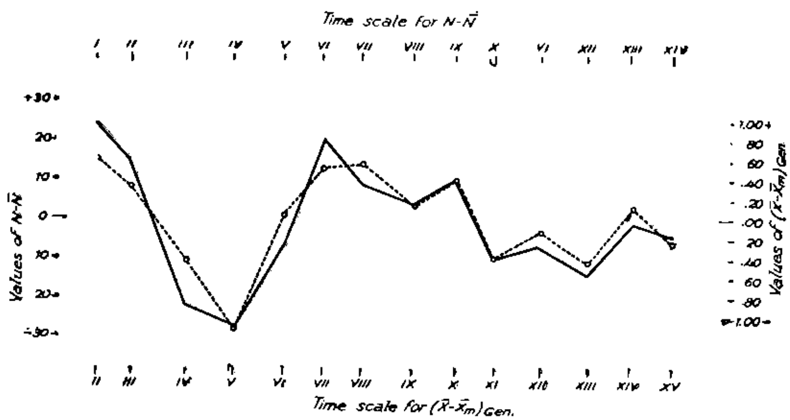

Fig. 12. Values of $N - \overline{N}$ *(continuous line) and* $(\overline{X} - \overline{X}_m)_{Gen}$ *(open circles).*

In this Figure it can be seen that those disagreements, appearing in the various areas, if they are taken separately, will vanish to a large extent; and that the course of the differences $(\overline{X}-\overline{X}_m)_{Gen}$ and $N-\overline{N}$ is more or less the same, the correlation coefficient taking a very high value ($r_{\overline{X},N} = 0.94 \pm 0.01$).

7. THE DIFFERENCE $T'_{eq}-T_{eq}$

The phase difference between Europe and the United States is observed not only in the values of $\overline{X}$ but also in the averages of the difference $T'_{eq}-T_{eq}$. The average $\overline{T'_{eq}-T_{eq}}$ of this difference for each period can be easily calculated from Tables IV, V, VII and VIII by the relation:

$$\overline{T'_{eq}-T_{eq}} = \overline{X} + \tfrac{1}{2}\,\overline{\Delta R}$$

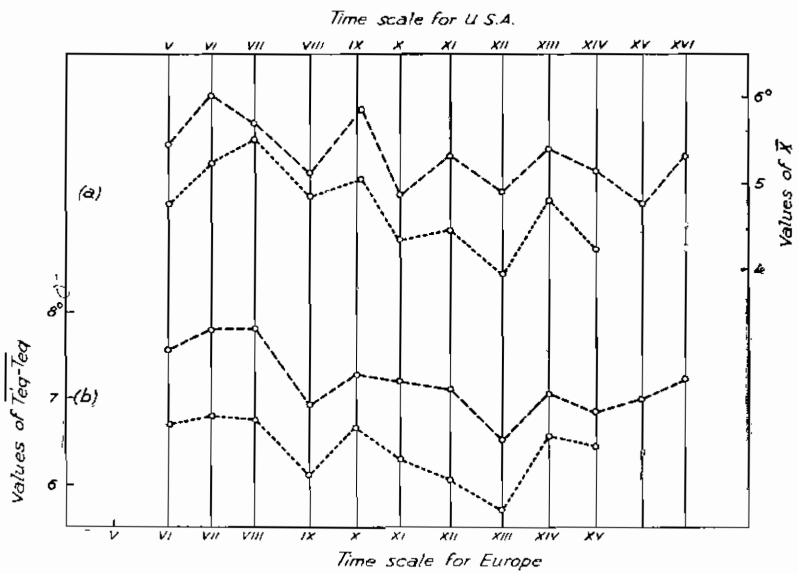

Fig. 13 a. Values of $\overline{X}$ for the United States (top) and Europe (bottom).
b. Values of $\overline{T'_{eq}-T_{eq}}$ for the United States (top) and Europe (bottom).

The above stated phase difference between the values of $\overline{X}$ and $\overline{T'_{eq}-T_{eq}}$ corresponding to Europe and the United States is represented by Fig. 13 a, b, where the upper circles in (a) and (b) represent respectively the values of $\overline{X}$ and $\overline{T'_{eq} - T_{eq}}$ for the United States and the lower circles the values of these quantities for Europe.

If we except the period VIII/VII during which a disagreement is observed between the values of $\overline{X}$ for Europe and the United States, in all the other periods the values of this quantity corresponding to these two areas present an almost analogons course. This analogy is even more striking if we consider the values of the quantity $\overline{T'_{eq} - T_{eq}}$. Indeed, for the periods V to XV the values of the ratio:

$$(\overline{T'_{eq} - T_{eq}})_{U.S.A.} : (\overline{T'_{eq} - T_{eq}})_{Eur.}$$

lie between 1.07 and 1.17. We therefore think it most improbable that this phase difference between Europe and U.S.A. should be fortuitous and without any physical significance.

This observed phase difference, which probably also exists between other regions of the Earth, discloses the fact that the atmosphere responds differently to the same impulse, from region to region. On the other band, if we consider that X represents, formally, the range of the differences $T_{13-i} - T_i$ and that these differences depend on the absorption by the lower atmosphere layers of the radiation of the Earth surface, then it is probable that this quantity X is directly or indirectly related to the amount of the water vapours and other elements present at each time in the lower atmosphere, on which its transparency coefficient depends. In this case the above stated relationship must arise from a relation between the solar activity expressed by the mean annual sunspot number and the atmospheric transparency.

But all these are no more than conjectures. The only thing certain for the present is the relationship between X and N given by the observation data for Europe and the United States, to the discovery of which we were led by the developments [3] and [4] of the mean monthly air temperatures. This relationship, as our preliminary investigations show, does not seem to hold within the 11-year sunspot cycle, where the quantity X generally presents a double fluctuation, with two or more maxima and minima. But in this case too, the course of the maxima and minima of X agrees satisfactorily with that of the mean annual sunspot number N corresponding to the successive periods, always with a phase difference of a whole period for the 13 stations of Europe and the Near East studied before. This problem is going to be the subject of special investigations on our part.

On the other hand, of the six constants on which the mean monthly air temperatures depend (see developments [3] and [4]), only the parameter P and the eccentricity e, as well as their functions Q_o and e_o seem to be correlated with sunspots. The other constants A, C, V and W do not present any correlation with the annual sunspot number. Consequently, the effect of the variations of e and P on the developments [3] and [4] from one 11-year period to the next is altered by the variations of the other constants that are not dependent on the sunspots. Perhaps we can so explain why all to date attempts at finding a clear relationship between sunspots and air temperature have failed.

Acknowledgments

The writer wishes to express his sincere thanks to the National Meteorological Services of the Countries to which the stations considered here belong for their kindness to put at his disposal the observation data of the air temperature from 1934 onwards.

Also to Mr. Harold Davies, M. A., Professor of English and Mrs. Ariadne Koumari, M. A., Lecturer in English at the Institute of Foreign Languages and Literatures, University of Thessaloniki, who kindly read and corrected the English text.

REFERENCES

Birkeland, B.	1949	*Old Meteorological Observations at Trondheim. Atmospheric Pressure and Temperature During 185 Years.* (Geofysiske Publikasjoner, ***XV*, 4**).
Clayton, H.	1944 1934 1947	*World Weather Records.* Smithson. Misc. Coll., ***79*, 90, 105.**
Hlavàč, G.	1940	*Met. Z.*, **57**, 267.
Manley, G.	1953	*Quart. J. R. Meteorol. Soc.*, **79**, 242.
Mavridis, L.	1954a	*Praktika de l'Acad. d'Athènes*, **29**, 97.
	1954b	*On the Mathematical Expression of the Annual Variation of Air Temperature.* (Meteorologica Univ. Thessaloniki, **3**).
Schindler, G.	1948	*Z. f. Met.* **2**, 280.
Stetson, H.	1947	*Sunspots in Action.* (New York: Ronald Press Co).
Xanthakis, J.	1948	*Relation Between the Mean Monthly Air Temperatures in the Temperate Zones.* (Thessaloniki: University of Thessaloniki Press). Also *Bull. Amer. Met. Soc.*, **29**, 550.
	1952	*Praktika de l'Acad. d'Athènes*, **27**, 168.
	1953	*New Relations Between the Mean Monthly Air Temperatures.* (Thessaloniki: University of Thessaloniki Press).