Sci. Annals, Fac. Phys. & Mathem., Univ. Thessaloniki, 14, 175, (1974)

STATE DEPENDENT RADIAL DISTRIBUTION FUNCTIONS FOR INFINITELY EXTENDED FERMI SYSTEMS

by

E. MAVROMMATIS

(Physics Department, N.R.C. «Demokritos», Aghia Paraskevi Attikis, Athens, Greece)

(Introduced by Prof. M. E. Grypeos)

(Received 14.6.1974)

Abstract: *A generalized expression of the state dependent radial distribution functions $G_1(r_{12})$ for infinitely extended fermi systems are given by employing a cluster expansion formalism. A comparison is also made between the derived second cluster term and the eorresponding term in the usual Iwamoto-Yamada formalism.*

The obtained results are mainly uscful in computing the three-body terms in the energy expression of fermi systems in their ground state, when state dcpendent two-body potentials are employed.

1. INTRODUCTION

The state independent radial distribution function for a uniform many particle system is generally defined by:

$$g(\vec{r}_1,\vec{r}_2) = \frac{\rho(\vec{r}_1,\vec{r}_2)}{\rho^2} = \frac{N(N-1)}{\rho^2} \sum_{\substack{\text{spin -}\\ \text{isospin}}} \int \Psi^*_N \Psi_N d\vec{r}_3 \dots d\vec{r}_N \qquad (1)$$

where ρ is the density of the system, Ψ_N its normalized wavefunction and $\rho(\vec{r}_1, \vec{r}_2)$ the pair distribution function which gives the probability of finding any two particles of the system at positions $\vec{r}_1$ and $\vec{r}_2$. If the probability for a particular two body spin-isospin state is needed, appropriate projection operators O_l are introduced between the wavefunctions and the state dependent pair distribution functions $\rho_l(\vec{r}_1, \vec{r}_2)$ and radial distribution functions $g_l(\vec{r}_1, \vec{r}_2)$ are obtained:

$$g_l(\vec{r}_1,\vec{r}_2) = \frac{\rho_l(\vec{r}_1,\vec{r}_2)}{\rho^2} = \frac{N(N-1)}{\rho^2} \sum_{\substack{\text{spin -}\\ \text{isospin}}} \int \Psi^*_N O_l \Psi_N d\vec{r}_3 \dots d\vec{r}_N \qquad (2)$$

The evaluation of $g(\vec{r}_1, \vec{r}_2)$ and $g_i(\vec{r}_1, \vec{r}_2)$, which are taken to depend upon $|\vec{r}_1 - \vec{r}_2| = r_{12}$ is not in general possible. If, however, a Jastrow-type wavefunction is assumed, the radial distribution functions can be clustered up using several formalisms [1-5], reviewed in a recent monograph [6]. Provided that good convergence of the cluster series can be achieved, the first few terms of them may approximate the corresponding g or g_i's quite satisfactorily. It is the purpose of this paper to give a generalized expression of the state dependent radial distribution functions in which the three body terms are included. This expression is derived in the next section. In the final section a comparison is made with the corresponding Iwamoto-Yamada expression and a discussion is given about the usefulness of the obtained results.

2. DERIVATION OF THE EXPRESSION FOR THE STATE DEPENDENT RADIAL DISTRIBUTION FUNCTIONS

As it was stated in the introduction, we deal with the derivation of g_i's for a fermion system in its ground state, described by the following trial wavefunction :

$$\Psi_N = \frac{\prod_{i<j} f(r_{ij})\,\Phi}{\left\{\int \prod_{b=1}^{N} dx_b \Phi^* \prod_{i<j} f^2(r_{ij})\Phi\right\}^{1/2}} \tag{3}$$

where x represents spatial, spin-isospin coordinates, $f(r_{ij})$ is the two-body correlation function and Φ is the Slater determinant of N single particle wavefunctions $\varphi_j(x)$, defined by

$$\varphi_j(x) = \frac{e^{j\vec{k}_j \cdot \vec{r}}}{\sqrt{\Omega}} X_{1j} \tag{4}$$

Ω is the volume occupied by the system and X_{1j} is the spin-isospin wavefunction. Following Aviles [1] we consider the corresponding G-distribution functions given by :

$$G(r_{12}) = \frac{N(N-1)}{\rho^2 f^2(r_{12})} \int \Psi^*_N \Psi_N d\vec{r}_3 \ldots d\vec{r}_N \tag{5a}$$

or equivalently

$$G(r_{12}) = \frac{1}{\rho^2 f^2(r_{12})} \int \prod_{b=1}^{N} dx'_b \Psi^*_N \left\{ \sum_{p',q'} \delta(\vec{r}_1 - \vec{r}_{p'})\delta(\vec{r}_2 - \vec{r}_{q'}) \right\} \Psi_N \tag{5b}$$

and

$$G_i(r_{12}) = \frac{1}{\rho^2 f^2(r_{12})} \int \prod_{b=1}^{N} dx_b' \Psi^*_N \{ \sum_{p',q'} \delta(\vec{r}_1 - \vec{r}_{p}') \delta(\vec{r}_2 - \vec{r}_{q}') D_i(p',q') \Psi_N \} \quad (6)$$

where $\mathbf{D}_i(p', q')$ represent projection operators to the i-spin-isospin state. In the case of nuclear matter, for example, we have the following states as i runs from 1 to 4 : singlet odd, singlet even, triplet odd and triplet even. The $\mathbf{D}_i$'s are expressed in terms of the spin and isospin operators $\vec{\sigma}$ and $\vec{\tau}$ respectively as :

$$\mathbf{D}_1 = \frac{1}{16} (1 - \vec{\tau}.\vec{\tau}).(1 - \vec{\sigma}.\vec{\sigma}), \ \mathbf{D}_2 = \frac{1}{16} (3 + \vec{\tau}.\vec{\tau})(1 - \vec{\sigma}.\vec{\sigma})$$
$$\mathbf{D}_3 = \frac{1}{16} (3 + \vec{\tau}.\vec{\tau})(3 + \vec{\sigma}.\vec{\sigma}), \ \mathbf{D}_4 = \frac{1}{16} (1 - \vec{\tau}.\vec{\tau})(3 + \vec{\sigma}.\vec{\sigma}) \quad (7)$$

The cluster expansion for $G(r_{12})$ is given in ref. (1)

$$G(r_{12}) = G^{(1)}(r_{12}) + G^{(2)}(r_{12}) + \dots \quad (8)$$

where

$$G^{(1)}(r_{12}) = 1 - \frac{l^2(k_F r_{12})}{s} \quad (9a)$$

$$G^{(2)}(r_{12}) = - \frac{2\rho}{s^2} l^2(k_F r_{12}) \int h(r_{13}) l^2(k_F r_{13}) d\vec{r}_3 - \frac{2\rho}{s} \int h(r_{13}) l^2(k_F r_{23}) d\vec{r}_3 +$$
$$+ \frac{4\rho}{s^2} l(k_F r_{12}) \int h(r_{13}) l(k_F r_{13}) l(k_F r_{23}) d\vec{r}_3 + \rho \int h(r_{13}) h(r_{23}). \Big[1 - \frac{1}{s} (l^2(k_F r_{12}) + \quad (9b)$$
$$+ l^2(k_F r_{13}) + l^2(k_F r_{23})) + \frac{2}{s^2} l(k_F r_{12}) l(k_F r_{13}) l(k_F r_{23}) \Big] d\vec{r}_3$$

In the above formulae $l(x) = \frac{3(\sin x - x\cos x)}{x^3}$, $h(r_{ij}) = f^2(r_{ij}) - 1$, k_F the fermi momentum and s the degeneracy of each orbital state.

The cluster expansion of G_i's in the FAHT formalism ([4]) is

$$G_i(r_{12}) = \frac{\sum_{k=1}^{N} {}_kC_N R_{ik}(r_{12})}{\rho^2 f^2(r_{12})} \quad (10)$$

where ${}_kC_N$ stands for $\binom{N}{k}$ and $R_{ik}(r_{12})$ is given by :

$$R_{1k}(r_{12}) = \sum_{p=2}^{k} \frac{k!(-1)^{k-p}}{p!(k-p)!} \sum_{\langle j_1 \cdots j_p \rangle} \sum_{\substack{\text{spin-} \\ \text{isospin}}} \int d\vec{r}_1{}' \ldots d\vec{r}_N{}' \Psi^*_{\langle j_1 \cdots j_p \rangle}(x') .$$

$$\cdot \left\{ \sum_{\substack{m',l' \\ m' \neq l'}} \delta(\vec{r}_1 - \vec{r}_{m'}) \delta(\vec{r}_2 - \vec{r}_{l'}) \, \mathbf{D}_i(m',l') \right\} \Psi_{\langle j_1 \cdots j_p \rangle}(x') \tag{11}$$

The $\langle j_1 \ldots j_p \rangle$ represents the arrangements of N objects into p and $\Psi_{< j_1 \ldots j_p >}(x')$ is the (pxp) Slater determinant of φ_{j_i}'s multiplied by an appropriate product of f's. The corresponding cluster expansion in AHT method, in which we are interested, will be obtained from the above expressions by letting $N \longrightarrow \infty$, (and $\Omega \longrightarrow \infty$, keeping ρ constant) since the two formalisms are identical for a uniform, infinitely extended, system.

The first cluster term has been calculated elsewhere explicitly [7,4]. We are interested in calculating the second cluster term which is obtained from (10) for $k = 3$. We have :

$$G_j^{(2)}(r_{12}) = \frac{{}_3C_N R_{13}(r_{12})}{\rho^2 f^2(r_{12})} \tag{12}$$

with

$$R_{13}(r_{12}) = (-1)\,3 \frac{\displaystyle\sum_{\langle j_1 j_2 \rangle} \frac{1}{2!} \int dx_1' dx_2' S_2^*(1',2') f^2(1'2) \left\{ \sum_{\substack{m'=1',l'=1' \\ m' \neq l'}}^{2'} \delta(\vec{r}_1 - \vec{r}_{m'}) \delta(\vec{r}_2 - \vec{r}_{l'}) \, \mathbf{D}_i(m',l') \right\} S_2(1',2')}{\displaystyle\sum_{\langle j_1 j_2 \rangle} \frac{1}{2!} \int dx_1' dx_2' f^2(1'2) \, [S_2(1',2')]^2} +$$

$$+ \frac{\displaystyle\sum_{\langle j_1 j_2 j_3 \rangle} \frac{1}{3!} \int dx_1' dx_2' dx_3' S_3^*(1',2',3') f^2(1'2) f^2(1'3) f^2(2'3) \cdot \left\{ \sum_{\substack{m'=1' \ l'=1' \\ m' \neq l'}}^{3'} \sum^{3'} \delta(\vec{r}_1 - \vec{r}_{m'}) \delta(\vec{r}_2 - \vec{r}_{l'}) \mathbf{D}_j(m',l') \right\} . S_3(1',2',3')}{\displaystyle\sum_{\langle j_1 j_2 j_3 \rangle} \frac{1}{3!} \int dx_1' dx_2' dx_3' [S_3(1',2',3')]^2 f^2(1'2) f^2(1'3) f^2(2'3)} \tag{13}$$

where

$$S_2(1',2') = \begin{vmatrix} \varphi_{j_1}(1') & \varphi_{j_1}(2') \\ \varphi_{j_2}(1') & \varphi_{j_2}(2') \end{vmatrix}, \quad S_3(1',2',3') = \begin{vmatrix} \varphi_{j_1}(1') & \varphi_{j_1}(2') & \varphi_{j_1}(3') \\ \varphi_{j_2}(1') & \varphi_{j_2}(2') & \varphi_{j_2}(3') \\ \varphi_{j_3}(1') & \varphi_{j_3}(2') & \varphi_{j_3}(3') \end{vmatrix}$$

For convenience, the numerator and denominator of the first and second term have been multiplied by $\frac{1}{2!}$ and $\frac{1}{3!}$ respectively and the $f(r_{ij})$ and $\varphi_{j_1}(x_i)$ have been denoted by $f(ij)$ and $\varphi_{j_i}(i)$. Suitably transforming the denominators of these expressions by taking into account that $\langle j_1j_2\rangle = {}_2C_N 2!$ and $\langle j_1j_2j_3\rangle = {}_3C_N 3!$ we obtain :

$$R_{13}(r_{12}) = \frac{1}{{}_3C_N}\left\{ \frac{\frac{{}_3C_N}{{}_2C_N}(-1)3\sum\limits_{\langle j_1j_2\rangle}\frac{1}{2!}\int dx_1'dx_2'S_2^*(1',2')f^2(1'2)}{1+\frac{2!}{N(N-1)}\sum\limits_{\langle j_1j_2\rangle}\frac{1}{2!}\int dx_1'dx_2'(f^2-1)[S_2(1',2')]^2} \right.\cdot$$

$$\left\{\sum_{\substack{m',1'\\m'\neq 1'}}^{2'}\delta'(\vec{r}_1-\vec{r}_{m}')\delta(\vec{r}_2-\vec{r}_1')\,\mathbf{D}_i(m',l')\right\}[S_2(1',2')] +$$

$$+\frac{\sum\limits_{<j_1j_2j_3>}\frac{1}{3!}\int dx_1'dx_2'dx_3'S_3^*(1',2',3')f^2(1'2)f^2(1'3)f^2(2'3)\,\left\{\sum\limits_{\substack{m',l'\\m'\neq l'}}^{3'}\delta(\vec{r}_1-\vec{r}_m')\delta(\vec{r}_2-\vec{r}_l')\,\mathbf{D}_i(m',l')\right\}\cdot S_3(1',2',3')}{1+\frac{3!}{N(N-1)(N-2)}\sum\limits_{\langle j_1j_2j_3\rangle}\frac{1}{3!}\int dx_1'dx_2'dx_3'[S_3(1',2',3')]^2\cdot[f^2(1'2)f^2(1'3)f^2(2'3)-1]}\right\}$$

The numerators $N_{i1}(r_{12})$, $N_{i2}(r_{12})$ and denominators DN_{i1}, DN_{i2} of the first and second term will be considered separately.

By inserting the form (4) for φ_{j_1}, performing the sum over $\langle j_1j_2\rangle$ and simplifying the sum over δ's by some symmetry arguments, the expression of the first numerator takes the form :

$$N_{i1}(r_{12}) = -\frac{(N-2)}{2}\frac{1}{\Omega^2}\sum\sum_{\vec{k}_j,\vec{k}_j l_{j_1},l_{j_2}=1}^{s}\int d\vec{r}_1'd\vec{r}_2'f^2(1'2)\delta(\vec{r}_1-\vec{r}_1')\delta(\vec{r}_2-\vec{r}_2').$$

$$[\langle Xl_{j_1}(1')Xl_{j_2}(2')|\mathbf{D}i(1'2')|Xl_{j_1}(1')Xl_{j_2}(2')\rangle - e^{i\vec{k}_{j_1}\cdot(\vec{r}_1'-\vec{r}_2')}e^{i\vec{k}_{j_2}\cdot(\vec{r}_2'-\vec{r}_1')}. \qquad (15)$$

$$\cdot \langle Xl_{j_1}(2')Xl_{j_2}(1')|\mathbf{D}_i(2'1')|Xl_{j_1}(1')Xl_{j_2}(2')\rangle - e^{i\vec{k}_{j_1}\cdot(\vec{r}_2'-\vec{r}_1')}\, e^{i\vec{k}_{j_2}\cdot(\vec{r}_1'-\vec{r}_2')} \cdot$$
$$\cdot \langle Xl_{j_1}(1')Xl_{j_2}(2')|\mathbf{D}_i(1'2')|Xl_{j_1}(2')Xl_{j_2}(1')\rangle +$$
$$+ \langle Xl_{j_1}(2')Xl_{j_2}(1')|\mathbf{D}_i(2'1')|Xl_{j_1}(2')Xl_{j_2}(1')\rangle]$$

Using the formula : $\sum_{k_i}^{\leqslant k_F} e^{i\vec{k}_i\cdot\vec{r}_{12}} = \frac{N}{s}\, l(k_F r_{12})$, this reduces to :

$$N_{l_1}(r_{12}) = -(N-2)\frac{\rho^2}{s^2} f^2(12)\left\{\Delta_i^{(d)} - \Delta_i^{(e)} l^2(k_F r_{12})\right\} \tag{16}$$

where the statistical factors $\Delta_i^{(d)}$ and $\Delta_i^{(e)}$ are defined as follows :

$$\Delta_i^{(d)} = \sum_{l_{j_1}, l_{j_2}} \langle Xl_{j_1}(1)Xl_{j_2}(2)|\mathbf{D}_i(12)|Xl_{j_1}(1)Xl_{j_2}(2)\rangle \tag{17a}$$

$$\Delta_i^{(e)} = \sum_{l_{j_1}, l_{j_2}} \langle Xl_{j_1}(1)Xl_{j_2}(2)|\mathbf{D}_i(12)|X_{j_2}(1)Xl_{j_1}(2)\rangle \tag{17b}$$

The values of $\Delta_i^{(d)}$ and $\Delta_i^{(e)}$ for $s = 4$ and $s = 2$ are given in Tables 1 and 2 respectively. It can be realized from expression (10), that the above evaluated quantity (16) divided by $f^2(12)$ is equal apart from a multiplicative constant to the first cluster term $G_i^{(1)}(r_{12})$:

$$G_i^{(1)}(r_{12}) = \frac{1}{s^2}\left\{\Delta_i^{(d)} - \Delta_i^{(e)} l^2(k_F r_{12})\right\} \tag{18}$$

By analyzing the sum in parentheses in the second numerator :

$$\sum_{\substack{m',l'=1\\ m'\neq l'}}^{3} \delta(\vec{r}_1 - \vec{r}_m{}')\delta(\vec{r}_2 - \vec{r}_l{}')\mathbf{D}_i(m',l') = 2!\left\{\delta(\vec{r}_1 - \vec{r}_1{}')\delta(\vec{r}_2 - \vec{r}_2{}')\mathbf{D}_i(1'2') + \right.$$
$$\left. + \delta(\vec{r}_1 - \vec{r}_1{}')\delta(\vec{r}_2 - \vec{r}_3{}')\mathbf{D}_i(1'3') + \delta(\vec{r}_1 - \vec{r}_2{}')\delta(\vec{r}_2 - \vec{r}_3{}')\mathbf{D}i(2'3')\right\}$$

and performing calculations only for the first term in the sum N_{l21}, in a way similar to that of the derivation of $N_{l1}(r_{12})$ one obtains :

$$N_{l21}(r_{12}) = \frac{\rho^3}{6s^2}\int d\vec{r}_1{}'d\vec{r}_2{}'d\vec{r}_3{}' f^2(1'2')\delta(\vec{r}_1 - \vec{r}_1{}')\delta(\vec{r}_2 - \vec{r}_2{}')\cdot[1 + h(1'3) +$$
$$(19)$$
$$+ h(2'3) + h(1'3)h(2'3)]\cdot[\Delta_i^{(d)} - \Delta_i^{(e)} l^2(k_F r'_{12}) - \frac{\Delta_i^{(d)}}{s} l^2(k_F r'_{13}) -$$

$$-\frac{\Delta_i^{(d)}}{s}\, l^2(k_F r'_{23}) + \frac{2\Delta_i^{(e)}}{s}\, l(k_F r'_{12}) \cdot l(k_F r'_{13}) l(k_F r'_{23})]$$

The two other terms in the sum can be deduced from the above formula by symmetry considerations and we are left with the same function of r_{12}. Using the formulae $\frac{\rho}{s}\int d\vec{r}_3 l^2(k_F r_{13}) = 1$ and

$$\frac{\rho}{s}\int d\vec{r}_3 l(k_F r_{13}) l(k_F r_{23}) = l(k_F r_{12})$$

we obtain for $N_{i2}(r_{12})$

$$N_{12}(r_{12}) = \frac{\rho^2}{s^2} f^2(r_{12}) \{(N-2)(\Delta_i^{(d)} - l^2(k_F r_{12})\Delta_i^{(e)}) + \rho\int d\vec{r}_3\, [h(r_{13}) + h(r_{23}) +$$

$$+ h(r_{13})h(r_{23})]\, [\Delta_i^{(d)} - \Delta_i^{(e)} l^2(k_F r_{12}) - \frac{\Delta_i^{(d)}}{s}(l^2(k_F r_{13}) + l^2(k_F r_{23})) + \quad (20)$$

$$+ \frac{2\Delta_i^{(e)}}{s} l(k_F r_{12}) l(k_F r_{13}) l(k_F r_{23})]\}$$

The first denominator DN_{i1} can be expanded in powers of $\left(\frac{1}{N^2}\right)$

$$DN_{i1} = 1 - \frac{\rho^2}{N(N-1)}\int d\vec{r}_1 d\vec{r}_2 h(r_{12}) \left[1 - \frac{l^2(k_F r_{12})}{s}\right] + O\left(\frac{1}{N^4}\right) \quad (21)$$

The second denominator DN_{i2} is also evaluated similarly to $N_{i2}(r_{12})$ and yields :

$$DN_{i2} = 1 - \frac{\rho^3}{N(N-1)(N-2)}\int d\vec{r}_1 d\vec{r}_2 d\vec{r}_3\, [h(r_{12}) + h(r_{13}) + h(r_{23}) +$$

$$h(r_{13})h(r_{13}) + h(r_{12})h(r_{23}) + h(r_{13})h(r_{23}) + h(r_{12})h(r_{23})h(r_{31})] \cdot \left[1 - \frac{l^2(k_F r_{12})}{s} - \quad (22)\right.$$

$$\left. - \frac{l^2(k_F r_{13})}{s} - \frac{l^2(k_F r_{23})}{s} + \frac{2}{s^2} l(k_F r_{12}) l(k_F r_{13}) l(k_F r_{23})\right] + O(1/N^6)$$

By combining formulae (16), (20), (21), (22) and using the fact that for infinitely extended systems

$$\frac{N-2}{N(N-1)}\rho^2\int d\vec{r}_1 d\vec{r}_2 h(r_{12}) \left(1 - \frac{l^2(k_F r_{12})}{s}\right) = \rho\int d\vec{r}_{12} h(r_{12}) \left(1 - \frac{l^2(k_F r_{12})}{s}\right) +$$

$$+ O(1/N)$$

we finally get for $G_i^{(2)}(r_{12})$, neglecting terms $O(1/N)$:

$$G_i^{(2)}(r_{12}) = \frac{1}{s^2}\left\{ -2\Delta_i^{(e)}\frac{l^2(k_F r_{12})}{s}\rho\int d\vec{r}_3 h(r_{13}) l^2(k_F r_{13}) - \right.$$

$$-2\rho\frac{\Delta_i^{(d)}}{s}\int d\vec{r}_3 h(r_{13}) l^2(k_F r_{23}) + 4\rho\frac{\Delta_i^{(e)}}{s} l(k_F r_{12})\int d\vec{r}_3 h(r_{13}) l(k_F r_{13}) l(k_F r_{23}) + \tag{23}$$

$$+\rho\int d\vec{r}_3 h(r_{13}) h(r_{23})\left[\Delta_i^{(d)} - \Delta_i^{(e)} l^2(k_F r_{12}) - \frac{\Delta_i^{(d)}}{s}[l(k_F r_{13}) + l(k_F r_{23})] + \right.$$

$$\left.\left. + \frac{2\Delta_i^{(e)}}{s} l(k_F r_{12}) l(k_F r_{13}) l(k_F r_{23})\right]\right.$$

Substitution of the values of $\Delta_i^{(d)}$ and $\Delta_i^{(e)}$ from Tables 1 and 2 yields the explicit forms of $G_i^{(2)}$ for different systems. Expression (23) is a generalization of the corresponding expression of ref. 4. As expected, the sum of G_i's equals the second term of the state independent radial distribution function $G^{(2)}(r_{12})$ as given by (9b) :

$$\sum_i G_i^{(2)}(r_{12}) = G^{(2)}(r_{12}) = -\frac{2\rho}{s^2} l^2(k_F r_{12})\int h(r_{13}) l^2(k_F r_{13}) d\vec{r}_3 - \frac{2\rho}{s}\int h(r_{13}) l^2(k_F r_{23}) d\vec{r}_3 +$$

$$+\frac{4\rho}{s^2} l(k_F r_{12})\int h(r_{13}) l(k_F r_{13}) l(k_F r_{23}) d\vec{r}_3 + \rho\int h(r_{13}) h(r_{23})\left[1 - \frac{1}{s}(l^2(k_F r_{12}) + l^2(k_F r_{13}) + \right.$$

$$\left. + l^2(k_F r_{23})) + \frac{2}{s^2} l(k_F r_{12}) l(k_F r_{13}) l(k_F r_{23})\right] d\vec{r}_3 \tag{24}$$

3. COMPARISON AND DISCUSSION·

It is useful to compare the derived expression (23) for $G_i^{(2)}(r_{12})$ with the second cluster term of $g_i(r_{12})$ in IY cluster expansion [3,6], which can be obtained either by a truncation of a cluster series for $g_i(r_{12})$ generated by IY's procedure or by isolation of the terms involving the potential $V_i(r_{12})$ in the energy series. Using this latter method in the energy expansion given by ref. (8) we get :

$$g_i^{(2)}(r_{12}) = \frac{f^2(r_{12})}{s^2}\left\{ -\frac{\rho}{s}\Delta_i^{(d)}\int d\vec{r}_3 h(r_{13}) l^2(k_F r_{23}) + \frac{\rho}{s}\Delta_i^{(e)} l(k_F r_{12})\int d\vec{r}_3 h(r_{13}) l(k_F r_{13}) l(k_F r_{23})\right.$$

$$+ \rho \int d\vec{r}_3 h(r_{13}) h(r_{23}) \Big\{ \Delta_i^{(d)} - \Delta_i^{(e)} l^2(k_F r_{12}) - \frac{\Delta_i^{(d)}}{s} [l^2(k_F r_{13}) + l^2(k_F r_{23})] + $$

$$+ \frac{2\Delta_i^{(e)}}{s} l(k_F r_{12}) l(k_F r_{13}) l(k_F r_{23}) \Big\} \cdot \qquad (25)$$

We realize, comparing expressions (23) and (25), that although the first cluster terms of the above expansions are identical, the second term in AHT expansion includes three more terms :

$$- \frac{2\rho \Delta_i^{(e)}}{s^3} l^2(k_F r_{12}) \int d\vec{r}_3 h(r_{13}) l^2(k_F r_{13})$$

$$- \frac{\rho}{s^3} \Delta_i^{(d)} \int d\vec{r}_3 h(r_{13}) l^2(k_F r_{23}) \qquad (26)$$

$$\frac{3\rho}{s^3} \Delta_i^{(e)} l(k_F r_{12}) \int d\vec{r}_3 h(r_{13}) l(k_F r_{13}) l(k_F r_{23})$$

The most direct use of the derived expressions for $G_i^{(2)}$'s is in the calculation of the second cluster term of the energy per particle, in certain cases. Assuming a Jastrow type correlation factor, the energy per particle for uniform and infinitely extended fermion systems interacting with two-body state dependent potentials can be expressed in Aviles formalism [1] as :

$$\frac{E}{N} = E_F + \frac{\hbar^2}{2m} \rho \int \Big\{ \frac{1}{2} \Big[\Big(\nabla f(r_{12}) \Big)^2 - f(r_{12}) \nabla^2 f(r_{12}) \Big] G(r_{12}) + \frac{m}{\hbar^2} \sum_i V_i(r_{12}) G_i(r_{12})$$

$$- \frac{1}{4} \vec{\nabla} f^2 . \vec{F}(\vec{r}_{12}) \Big\} d\vec{r}_{12} = E_F + E^{(1)} + E^{(2)} + \dots \qquad (27)$$

where E_F is the Fermi energy, and $\vec{F}(\vec{r}_{12})$ is :

$$\vec{F}(\vec{r}_{12}) = \frac{N(N-1)}{\rho^2 f^2(r_{12})} \frac{\int \prod_{i<j} f^2(r_{ij}) \vec{\nabla}_1 (\Phi^* \Phi) d\vec{r}_3 \dots d\vec{r}_N}{\int \prod_{i<j} f^2(r_{ij}) \Phi^* \Phi d\vec{r}_1 \dots d\vec{r}_N} \qquad (28)$$

The sum of the zeroth, first and second cluster terms may be quite a good approximation to the energy per particle. When this condition is satisfied, the derived expressions for the G_i's can be used reliably in a variety of calculations. This sort of calculation is in progress.

I would like to thank Professor M. Grypeos for suggesting the above subject and for useful discussions.

TABLE 1. *Statistical factors for s = 4 systems*

i	$\Delta_i^{(d)}$	$\Delta_i^{(e)}$
1 (singlet odd)	1	1
2 (singlet even)	3	—3
3 (triplet odd)	9	9
4 (triplet even)	3	—3

TABLE 2. *Statistical factors for s = 2 systems*

i	$\Delta_i^{(d)}$	$\Delta_i^{(e)}$
1 (singlet even)	1	—1
2 (triplet odd)	3	3

REFERENCES

1) J. B. Aviles, Ann. Phys. 5(1958) 251.
2) C. D. Hartogh and H. A. Tolhoek, Physica 24(1958) 721, 875, 896.
3) F. Iwamoto, M. Yamada, Progr. Theor. Phys. 18(1957) 345.
4) P. Westhaus, Ph. D. thesis, Washington University, 1966 (unpublished).
5) J. W. Clark, P. Westhaus, J. Mathem. Phys. 9(1968) 131, 149.
6) E. Feenberg, The Theory of Quantum Fluids, Academic Press, New York and London (1969).
7) V. J. Emery, Nucl. Phys. 6, 585 (1958).
8) D. Chakkalakal, Ph. D. thesis, Washington University, 1968 (unpublished).

ΠΕΡΙΛΗΨΙΣ

ΑΚΤΙΝΙΚΑΙ ΣΥΝΑΡΤΗΣΕΙΣ ΚΑΤΑΝΟΜΗΣ ΕΞΗΡΤΗΜΕΝΑΙ ΕΚ ΤΗΣ ΚΑΤΑΣΤΑΣΕΩΣ ΔΙΑ ΣΥΣΤΗΜΑΤΑ ΦΕΡΜΙΟΝΙΩΝ ΑΠΕΡΙΟΡΙΣΤΟΥ ΕΚΤΑΣΕΩΣ

ὑπὸ

Ε. ΜΑΥΡΟΜΜΑΤΗ

(Δ/νσις Φυσικῆς, ΚΠΕ Δημόκριτος, Ἁγία Παρασκευὴ Ἀττικῆς)

Εἰς τὴν παροῦσαν ἐργασίαν, λαμβάνονται γενικευμέναι ἐκφράσεις τῶν ἐξηρτημένων ἐκ τῆς καταστάσεως ἀκτινικῶν συναρτήσεων κατανομῆς $Gi(r_{12})$, εἰς περίπτωσιν συστήματος φερμιονίων ἀπεριορίστου ἐκτάσεως, διὰ χρήσεως ἀναπτύγματος κατὰ «clusters». Οἱ οὕτω προκύπτοντες ὅροι β′ τάξεως συγκρίνονται μὲ τοὺς ἀντιστοίχους ὅρους β′ τάξεως εἰς τὸν συνήθη φορμαλισμὸν τῶν Iwamoto-Yamada. Τὰ λαμβανόμενα ἀποτελέσματα εἶναι κυρίως χρήσιμα εἰς τὸν ὑπολογισμὸν τῶν ὅρων τριῶν σωμάτων τῆς ἐνεργείας τῆς θεμελιώδους καταστάσεως συστήματος φερμιονίων ὅταν χρησιμοποιοῦνται δυναμικὰ ἐξηρτημένα ἐκ τῆς καταστάσεως.