

## A SOLUTION OF RADIATION PATTERN SYNTHESIS OF ARBITRARY ANTENNAS

*(One, Two and Three Dimensional Antennas)*

by

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**Abstract :** *A method for determining the excitation of an arbitrary antenna is presented. The integral form of the radiation pattern is approximated, with the aid of Chebyshev analysis, by a sum exactly similar to the array factor of a non-uniformly spaced array. The synthesis of the resulting array factor is carried out by the orthogonal method, whereas the distribution function is expressed analytically with the aid of Chebyshev analysis. The problem is solved theoretically for one, two and three dimensional sources, and examples are given for the corresponding cases.*

### I. INTRODUCTION

The critical point for an antenna to give a pattern sought is its distribution function. The problem of finding the distribution function has not been touched in the general case, but some successful methods have been applied in many special cases.

A series of papers by *Stutzman*<sup>1,2,3,4,5</sup>, using the iterative sampling method, showed that this method cannot be uncautiously applied in the cases examined.

The Fourier analysis<sup>6, 7, 8, 9</sup> as well as synthesis, by imposing some error criteria<sup>10, 11, 12</sup>, have been used for some special array forms.

In the paper the problem is solved with the aid of Chebyshev analysis resulting in an approximation as good as one wishes.

### II. FORMULATION

The radiation pattern of an arbitrary apparatus of sources is known to be given by

$$F(\varphi, \theta) = \underset{\text{apparatus}}{\int \int \int} g(x, y, z) \exp\left[j \frac{2\pi}{\lambda} (x \cos \theta \cos \varphi + y \sin \theta \cos \varphi + z \cos \theta)\right] dx dy dz \quad (1)$$

The determination of the distribution function,  $g(x, y, z)$ , from (1), when  $F(\varphi, \theta)$  is known, can be accomplished by many approximating methods, depending on the researcher's angle of attack. One can get a better insight by separating the problem into three subcases: Linear, planar and spatial.

*1. One - dimensional problem (Fig. 1).*

The radiation pattern of a linear source antenna of length  $2L$  is given by

$$F(u) = \int_{-L}^L g(x) \exp(j2\pi x u / \lambda) dx \quad (2)$$

which is equivalent to

$$F(u) = \int_{-1}^1 g(p) \exp(j2\pi L p u / \lambda) dp \quad (3)$$

where  $p = x/L$  and  $u = \sin \theta \cos \varphi$

Recalling that every function  $f(p) = g(p) \exp(j2\pi L p u / \lambda)$  can be approximated by a Chebyshev series of the form\*

$$f(p) = \sum_{s=0}^n q_r(u) T_r(p), \text{ where}$$

$$q_r(u) = \frac{2}{n} \sum_{s=0}^n f\left(\cos \frac{\pi s}{n}\right) \cdot \cos \frac{\pi r s}{n} = \frac{2}{n} \sum_{s=0}^n g\left(\cos \frac{\pi s}{n}\right) \exp(j2\pi / \lambda L u \cdot \cos \frac{\pi s}{n}) \cos \frac{\pi r s}{n},$$

the integral equation (3) can be expressed as

$$F(u) = \sum_{r=0}^n q_r(u) \int_{-1}^1 T_r(p) \cdot dp \quad (5)$$

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(\* The symbol  $\Sigma''$  represents sums having the first and last coefficient multiplied by  $1/2$ ).

Now, since

$$\int_{-1}^1 T_0(p) dp = 2 \quad , \quad \int_{-1}^1 T_1(p) dp = 0$$

and

$$\int_{-1}^1 T_r(p) \cdot dp = \frac{1}{2} \left[ \frac{1 - (-1)^{r+1}}{r+1} - \frac{1 - (-1)^{r-1}}{r-1} \right] , \quad r > 1 \quad (6)$$

one can write |13| ,

$$F(u) = \sum_{k=0}^{n/2} \left( \frac{1}{2k+1} - \frac{1}{2k-1} \right) \cdot q_{2k}(u) = - \frac{4}{n} \sum_{k=0}^{n/2} \sum_{s=0}^n \frac{\cos \frac{2k\pi s}{n}}{4k^2-1} g(\cos \frac{\pi s}{n}) \exp(j2\pi/\lambda \text{Lucos} \frac{\pi s}{n}) \quad (7)$$

Eq. (7) shows that  $F(u)$  can be approximated by a non-uniform array, having a pattern of the form

$$F(u) = \sum_{s=0}^n a_s \exp(j2\pi/\lambda x_s u) \quad (8)$$

where

$$x_s = L \cdot \cos \frac{\pi s}{n} \quad \text{and}$$

$$\alpha_0 = - \frac{2}{n} \sum_{k=0}^{n/2} \frac{g(1)}{4k^2-1} \quad (9)$$

$$\alpha_n = - \frac{2}{n} \sum_{k=0}^{n/2} \frac{g(1)}{4k^2-1}$$

$$\alpha_s = - \frac{4}{n} \sum_{k=0}^{n/2} \frac{g(\cos \frac{\pi s}{n})}{4k^2-1}$$

$$s \neq 0, n$$

The synthesis of a non-uniform array can be carried out by several methods; one of them is the orthogonal, which has been extensively used by the author<sup>14, 15, 16</sup>.

Of course, if the apparatus is non-continuous, then (3) can be turned into (8) right from the start.

## 2. Two-dimensional problem (Fig. 2).

In this case, a two-dimensional apparatus gives:

$$F(u, v) = \int_{\text{apparatus}} \int g(x, y) \exp[j2\pi/\lambda(xu + yv)] dy dx \quad (10)$$

The integral equation (10) can be generally written as

$$F(u, v) = \int_{\alpha}^b \int_{f_1(x)}^{f_2(x)} g(x, y) \exp [j2\pi/\lambda(xu + yv)] dy dx \quad (11)$$

We now perform the following transformations:

$$y = \frac{1}{2} \{ [f_2(x) + f_1(x)] + [f_2(x) - f_1(x)]p \}$$

and

$$x = \frac{1}{2} \{ (b + \alpha) + (b - \alpha)q \}$$

Thus, (11) takes on the form

$$F(u, v) = \int_{-1}^1 \int_{-1}^1 G(p, q) \cdot \left| \frac{\partial(x, y)}{\partial(p, q)} \right| dp dq \quad (13)$$

where  $\left| \frac{\partial(x, y)}{\partial(p, q)} \right|$  is the Jacobian of the transformation, and  $G(p, q)$  is the function resulting from  $g(x, y) \exp [j2\pi/\lambda (xu + yv)]$  when  $x$  and  $y$  are substituted from (12)

Integrating the relation by Chebyshev we get, like in the linear case,

$$F(u, v) = \frac{16}{n^2} \sum_{k=0}^{n/2} \sum_{s=0}^{n/2} \sum_{l=0}^{n/2} \sum_{m=0}^{n/2} \frac{\cos \frac{2k\pi s}{n} \cos \frac{2m\pi l}{n}}{(4k^2 - 1)(4l^2 - 1)} G\left(\cos \frac{\pi s}{n}, \cos \frac{\pi l}{n}\right) \quad (14)$$

By (12),

$$F(u, v) = \frac{16}{n^2} \sum_{k=0}^{n/2} \sum_{s=0}^{n/2} \sum_{l=0}^{n/2} \sum_{m=0}^{n/2} \frac{\cos \frac{2k\pi s}{n} \cos \frac{2m\pi l}{n}}{(4k^2-1)(4l^2-1)} g(x_s, y_{sl}) \exp[j2\pi/\lambda(x_s u + y_{sl} v)] \quad (15)$$

where

$$x_s = \frac{1}{2} \left[ (b+a) + (b-a) \cos \frac{\pi s}{n} \right]$$

$$y_{sl} = \frac{1}{2} \{ [f_2(x_s) + f_1(x_s)] + [f_2(x_s) - f_1(x_s)] \cos \frac{\pi l}{n} \} \quad (16)$$

Eq. (15) shows that the case of the two-dimensional problem can also be traced to a planar non-uniform array, having  $(n+1)^2$  elements and expressed by

$$F(u, v) = \sum_{i=1}^{(n+1)^2} \alpha_i \exp[j2\pi/\lambda(x_i u + y_i v)] \quad (17)$$

For an apparatus of non-continuous arrays, one can reach (17) in two ways:

For a non-continuous array consisting of discrete elements, eq. (17) is straightforwardly arrived at.

For  $m$  continuous linear sources placed on a plane,  $F(u, v)$  takes the form

$$F(u, v) = \sum_{s=0}^{m \cdot n} \alpha_s \exp[j2\pi/\lambda (x_{sm} u + y_{sm} v)] \quad (18)$$

where

$$x_{sl} = L_1 \cos \frac{\pi s}{n} \cos \varphi_l \quad \text{and} \quad y_{sl} = L_1 \cos \frac{\pi s}{n} \sin \varphi_l \quad (19)$$

### 3. Three dimensional problem (Fig. 3).

A spatial apparatus has a pattern of the form

$$F(\varphi, \theta) = \int_{\alpha}^b \int_{f_1(z)}^{f_2(z)} \int_{\varphi_1(y,z)}^{\varphi_2(y,z)} g(x, y, z) \exp [j2\pi/\lambda (x \sin \theta \cos \varphi + y \sin \theta \sin \varphi + z \cos \theta)] dx dy dz \quad (20)$$

This equation can be integrated in a similar manner to give

$$F(\varphi, \theta) = -\frac{64}{n^3} \sum_{k=0}^{n/2} \sum_{s=0}^n \sum_{l=0}^{n/2} \sum_{n=0}^n \sum_{t=0}^{n/2} \sum_{h=0}^n \frac{\cos \frac{2k\pi s}{n} \cos \frac{2l\pi m}{n} \cos \frac{2t\pi h}{n}}{(4k^2-1)(4l^2-1)(4t^2-1)} \cdot g(x_s, y_{sm}, z_{smh}) \exp[j2\pi/\lambda(x_s \sin\theta \cos\varphi + y_{sm} \sin\theta \sin\varphi + z_{smh} \cos\theta)] \quad (21)$$

Eq.(21) represents a general non-uniformly spaced array of the form

$$F(\varphi, \theta) = \sum_{i=1}^{(n+1)^2} a_i \exp[j2\pi/\lambda(x_i \sin\theta \cos\varphi + y_i \sin\theta \sin\varphi + z_i \cos\theta)] \quad (22)$$

For a non-continuous array one can be led directly to (22), whereas for an array of  $m$  linear sources one gets a formula similar to (22):

$$F(\varphi, \theta) = \sum_{k=1}^{m(n+1)^2} a_k \exp[j2\pi/\lambda(x_{mk} \sin\theta \cos\varphi + y_{mk} \sin\theta \sin\varphi + z_{mk} \cos\theta)] \quad (23)$$

where

$$\begin{aligned} x_{si} &= L_i \cos \frac{\pi s}{n} \cos\varphi_i \sin\theta_i \\ y_{si} &= L_i \cos \frac{\pi s}{n} \sin\varphi_i \sin\theta_i \\ z_{si} &= L_i \cos \frac{\pi s}{n} \cos\theta_i \end{aligned} \quad (24)$$

Complicated though the relations arrived at may be, they are recommended when high accuracy is sought.

#### 4. Formulae in non-uniformly spaced array synthesis (Fig. 4).

Regarding the orthogonal method, the general formulae for the synthesis of a general non-uniformly spaced array have been given by the author [15]: The amplitudes,  $A_i$ , of  $F(\varphi, \theta)$ , which is of the form

$$F(\varphi, \theta) = \sum_{l=1}^N A_l \exp[j2\pi/\lambda(x_l \sin\theta \cos\varphi + y_l \sin\theta \sin\varphi + z_l \cos\theta)] \quad (25)$$

are given by

$$A_l = \sum_{l=1}^N B_l C_l^{(l)} \quad (26)$$

where

$$B_l = \langle F, \Psi_l^* \rangle$$

$$\Psi_j = \sum_{l=1}^j C_l^{(j)} \exp[j2\pi/\lambda(x_l \sin\theta \cos\varphi + y_l \sin\theta \sin\varphi + z_l \cos\theta)]$$

$$C_k^{(n)} = -\frac{4\pi}{D_n} \sum_{j=k}^{n-1} C_k^{(j)} \left( \sum_{l=1}^j C_l^{(j)} S_{nl} \right) \quad (27)$$

$$C_n^{(n)} = \frac{1}{D_n}$$

$$D_n = \{4\pi - 16\pi^2 \sum_{j=1}^{n-1} \left( \sum_{l=1}^j C_l^{(j)} S_{nl} \right)^2\}^{1/2}$$

$$S_{nl} = \frac{\sin 2\pi/\lambda r_{nl}}{2\pi/\lambda r_{nl}}$$

### III. COMPUTATION OF THE DISTRIBUTION FUNCTION

Since the formulae we use are rather complicated, great care was taken in every step, regarding the accuracy of the computation.

The program, in FORTRAN IV, and its running in a UNIVAC 1106 gave acceptable results only after many efforts. The experience the author acquired can be epitomized in two main points:

i) The number of harmonics,  $n$ , used in the series must be analogous to the length of the greatest dimension of the apparatus and it is suggested that it reaches a value of 5 or more per wavelength, depending on the complexity of the pattern sought.

ii) The accuracy of the values of  $S_{ni}$  must be checked, because the  $S_{ni}$ 's constitute the basis of the computation. The author used as a check the condition

$$|S| |S|^{-1} = |I| \quad (28)$$

The distribution function is given analytically. This is done because one first finds

$$G\left(\cos \frac{\pi s}{n}, \cos \frac{\pi m}{n}, \cos \frac{\pi h}{n}\right) = g(x_s, y_{sm}, z_{smh}).$$

This function can then give  $G(p, q, w)$  with the aid of Chebyshev analysis, in the form

$$G(p, q, w) = \frac{8}{n^3} \sum_{r=0}^n \sum_{l=0}^n \sum_{s=0}^n \sum_{m=0}^n \sum_{t=0}^n \sum_{h=0}^n \left( G \cos \frac{\pi s}{n}, \cos \frac{\pi m}{n}, \cos \frac{\pi h}{n} \right) \cdot \cos \frac{\pi r s}{n} \cos \frac{\pi l m}{n} \cos \frac{\pi t h}{n} \cdot T_r(p) T_l(q) T_h(w). \quad (29)$$

#### IV. EXAMPLES

The computer output gives the distribution function and the pattern thus produced. Here are some examples where Chebyshev analysis was applied:

i) A noteworthy case: Consider a non-uniform linear array of  $2N+1$  elements; if its total length is  $L$  then, to produce a pattern of the form

$$F(u) = \frac{\sin\left(\frac{\pi L u}{\lambda}\right)}{\frac{\pi L u}{\lambda}},$$

one must place the elements in distances of

$$\frac{L}{2} \cos \frac{\pi i}{2N} \quad (i = 0, 1, \dots, 2N)$$



and their amplitudes must be

$$a_i = -\frac{4}{n} \sum''_{k=0}^n \frac{\cos \frac{k\pi i}{n}}{4k^2 - 1} \quad i \neq 0, 2n$$

and

$$\alpha_0 = \alpha_{2n} = -\frac{2}{n} \sum''_{k=0}^n \frac{1}{4k^2 - 1}$$

This is an important result, and the numerical results show a remarkable convergence.

Figs 5a and 5b refer to two cases, for  $L = 3.5\lambda$  and  $N = 10$ , and for  $L = 5\lambda$  and  $N = 10$ , respectively.

From the above discussion, and given that  $F(u)$  represents the pattern of a linear source antenna, of reduced length  $L/\lambda$ , with uniform excitation, one can deduce that such an antenna can be approximated by a nonuniformly spaced linear array, having a non-uniform excitation.

ii) Consider the problem of synthesizing a linear source antenna, having a length of  $L = 3.5 \lambda$ ; one wishes to get a pattern showing a ratio of 30 of main - to-side-lobe.

Approximating the pattern by a pulse function, we reduce the problem, by eq. (8), to that of a discrete array. By the orthogonal method we find, for the elements' distances,  $1.75 \cos \frac{\pi i}{N}$  ( $i = 0, 1, \dots, N$ ), as before; the amplitudes are as follows for 11 and 21 elements respectively.

11 Elements	21 Elements
$\alpha_0 = \alpha_{10} = .023$	$\alpha_0 = \alpha_{20} = .011$
$\alpha_1 = \alpha_9 = .129$	$\alpha_1 = \alpha_{19} = .073$
$\alpha_2 = \alpha_8 = .084$	$\alpha_2 = \alpha_{18} = .122$
$\alpha_3 = \alpha_7 = .46$	$\alpha_3 = \alpha_{17} = .060$
$\alpha_4 = \alpha_6 = .68$	$\alpha_4 = \alpha_{16} = .078$
$\alpha_5 = 1.$	$\alpha_5 = \alpha_{15} = .189$
	$\alpha_6 = \alpha_{14} = .436$
	$\alpha_7 = \alpha_{13} = .582$
	$\alpha_8 = \alpha_{12} = .656$
	$\alpha_9 = \alpha_{11} = .901$
	$\alpha_{10} = 1.$

Fig. 6 shows how the pattern for  $N = 21$  converges to the theoretical one sought. Based on the amplitudes, and reducing (29) in one dimension, we are led to the current distribution of Fig. 7.

iii) Consider a planar rectangular source antenna of dimensions  $2a \times 2b$  (Fig. 8a). To find the current distribution one must, as has been explained, find the amplitudes of a nonuniform planar array. If one wishes to produce a pattern having the form of a pluse, with  $M/S = 25$ , then, for length  $= 3.5\lambda$  and width  $= 5\lambda$ , one ends up with the distribution shown in Fig. 8a, where there are  $51 \times 41$  sources. The pattern produced is shown in Fig. 8b.

We should not fail, here, to examine the case of a nonuniform planar array, producing a pattern similar to one produced by a planar source antenna with unity distribution function.

For this case, and for  $11 \times 11$ , sources, placed in positions given by  $x_s = a \cos \frac{\pi s}{10}$ ,  $y_m = b \cos \frac{m\pi}{10}$ , we get the following amplitudes:

$A/\frac{m}{s}$	0/10	1/9	2/8	3/7	4/6	5
0/10	0.1487	.4045	.5231	.6213	.6780	.7033
1/9	.2022	.5498	.7112	.8445	.9217	.9561
2/8	.2615	.7112	.9179	1.0923	1.1921	1.2366
3/7	.1606	.8445	1.0920	1.2970	1.4457	1.4687
4/6	.3389	.9217	1.1921	1.4157	1.5450	1.6028
5	.3516	.9562	1.2366	1.4687	1.6028	1.6627

iv) Finally, let us consider a three dimensional apparatus consisting of two rectangular planar sections, forming an angle of  $30^\circ$ . Each section has dimensions  $3.5\lambda \times 5\lambda$ . To get a pattern of a beamwidth of  $20^\circ$  for the main lobe, produced on the basis of a directive pattern having the form

$$f(\varphi, \theta) = \exp(p \cos \varphi \sin \theta + q \cos \theta) \cos(b \sin \theta),$$

one must have a distribution function of the form of Fig. 9a, resulting in the pattern shown in 9b.

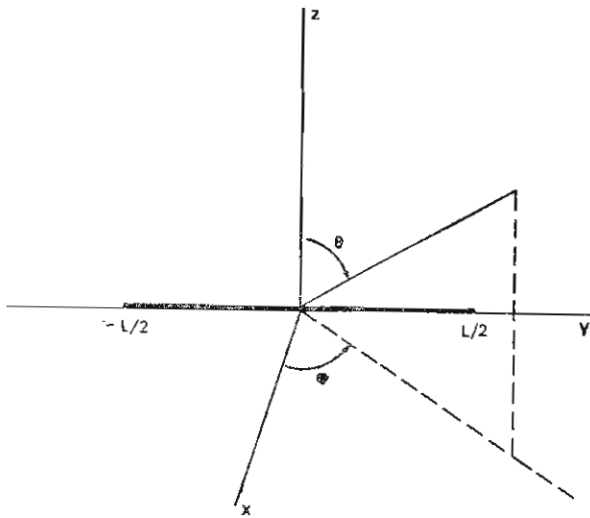


Fig. 1. Linear source Antenna

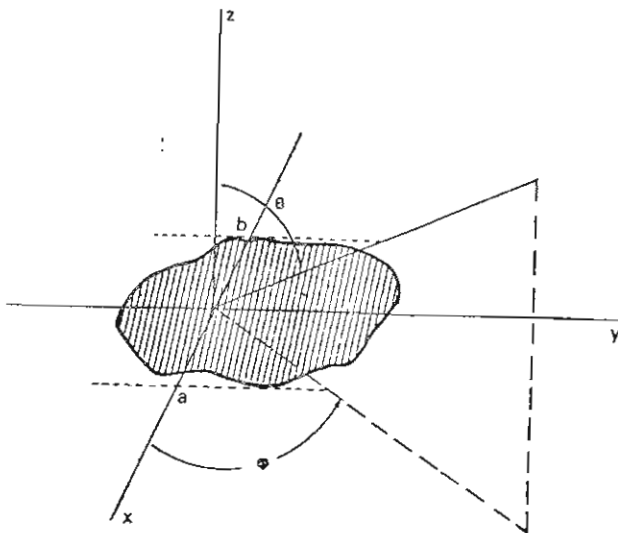


Fig. 2. Two - dimensional antenna apparatus

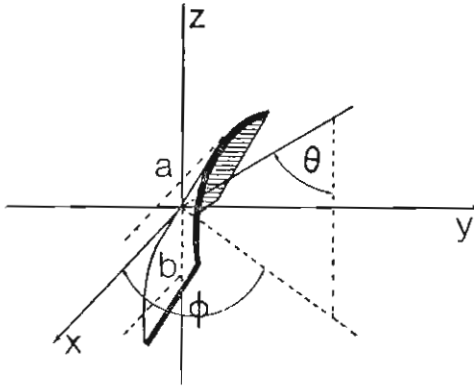


Fig 3. Three-dimensional antenna apparatus

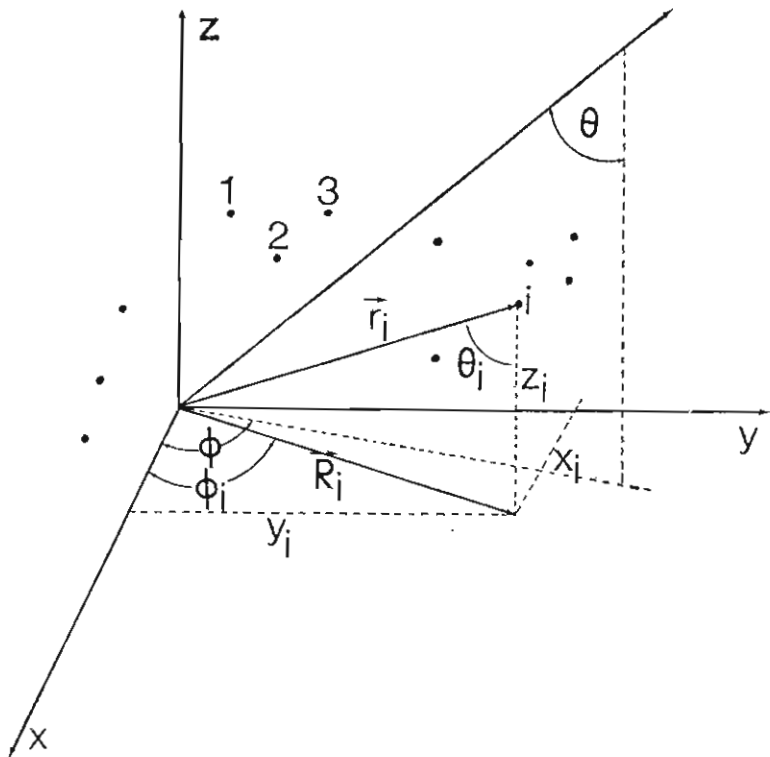


Fig. 4. General Nonuniformly spaced antenna array.

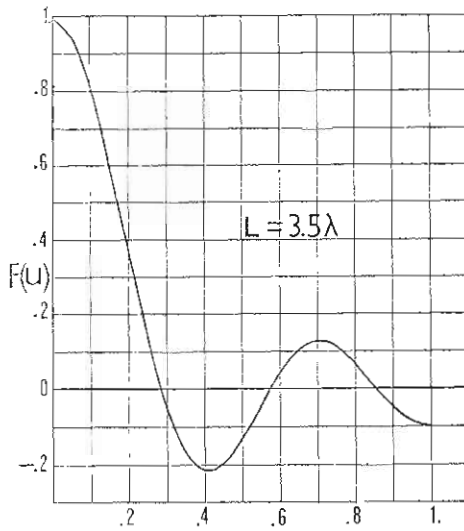


Fig. 5a. Radiation Pattern of Nonuniform Linear Array which is the same as a line source antenna with unity excitation ( $L = 3.5\lambda$ ,  $N = 10$ ).

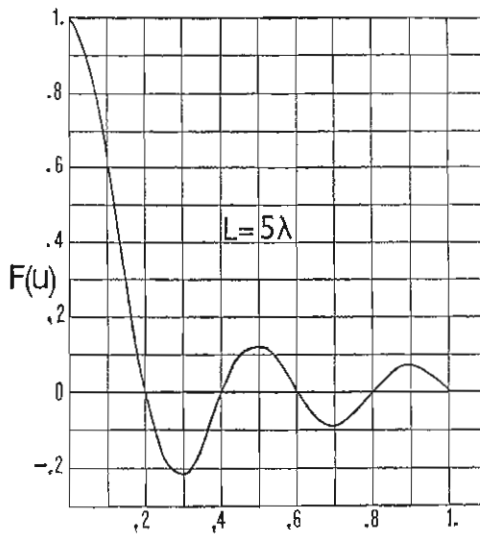


Fig. 5b. Radiation Pattern of Nonuniform Linear Array which is the same as a line source antenna with unity excitation ( $L = 5\lambda$ ,  $N = 10$ ).

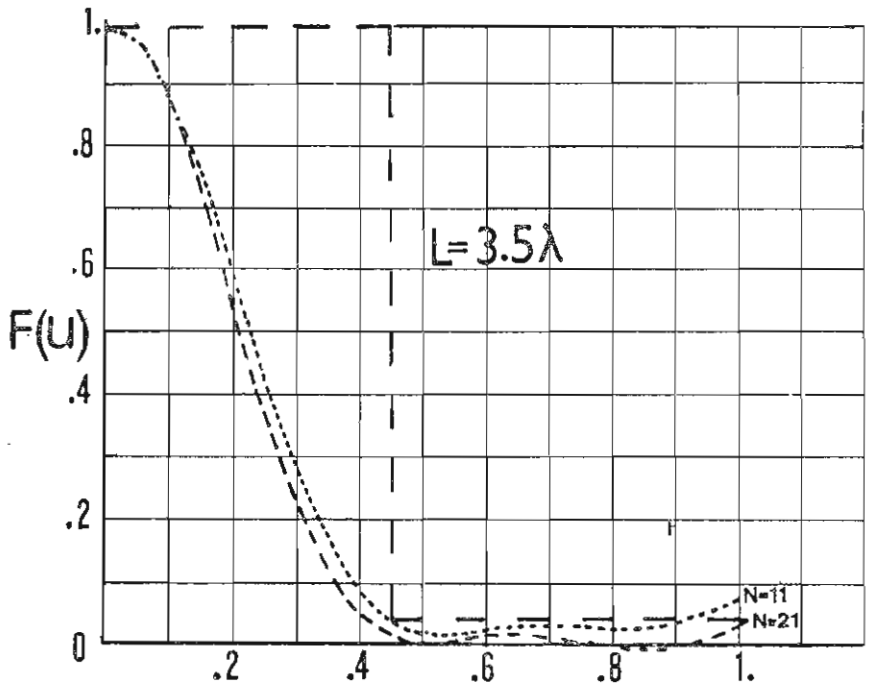


Fig. 6. Radiation Pattern of a line source antenna with main-to-side-lobe 30 and length  $3.5\lambda$ .

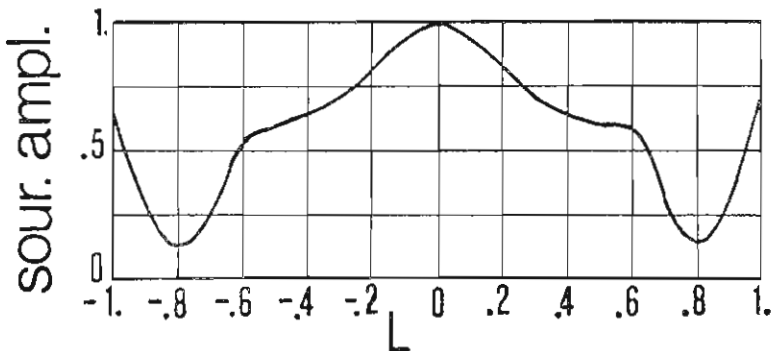


Fig. 7. The current distribution of a line source antenna with radiation pattern of Fig. 6.

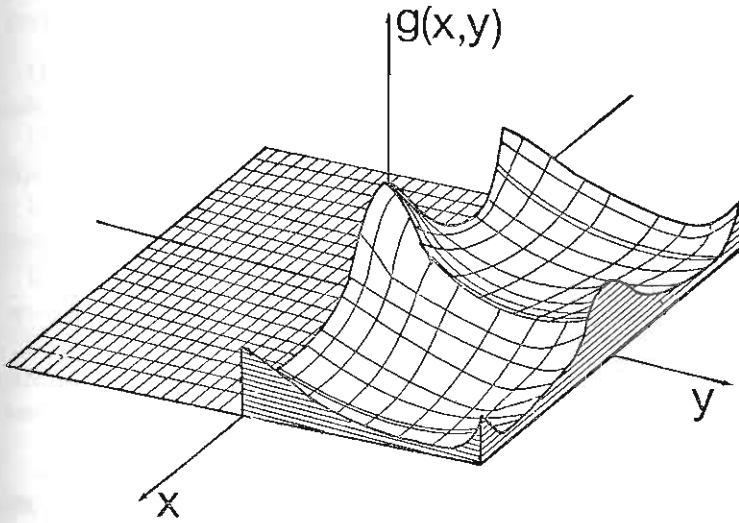


Fig. 8a. A planar source antenna of dimensions  $3.5\lambda \times 5\lambda$  with the current distribution.

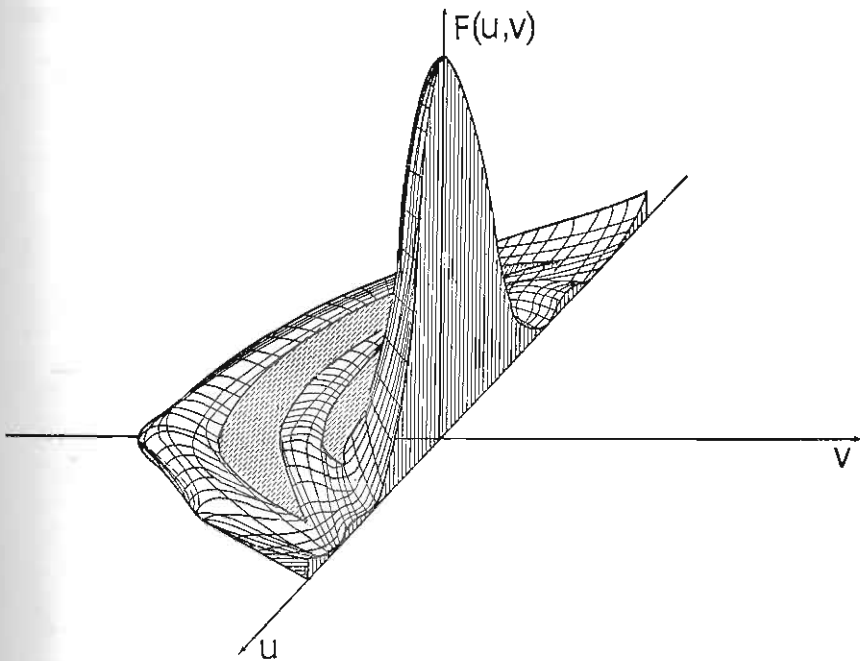


Fig. 8b. The radiation pattern of a planar rectangular source antenna with current distribution of Fig. 8a.

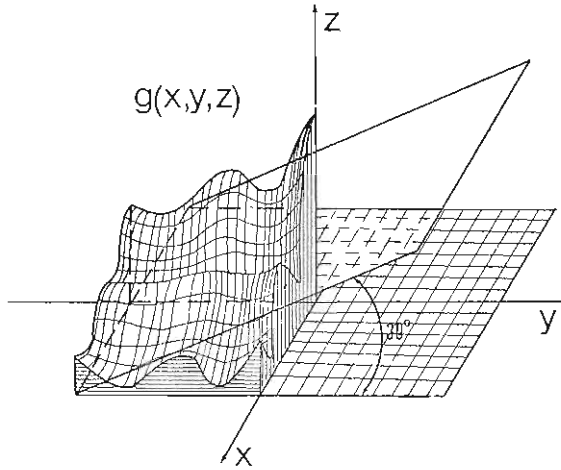


Fig. 9a. Three dimensional antenna apparatus consisting of two rectangular planar sections, forming an angle of  $30^\circ$ , with the current distribution.

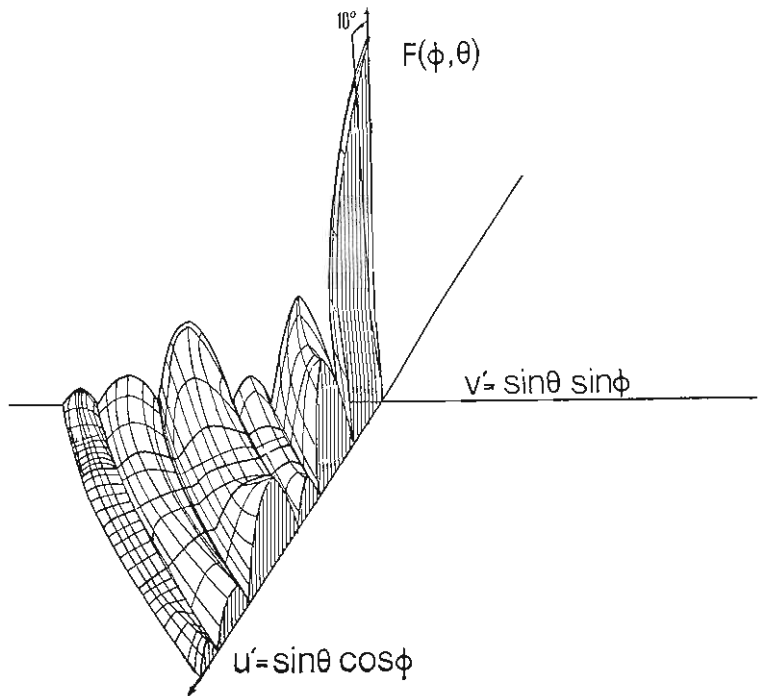


Fig. 9b. Radiation pattern of a three dimensional antenna apparatus with the current distribution of Fig. 9a.



## V. CONCLUSION

On the basis of the preceding discussion one can draw the following conclusions:

- i) It is possible to have a correspondence between an arbitrary continuous source apparatus and a nonuniform array.
- ii) The amplitudes of the nonuniform array can determine the distribution function of the source apparatus.
- iii) Chebyshev analysis can give results of acceptable accuracy in every case.
- iv) The procedure outlined above generalizes the problem of synthesizing nonuniform arrays and presents a way to determine the position of the sources.

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## ΠΕΡΙΛΗΨΙΣ

### ΛΥΣΙΣ ΤΟΥ ΠΡΟΒΛΗΜΑΤΟΣ ΣΥΝΘΕΣΕΩΣ ΔΙΑΓΡΑΜΜΑΤΟΣ ΑΚΤΙΝΟΒΟΛΙΑΣ ΚΕΡΑΙΑΣ

(Κεραΐαι μιᾶς, δύο καὶ τριῶν διαστάσεων)

Ἰπὸ

I. ΣΑΧΑΛΟΥ

Εἰς τὴν ἐργασίαν ταύτην μελετᾶται ἡ εὐρεσις τῆς ρευματικῆς κατανομῆς εἰς κεραΐας οἰωνδήποτε διαστάσεων καὶ σχημάτων.

Ἡ ὅλη ἀνάλυσις στηρίζεται εἰς τὴν εὐρεσιν προσεγγιστικῶν τύπων τῆ βοηθείᾳ τῆς ἀναλύσεως Chebyshev δι' ἀπλᾶς, διπλᾶς καὶ τριπλᾶς ὀλοκληρώσεις, ἀπαραιτήτους εἰς τὰς περιπτώσεις σχεδιάσεως μιᾶς κεραΐας τυχόντος σχήματος.

Ἡ ἐπίλυσις τοῦ προβλήματος χωρίζεται εἰς τρεῖς περιπτώσεις διὰ κεραΐας μιᾶς, δύο καὶ τριῶν διαστάσεων.

Ἐν συνδυασμῷ πρὸς τὴν μέθοδον ὀρθοκανονικοποιήσεως ἐπιτυγχάνεται ἡ εὐρεσις τῆς ρευματικῆς κατανομῆς εἰς ἀπάσας τὰς περιπτώσεις.

Εἰς τὰς ἐφαρμογὰς δίδεται ἰδιαιτέρον βάρος, δοθέντος ὅτι ἡ μέθοδος δύναται νὰ χρησιμοποιηθῆ εἰς οἰωνδήποτε περίπτωσιν.

Εἰς τὴν πρώτην ἐφαρμογὴν δίδονται ἀριθμητικὰ ἀποτελέσματα εἰς τὰ ὅποια δεικνύεται διὰ πρώτην φοράν ὅτι μιὰ διακεκριμένη ἀνομοιόμορφος κατανομή δύναται νὰ ἀποδόσῃ ἀπολύτως τὰ αὐτὰ ἀποτελέσματα πρὸς μία συνεχῆ τοιαύτην διαρροεμένην ὑπὸ ὁμοιομόρφου ρεύματος.

Εἰς τὴν δευτέραν ἐφαρμογὴν εὐρίσκεται ἡ ρευματικὴ κατανομὴ γραμμικῆς κεραΐας πρὸς ἐπίτευξιν λόγου κυρίου πρὸς δευτερεύοντα λοβὸν ἴσου πρὸς 30.

Εἰς τὴν τρίτην ἐφαρμογὴν εὐρίσκεται ἡ ρευματικὴ κατανομὴ ἑνὸς ἐπιπέδου ἀγωγίμου ὑλικοῦ διαστάσεων  $3.5 \lambda \times 5.0 \lambda$  πρὸς παραγωγὴν διαγράμματος ἀκτινοβολίας μὲ λόγον κυρίου πρὸς δευτερεύοντα λοβὸν ἴσον πρὸς 25. Συγχρόνως παρατίθενται πίναξ ἀριθμητικῶν ὑπολογισμῶν πρὸς ἀναγωγὴν τῆς συνεχοῦς ἐπιπέδου κεραΐας εἰς τὴν ἀντίστοιχον ἐπίπεδον ἀνομοιόμορφον κατανομὴν στοιχείων.

Εἰς τὴν τετάρτην τέλος ἐφαρμογὴν ἐξετάζεται μιὰ κεραΐα ἀποτελουμένη ἐκ δύο τεμνομένων ἐπιπέδων καὶ εὐρίσκεται ἡ ρευματικὴ κατανομὴ δι' ἑν προκαθωρισθὲν κατευθυντικὸν διάγραμμα ἀκτινοβολίας.

Εἰς τὴν ἐργασίαν παρατίθενται καὶ τὰ ἀπαραίτητα σχήματα ὡς καὶ μιὰ ἀναφορὰ εἰς τοὺς ὑπολογισμοὺς τοὺς πραγματοποιηθέντας διὰ τοῦ Ἡλεκτρονικοῦ ὑπολογιστοῦ, εἰς τὴν ὁποίαν ἀναπτύσσονται ὀρισμένα ἐμπειρικὰ συμπεράσματα.