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POST-GALILEAN INVARIANCE OF POST-NEWTONIAN HYDRODYNAMICS

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Abstract: *Transforming directly the post-Newtonian Eulerian equations of hydrodynamics we prove that the most general post-Galilean transformation resulting in their functional form invariance is identical with the corresponding transformation, derived independently by previous authors, which leaves unchanged the functional form of the metric tensor. In this way we generalize the meaning of the post-Galilean invariance by establishing its uniqueness and by extending the range of its validity.*

§ (1): INTRODUCTION - THE POST-GALILEAN INVARIANCE OF THE METRIC TENSOR

The post-Newtonian general relativistic perfect-fluid metric tensor has been derived by Chandrasekhar (1965) with the aid of the so-called method of the post-Newtonian approximations (PNA), which consists in solving the field equations for the metric tensor in the form of a power series of c^{-1} (c is the velocity of light), and is based on the assumption of weak fields and low velocities.

The metric tensor so obtained is invariant under linear time translations, three dimensional linear translations, and three dimensional rotations.

The above symmetries, however, are not enough for the metric tensor, obtained as a solution of the field equations, to be a «physically accepted» metric tensor. A further and equally important symmetry must be guaranteed for the metric tensor, and this is its *post-Galilean invariance* (PGI), namely its functional form invariance under the most general *post-Galilean transformation* (PGT).

As it is well known [Chandrasekhar and Contopoulos (1967), hereafter abbreviated as CC)], a PGT is a transformation of the space-time coordinates, which reduces to a Lorentz transformation of (constant) velocity much smaller than c in the asymptotically flat region of space time far from the source generating the metric tensor, and which preserves the post-Newtonian metric tensor in its standard from. In view of this definition, we can explain the physical meaning of the PGI as follows (Will 1971, see footnote 1): Each one of two observers, who set out to calculate the metric tensor due to the same bounded perfect-fluid source, uses a global coordinate frame, which becomes inertial asymptotically at large distances from the source. If the form of the metric tensor is preserved in these two approximately inertial frames, then according to the above definition they are related by a PGT. If furthermore the two observers compare the metric tensors, as each one computes it using the fluid variables determined in his own frame, it is obvious in view of the covariance principle that the results of their physical measurements must be physically equivalent. Also it is obvious that the results of the physical measurements can depend only on physically measurable velocities, like the velocities of the fluid elements relatively to each other or to the fluid's center of mass or the velocity of the fluid's center of mass relatively to the Universe's mean rest frame (for more on this frame see Will (1973)). In any case, the results *cannot* depend on the arbitrary velocity of the observer's frame, and the only way to guarantee this is to demand that the metric has the same functional form independent of the velocity of the frame, in which it is calculated, (and hence independent of the relative velocity of the two observers). In other words, since the two frames are asymptotically inertial, the metric tensor must be functional form invariant under an asymptotically Lorentzian transformation of low velocity. The above property of the metric tensor has been called its PGI.

Now it becomes apparent that in order to find the form of the PGT, we have to expand the usual Lorentzt ransformation in powers of the quantity (relative velocity) / c, and then adding properly arbitrary functions, to generalize it, so that to take into account gravitation-induced deviations from the pure Lorentz transformation. The arbitrary functions finally are evaluated with the aid of the usual transformation law for tensors.

Footnote 1: This explanation is not restricted to the PGI of to the post-Newtonian limit of the general theory of relativity, but applies also to the PGI of the post-Newtonian limit of every *metric* theory of gravity.

The form of the most general PGT, so derivable, resulting in the functional form invariance of the metric tensor (this transformation is abbreviated here as T_m) has been given for the first time in CC in the case of a system of point-masses. The same method was applied later by Will (1971) in the perfect-fluid case in the context of the parametrized post-Newtonian formalism.

§ (2): THE POST-GALILEAN INVARIANCE OF THE POST-NEWTONIAN EULERIAN EQUATIONS OF HYDRODYNAMICS

The post-Newtonian Eulerian equations of motion have been derived directly from the vanishing of the covariant derivative of the perfect-fluid energy-momentum tensor (Chandrasekhar 1965).

In spite of the fact that these equations are very complicated functions of the metric tensor and its derivatives, it can be proved in a straightforward manner that the transformarion T_m leaves unchanded their functional form as well. This result, however, although, unfortunately, it is commonly accepted so, does *not* mean at all that the equations of motion are post-Galilean invariant. It means simply that they are functional form invariant under the most general PGT, which leaves unchanged the metric tensor. Obviously, this property of the equations of motion does not prove their PGI, because it does not answer to the following crucial question: Which is the most general PGT resulting in the functional form invariance of the Eulerian equations of hydrodynamics (this transformation is abbreviated here as T_e) and, if such a transformation exists, how is it related to the transformation T_m?

In attempting to answer questions like these, one might argue on purely physical considerations. Thus we consider two different frames in which the post Newtonian Eurelian equations have identically the same form. Then physical systems (like a bounded perfect-fluid mass) with identically the same initial conditions in the two frames will evolve in identically the same way. Now, since all aspects of the metric are probed by and measurable in terms of geodesic motions, and since furthermore the geodesic motions are a subset of hydrodynamical motions, the metrics, from which the motions in the two physical systems are derived, must be the same (to post-Newtonian order). In other words, the transformation T_e leaves unchanged the functional form of the metric tensor. Since, moreover, as it has alerady been said, the transformation T_m leaves unchanged the functional form of the metric, we conclude that the transformations T_m and T_e are equivalent.

Now a mathematical answer to the above questions, which would prove not simply the equivalence but in fact the *identity* of the trans formations T_m and T_e is not so obvious. Most important, it requires a *direct* transformation of the Eulerian equations of motion themselves, in the sense that any *indirect* method for *proving* the functional form invariance of equations of motions (not necessarily only the perfect-fluid ones) cannot be used. Such a method is described for example in part 7 of CC, where use is made of the following theorem in analytical dynamics: «a necessary and sufficient condition for the equations of motion to have the same forms in two sets of variables is that the corresponding Lagrangians in these two sets differ by a total time derivative». In using this theorem the above authors transform the point-mass Lagrangian (their eq. (7)) from (in our notation) the unbarred to the barred variables and deduce that the Lagrangians in the two sets of variables differ by the total time derivative with respect to the *barred* time-coordinate of a certain function (given by their eq. (132)). To this end, however, it is necessary to *assume* that the conservation laws and the conserved quantities in the barred frame are of exactly the *same* form as the corresponding ones in the unbarred frame. But this means that the metrics and consequently the equations of motion in the two frames are of the same form. We conclude therefore that in this way the PGI of the equations of motion is *not proved*, but that, as it is clearly stated in CC, again it is simply *verified* that under the similar invariance of the metric tensor, the equa tions of motion are also invariant. In this sense, wo cannot use for our purposes the perfect-fluid Lagrangian derived by Plebanski and Bazanski (1959) or the Eulerian variational principle developed for relativistic hydrodynamics by Tam (1966) and by Tam and O'Hanlon (1969).

The main results of the *direct* transformation of the Eulerian equations have been reported in the form of a note (Spyrou 1976). It is the purpose of this paper to consider the whole problem of the PGI of the Eulerian equations in some detail and at the same time to prove mathematically the identity of the transformations T_m and T_e. This identity means that the metric tensor and the equations of motion independently of each other, are functional form invariant under the *same* most general PGT. This property of the post-Newtonian limit of the general relativistic hydrodynamics we shall call *post-Galilean invariance of the post-Newtonian hydrodynamics.*

We believe that this property of the post-Newtonian limit of general relativity (and more generally of every *metric* theory of gravity) is as fundamental as the PGI of the metric tensor alone, and that it generalizes the meaning of the post-Galilean invariance by establishing its uniqueness on the one hand, and by extending its range of validity on the other hand.

In the next paragraph we shall find the form of the transformation T_0.

§ (3): OUTLINE OF THE METHOD

In this paper we shall consistently use the standard notation used in the theory of the PNA. Thus in the approximately inertial frame $O(x^\alpha, x^0=ct)$ (set up by the one of the two previously mentioned observers) the Eulerian equations of hydrodynamics are provided by the relations

$$R \equiv \frac{\partial \rho^*}{\partial t} + \frac{\partial}{\partial x^\alpha}(\rho^* \upsilon^\alpha) = 0, \qquad (3.1)$$

and

$$S^\alpha \equiv \rho^* \frac{d\upsilon^\alpha}{dt} - \rho^* \frac{\partial U}{\partial x^\alpha} + \frac{\partial}{\partial x^\alpha}\left[\left(1 + \frac{3U}{c^2}\right) p\right]$$

$$+ \frac{1}{c^2}\left[- \frac{\partial p}{\partial x^\alpha}\left(\frac{\upsilon^2}{2} + \Pi + \frac{p}{\rho}\right) + 4\rho \frac{d}{dt}(\upsilon^\alpha U - U_\alpha)\right.$$

$$- \upsilon^\alpha \left(\frac{\partial U}{\partial t} - \frac{\partial p}{\partial t}\right) + \frac{1}{2}\rho \frac{\partial}{\partial t}(U_\alpha - U_{\beta;\alpha\beta})$$

$$\left. + 4\rho\upsilon^\beta \frac{\partial U_\beta}{\partial x^\alpha} - 2\rho \frac{\partial \Phi}{\partial x^\alpha} - \rho\left(\upsilon^2 + \frac{3p}{\rho}\right)\frac{\partial U}{\partial x^\alpha}\right] = 0 \qquad (3.2)$$

In equations (3.1) and (3.2) Greek indices run from 1 to 3 corresponding to the three spatial coordinates, and the Einstein summation convention is assumed to hold over repeated indices. Also ρ, p, Π and

$$\upsilon^\alpha = \frac{dx^\alpha}{dt}, \left(\frac{d}{dt} = \frac{\partial}{\partial t} + \upsilon^\alpha \frac{\partial}{\partial x^\alpha}\right) \qquad (3.3)$$

are the proper density, pressure, internal energy density and the components of the coordinate three-velocity, respectively, of a fluid's element, while the post-Newtonian density ρ^* is defined by the relation

$$\rho^*(\mathbf{x}, \mathbf{t}) = \rho(\mathbf{x}, \mathbf{t}) \left\{ 1 + \frac{1}{c^2} \left[\frac{1}{2} \upsilon^2(\mathbf{x}, \mathbf{t}) + 3U(\mathbf{x}, \mathbf{t}) \right] \right\} \tag{3.4}$$

with U, the Newtonian gravitational potential, and further the «potentials» U_α, $U_{\alpha;\beta\gamma}$ and Φ being given in integtal form by the following well-known relations, respectively,

$$U(\mathbf{x}, \mathbf{t}) = G \int_V \rho(\mathbf{x}', \mathbf{t}) \, |\mathbf{x}-\mathbf{x}'|^{-1} \, d^3x',$$

$$U_\alpha(\mathbf{x}, \mathbf{t}) = G \int_V \rho(\mathbf{x}', \mathbf{t}) \, \upsilon^\alpha \, (\mathbf{x}', \mathbf{t}) \, |\mathbf{x}-\mathbf{x}'|^{-1} \, d^3x',$$

$$U_{\alpha;\beta\gamma}(\mathbf{x}, \mathbf{t}) = G \int_V \rho(\mathbf{x}, \mathbf{t}) \, \upsilon^\alpha \, (\mathbf{x}', \mathbf{t}) \, (x^\beta - x'^\beta) \, (x^\gamma - x'^\gamma) \, |\mathbf{x}-\mathbf{x}'|^{-3} \, d^3\mathbf{x}$$

and

$$\Phi(\mathbf{x}, \mathbf{t}) = G \int_V \rho(\mathbf{x}', \mathbf{t}) [\upsilon^2(\mathbf{x}', \mathbf{t}) + U(\mathbf{x}', \mathbf{t}) + \frac{1}{2} \Pi(\mathbf{x}', \mathbf{t}) + \frac{3}{2} \frac{p(\mathbf{x}', \mathbf{t})}{\rho(\mathbf{x}', \mathbf{t})}] \, |\mathbf{x}-\mathbf{x}'|^{-1} \, d^3x'.$$

In these relations G is the gravitational constant, $\mathbf{x}$ and $\mathbf{x}'$ are the «field-point», and «source-point» respectively, d^3x' is the coordinate three-dimensional volume element and the integrations are carried out at time t over the three-dimensional volume V occupied by the fluid.

Now let $\overline{O}(\overline{x}^\alpha, \overline{x}^0 = c\overline{t})$ be the approximately inertial frame associated with the second observer, and let the observer in the frame O move with a uniform velocity V with respect to the one in the frame $\overline{O}$ (see footnote 2). Expanding the pure Lorentz transformation connecting the variables $\mathbf{x}$, t and $\overline{\mathbf{x}}$, $\overline{t}$ in powers of V/c, retaining terms of order $1/c^2$, and generalizing the result by adding arbitrary functions, we write, in accordance with the reasoning of § (1), the required PGT in the form

$$\mathbf{x} = \overline{\mathbf{x}} - \mathbf{V}\overline{t} + \frac{1}{c^2} \left[-\frac{1}{2} V^2 \mathbf{V} \overline{t} + \frac{1}{2} (\overline{\mathbf{x}} \cdot \mathbf{V}) \mathbf{V} + \mathbf{Y}(\overline{\mathbf{x}}, \overline{t}) \right],$$

Footnote 2: For a discussion on the role played by **V** see part 3 of CC.

$$t=\bar{t}+\frac{1}{c^2}\left[\frac{1}{2}V^2\bar{t}-\bar{x}\cdot Y+Z(\bar{x},\bar{t})\right], \tag{3.6}$$

where Y and Z are the arbitrary functions of $\bar{x}$, $\bar{t}$ to be determined, describing the gravitation-induced deviations from the pure Lorentz transformation. Equations (3.6) are the same as equations (19) and (20) of CC.

For the evaluation of the explicit form of Y and Z we shall use the functional form invariance of the rest mass element and of the Eulerian equations of motion. However, no assumption concerning PGI of the metric tensor will be introduced. Hence the transformation T_e which will be evaluated in this way, is independent of the transformation T_m.

§ (4): THE SOLUTION FOR THE FUNCTIONS Y AND Z

In what follows all the barred quantities are assumed to be expressed in terms of the barred variables $\bar{x}$ and $\bar{t}$ in exactly the same way as the corresponding unbarred quantities are expressed in terms of the unbarred variables x and t. Especially, we notice that, since by definition the fluid variables ρ, p and Π are measured in a frame comoving with the fluid-element, they will be equal to their corresponding barred analogs irrespectively of the transformation of the coordinates.

4.1 The invariance of the rest-mass element and its consequences.

The invariance of the rest-mass element allows us to write (Landau and Lifshitz (1971), Robertson and Noonan (1968))

$$\rho^* d^3x=\bar{\rho}^* d^3\bar{x}=\text{amount of rest-mass in the corresponding volume elements } d^3x \text{ and } d^3\bar{x}, \tag{4.1}$$

where in analogy to equation (3.4)

$$\bar{\rho}^*(\bar{x},\bar{t})=\bar{\rho}(\bar{x},\bar{t})\left\{1+\frac{1}{c^2}\left[\frac{1}{2}\bar{\upsilon}^2(\bar{x},\bar{t})+3\,\bar{U}(\bar{x},\bar{t})\right]\right\} \tag{4.2}$$

with

$$\bar{\upsilon}^\alpha=\frac{d\bar{x}^\alpha}{d\bar{t}}=\dot{\bar{x}}^\alpha . \tag{4.3}$$

(Total time derivarives with respect to $\bar{t}$ will be denoted by a dot and partial derivatives by a comma, namely, $_{,\alpha} \equiv \frac{\partial}{\partial \bar{x}^\alpha}$, $_{,0} = \frac{\partial}{\partial \bar{t}}$ e.t.c., so that $\frac{d}{d\bar{t}}$ $(\ldots) = (\ldots)_{,0} + \bar{\upsilon}^\alpha (\ldots)_{,\alpha}$).

The relation between the velocities $\mathfrak{v}$ and $\bar{\mathfrak{v}}$ is easily found, with the aid of equations (3.3), (3.5), (3.6) and (4.3), to be

$$
\begin{aligned}
\upsilon^\alpha &= \bar{\upsilon}^\alpha - V^\alpha \\
&+ \frac{1}{c^2} \Big\{ Y^\alpha{}_{,0} + V^\alpha Z_{,0} \\
&+ \Big[Y^\alpha{}_{,\beta} + V^\alpha Z_{,\beta} - \delta^{\alpha\beta} Z_{,0} - \frac{1}{2} (V^2 \delta^{\alpha\beta} + V^\alpha V^\beta) \Big] \bar{\upsilon}^\beta \\
&- (Z_{,\beta} - V^\beta) \bar{\upsilon}^\alpha \bar{\upsilon}^\beta \Big\}
\end{aligned}
\tag{4.4}
$$

while the relation between the volume elements in equation (4.1), as it is deduced from the invariance of the four-dimensional volume element (Landau and Lifshitz (1971)) with the aid of equations (3.5) and (4.4) (in its Newtonian limit), is

$$
d^3x = \Big\{ 1 + \frac{1}{c^2} \Big[Y^\alpha{}_{,\alpha} + V^\alpha Z_{,\alpha} - \frac{V^2}{2} - (Z_{,\alpha} - V^\alpha) \bar{\upsilon}^\alpha \Big] \Big\} d^3\bar{x} \tag{4.5}
$$

A direct consequence of equations (3.4), (4.2), (4.4) (in its Newtonian limit) and (4.5) is the relation

$$
\rho^* d^3x - \bar{\rho}^* d^3\bar{x} = \frac{1}{c^2} \bar{\rho} d^3\bar{x} (Y^\alpha{}_{,\alpha} + V^\alpha Z_{,\alpha} - \bar{\upsilon}^\alpha Z_{,\alpha}) \tag{4.6}
$$

Comparing now equations (4.1) and (4.6) we see that the demand of the invariance of the rest-mass element is equivalent to

$$
Y^\alpha{}_{,\alpha} + V^\alpha Z_{,\alpha} - \bar{\upsilon}^\alpha Z_{,\alpha} \equiv 0 \tag{4.7}
$$

Since furthermore equation (4.7) must be identically satisfied, namely for all values of $\bar{\mathfrak{v}}$, we finally obtain

$$
Y^\alpha{}_{,\alpha} = 0, \tag{4.8}
$$

$$
Z_{,\alpha} = 0. \tag{4.9}
$$

4.2 The transformation of the Eulerian equation of hydrodynamics.

In order to reexpress equations (3.1) and (3.2) in the frame $\bar{O}$, we have to transform into the barred variables the potentials U, U_α, $U_{\alpha;\beta\gamma}$ and Φ. The required transformation is provided by equations (52)—(56) of Will (1971). With the aid of these equations as well as of equations (3.4), (3.6) (4.2) and (4.4)-(4.6) and following steps analogous to the ones for the derivation of equation (4.6) we find that the demand of the functional form invariance of equations (3.1) and (3.2) is equivalent to

$$R-\bar{R}=\frac{1}{c^2}\rho\frac{\bar{d}}{dt}\left(Y^\alpha{}_{,\alpha} + V^\alpha Z_{,\alpha} - \bar{\upsilon}^\alpha Z_{,\alpha} \right) \equiv 0 \qquad (4.10)$$

and

$$S^\alpha - \bar{S}^\alpha = \frac{1}{c^2}\bar{\rho}\left(Q^{\alpha\beta}\dot{\bar{\upsilon}}^\beta + Q^\alpha \right) \equiv 0, \qquad (4.11)$$

where $Q^{\alpha\beta}$ ans Q^α are functions of $\mathbf{x}$, $\mathbf{t}$ and $\bar{\boldsymbol{\upsilon}}$ (but not of $\dot{\bar{\boldsymbol{\upsilon}}}$), defined by the following relations, respectively,

$$Q^{\alpha\beta} = Y^\alpha{}_{,\beta} + Y^\beta{}_{,\alpha} - 2\delta^{\alpha\beta}Z_{,0} + V^\beta Z_{,\alpha} + V^\alpha Z_{,\beta} - \bar{\upsilon}^\gamma(Z_{,\beta}\delta^{\alpha\gamma} + 2Z_{,\gamma}\delta^{\alpha\beta}), \quad (4.12)$$

and

$$Q^\alpha = Y^\alpha{}_{,00} + \left\{ G\int_V \frac{\bar{\rho}'}{|\bar{x}-\bar{x}'|^3} [(\bar{x}-\bar{x}').(Y-Y')]d^3\bar{x}' \right\}_{,\alpha}$$

$$+ V^\alpha Z_{,00} + (\bar{\rho}\bar{U}_{,0} - \bar{P}_{,0})Z_{,\alpha}$$

$$- \left\{ G\int_V \frac{\bar{\rho}'}{|\bar{x}-\bar{x}'|^3}[(\bar{x}-\bar{x}').(\bar{\upsilon}'-V)](Z-Z')d^3x' \right\}_{,\alpha}$$

$$+ \bar{\upsilon}^\beta(2Y^\alpha{}_{,\beta} + 2V^\alpha Z_{,\beta} - \delta^{\alpha\beta}Z_{,0})_{,0}$$

$$+ \bar{\upsilon}^\beta\bar{\upsilon}^\gamma(Y^\alpha{}_{,\beta} - 2Z_{,0}\delta^{\alpha\beta} + V^\alpha Z_{,\beta})_{,\gamma}$$

$$+ \bar{\upsilon}^\alpha\bar{\upsilon}^\beta\bar{\upsilon}^\gamma Z_{,\beta\gamma}, \qquad (4.13)$$

with a primed quantity under the integral sign beign evaluated at the «source-point» $\bar{x}'$ at the time $\bar{t}$.

Now, conditions (4.10) and (4.11) can be considerably simplified, if use is made of the invariance of the rest-mass element. Thus, it is obvious in view of equation (4.7) that equation (4.10) is identically satisfied. In other words *the invariance of the rest-mass element is sufficient for the invariance of the continuity equation* (3.1).

Furthermore, since equation (4.11) must be satisfied identically, namely for all the values of $\vec{\upsilon}$, both Q^{α} and $Q^{\alpha\beta}$ independently of each other must vanish identically, namely for all the values of $\vec{\upsilon}$. Thus using equations (4.8) and (4.9) we first prove that the vanisbing of the contracted version of equation (4.12) reduces to

$$Z_{,0} = 0, \tag{4.14}$$

and then we easily find the relations

$$Y^{\alpha}{}_{,\beta} + Y^{\beta}{}_{,\alpha} = 0 \tag{4.15}$$

and

$$Y^{\alpha}{}_{,00} + \left\{ G \int_V \frac{\bar{\rho}'}{|\bar{x}-\bar{x}'|^3} \, [(\bar{x}-\bar{x}') \, . \, (\mathbf{Y}-\mathbf{Y}')] d^3\bar{x}' \right\}_{,\alpha} = 0,$$

$$Y^{\alpha}{}_{,\beta 0} = 0,$$

$$Y^{\alpha}{}_{,\beta\gamma} = 0. \tag{4.16}$$

We observe that the pair or equations (4.9) and (4.14) is equivalent to equation (3.9) of CC the solution being

$$Z = 0 \tag{4.17}$$

Similarly equation (4.15) is exactly the same as equation (4.8) of CC. Its solution is

$$\mathbf{Y}(\bar{x}, \bar{t}) = \mathbf{a}(\bar{t}) + \bar{x} \times \mathbf{b}(\bar{t}).$$

and equations (4.16) can now be used in a straightforward manner to prove that this solution can finally be put in the form

$$\mathrm{Y}(\bar{x}, \bar{t}) = \mathbf{C} + \bar{x} \times \mathbf{B} + \mathbf{A}\,\bar{t}, \tag{4.18}$$

where **A**, **B** and **C** are certain constant vectors. Equations (4.17) and (4.18) provide the required solutions for the unknown functions **Y** and Z.

§ (5): CONCLUDING REMARKS

In this paper it is pointed out that the post-Galilean invariance of the Eulerian equations of post-Newtonian hydrodynamical motions can not be considered as a consequence of the corresponding invariance of the metric tensor, and that it has to be proved independently.

The direct transformation of the equations of motion revealed that they are functional form invariant under the same most general post-Galilean fransformation as the metric tensor. In fact equations (3.6) supplied with equations (4.17) and (4.18) are exactly the same as equations (40) and (59) of CC.

The identity of the above two transformations, which have been derived independently of each other, generalizes the meaning of the term post-Galilean invariance, so that the latter applies to the wider range of the post-Newtonian general relativistic hydrodynamics, and not simply to the metric tensor.

However, it has to be particularly emphasized that, unlikely to the method followed in CC, in the context of the method followed here it is not possible to evaluate the term of order $1/c^4$ in the second of equations (3.6). Obviously this is due to the fact that the transformed equations (3.1) and (3.2) contain terms of order at most $1/c^2$. If, therefore, the evaluation of the above term is necessary in the context of the same method, we must transform the Eulerian equations of motion in the 2nd PNA, given by equations (52) - (54) of Chandrasekhar and Nutku (1969). In this case, of course, the term of order $1/c^4$ in the first of equations (3.6) could also be evaluated.

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ΠΕΡΙΛΗΨΗ

ΜΕΤΑ-ΓΑΛΙΛΑΪΚΗ ΑΜΕΤΑΒΛΗΤΟΤΗΤΑ ΤΗΣ ΜΕΤΑΝΕΥΤΩΝΕΙΑΣ ΥΔΡΟΔΥΝΑΜΙΚΗΣ

Ὑπὸ

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Μὲ ἀπ' εὐθείας μετασχηματισμὸ τῶν ἐξισώσεων Euler τῆς ὑδροδυναμικῆς στὴν μετανευτώνεια προσέγγιση τῆς Γενικῆς Θεωρίας τῆς Σχετικότητας ἀποδεικνύεται ὅτι ὁ πιὸ γενικὸς μεταγαλιλαιϊκὸς μετασχηματισμὸς ποὺ ἀφήνει ἀμετάβλητη τὴ συναρτησιακὴ μορφὴ τῶν ἐξισώσεων αὐτῶν εἶναι ὁ ἴδιος μὲ τὸν ἀντίστοιχο μετασχηματισμὸ ποὺ ἀφήνει ἀμετάβλητη τὴ συναρτησιακὴ μορφὴ τοῦ μετρικοῦ τανυστῆ. Μ' αὐτὸ τὸν τρόπο γενικεύεται ἡ ἔννοια τῆς μεταγαλιλαιϊκῆς ἀμεταβλητότητας, ἀποδεικνύεται ἡ μοναδικότητά της καὶ ἐκτείνεται ἡ περιοχή, ὅπου αὐτὴ ἰσχύει.