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THE FORM FACTOR AND CHARGE DENSITY DISTRIBUTION OF THE ^{4}He NUCLEUS IN THE QUARATI AND WATT APPROXIMATION

by

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Abstract: *The expression for the elastic electron scattering form factor of the* ^{4}He *nucleus (derived by means of a simple correlation function), which was given originally by P. Quarati and A. Watt, is considered. A test of the validity of their basic approximation is made and an approximate analytic expression for the charge density distribution of* ^{4}He *and its r.m.s. radius is obtained. Computations are also performed and comments are made on the results obtained.*

1. *Introduction*

A number of authors have tried to explain the elastic electron scattering form factor behaviour of the ^{4}He nucleus by considering the effects of short-range dynamical correlations[4,2,11,8]. In many cases, these are included in the nuclear wave function, by using the Jastrow method[7], which consists of replacing a shell-model wave function Φ_{SM} by

$$\Psi = \left[\prod_{i<j}^{A} (1-f(r_{ij})) \right]^{1/2} \Phi_{SM} \tag{1}$$

The correlation function f is supposed to have the following properties:

$$\begin{aligned} &f(0) = 1 \\ &f(r) \simeq 0 \text{ for large interparticle distances} \end{aligned} \tag{2}$$

In order to calculate the charge form factor, in Born approximation[3]

$$F_{ch}(q) = f_p(q^2) f_{CM} \frac{1}{A} \frac{\langle \Psi \mid \sum_{k=1}^{A} e^{i\vec{q}\cdot\vec{r}_k} \mid \Psi \rangle}{\langle \Psi \mid \Psi \rangle}$$

$$= f_p(q^2) f_{CM} \frac{\langle \Psi \mid e^{i\vec{q}\cdot\vec{r}_1} \mid \Psi \rangle}{\langle \Psi \mid \Psi \rangle} \tag{3}$$

(where $f_p(q^2)$ is the proton form factor and f_{CM} is the correction for the centre of mass motion), it is usual to expand the product $\prod_{i<j}^{A} (1-f(r_{ij}))$ in a series of the form

$$\prod_{i<j}^{A} (1-f(r_{ij})) = 1 - \sum_{i<j}^{A} f(r_{ij}) + \sum_{\substack{i<j \\ k<l}}^{A} f(r_{ij}) f(r_{kl}) - 0(f^3) + \ldots \tag{4}$$

where A is equal to 4, in the case of ^{4}He.

In most calculations, terms containing products of two or more correlation functions are neglected. If a more detailed study is desirable, however, correlations involving more than two particles must be taken into account. This can be done exactly for ^{4}He if a simple correlation function is used[2,11]. On the other hand, Quarati and Watt have proposed a simple approximate scheme for this purpose[9]. This is discussed in the next section.

2. *The approximate expression of Quarati and Watt*

Quarati and Watt bave investigated, analytically, correlations of all orders in ^{4}He for the following functions a)a Gaussian: $f(r) = \exp(-r^2/b^2)$ and b)a step function: $f(r) = 1$ for $r<r_c$ and $f(r) = 0$ for $r>r_c$.

They have made the usual assumption that Φ_{SM} is built entirely from the harmonic-oscillator 1s shell single-particle wave functions. Furthermore, they have assumed that the function f is much different from zero only in a region of space much smaller than the nuclear volume. Tbis means that we may, to a good approximation, separate any integral of correlation functions and single-particle wave functions into an integral of correlation functions and an integral of single-particle wave functions.

Using the Gausian form for f, Quarati and Watt have obtained the following approximate expression for the form factor

$$
\begin{aligned}
F(q) = f_p(q^2)f_{CM}\frac{1}{N}[e^{-x} &- \{3\gamma 2^{-3/2}(e^{-x}+e^{-x/2})\} \\
&+ \{9\gamma^2 3^{-3/2}e^{-x/3} + \frac{3}{8}\gamma^2 e^{-x/2} + 3^{-1/2}\gamma^2 e^{-x}\} \\
&- \{2\gamma^3 e^{-x/4} + \frac{1}{9}\gamma^2 e^{-x/3} + \frac{\gamma^2}{27}e^{-x}\} \\
&+ \{\frac{1}{2}3^{-1/2}\gamma^3 e^{-x/4} + \frac{3}{64}\gamma^3 e^{-x/4}\} \\
&- \{6\gamma^3(32)^{-3/2}e^{-x/4}\} \\
&+ \{\frac{\gamma^3}{512}e^{-x/4}\}] \qquad (5)
\end{aligned}
$$

where $x = q^2\alpha^2/4$, $\gamma = (b/\alpha)^3$, a is the harmonic ascillator parameter and N is the normalization factor which may be easily obtained from (5), since $F(0) = 1$. The terms in curly brackets are the contributions from products of 1,2, . . . 6 correlation functions and we shall refer to them as $\tilde{F}^{(2)}(q)$, $\tilde{F}^{(3)}(q), \ldots \tilde{F}^{(7)}(q)$, and $\tilde{F}^{(1)}(q)$ being the term e^{-x} (resulting from the unity in (1)), the usual harmonic-oscillator (h.o.) shell-model form factor in Born approximation. The corresponding quantities resulting from them if the finite size of the proton and the centre of mass motion correction are taken into account, will be referred to as $F^{(1)}(q)$, $F^{(2)}(q) \ldots F^{(7)}(q)$.

3. Test of the validity of the approximation

It is possible to make a test of the validity of the approximation of Quarati and Watt, if we compare the results based on it with those corresponding to the exact calculation of the integrals. This has been done for the integrals containing one correlation function, where the exact calculation can be easily obtained and is in fact known[3]. The approximate expression for the form factor if we consider the contribution from one correlation function is

$$F^{(1,2)}(q) = f_p(q^2)f_{CM}\frac{1}{N^{(1,2)}}[e^{-x} - \{3\gamma 2^{-3/2}(e^{-x}+e^{-x/2})\}] \qquad (6)$$

as it is clear from expression (5). The corresponding exact expression is the following:

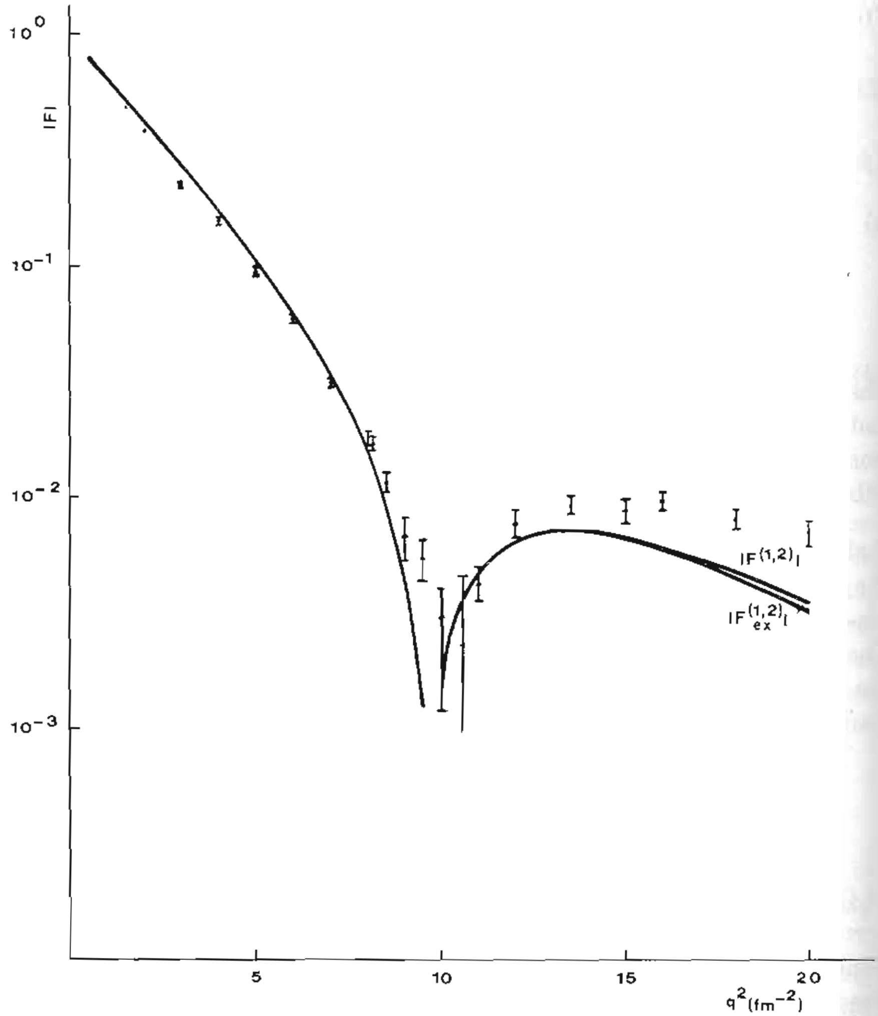

Fig. 1. The two-body form factor (absolute values) computed from the exact and the approximate expression as a function of q^2.

$$F_{ex}^{(1,2)}(q) = f_p(q^2)\, f_{CM} \frac{1}{N_{ex}^{(1,2)}} [e^{-x} - 3(1 + 2\gamma^{-2/3})^{-3/2} \quad (7)$$

$$(e^{-x} + e^{-\frac{x}{2}(1 + \frac{1}{1+2\gamma^{-2/3}})})]$$

By comparing expressions (6) and (7) we may easily conclude that $F^{(1,2)}$ (q) is a good approximation to $F^{(1,2)}(q)$, as long as the following condition is satisfied:

$$\frac{1}{2} (\frac{b}{\alpha})^2 << 1 \quad (8)$$

It is interesting to see whether this is indeed the case if the corresponding best fit values are used in the computation. The best values obtained with $F^{(1,2)}(q)$ are: $\alpha = 1.198$ fm and $b = 0.623$ fm, while those with $F_{ex}^{(1,2)}(q)$ are $\alpha = 1.169$ fm and $b = 0.734$ fm. In the former case the value of $1/2\,(b/\alpha)^2$ is therefore 0.135, while in the latter $1/2(b/\alpha)^2 = 0.197$. It is seen that the required condition is not fully satisfied, with the effect of obtaining quite large differences between the values of $|F^{(1,2)}(q)|$ and $|F_{ex}^{(1,2)}(q)|$ (owing to the exponentials involved in the formulae), as we observe from tables I and II, where the quantities $F^{(2)}$, $F_{ex}^{(2)}$, $F^{(1,2)}$, $F_{ex}^{(1,2)}$, together with the experimental values and errors, are given for various values of the momentum transfer q.

This does not mean, of course, that the Quarati and Watt expression is not expected to be appropriate for obtaining a reasonable approximation to the exact expression. This is due to the fact that when the corresponding best fit values are used in comparing the values of $F^{(1,2)}$ (q) and $F_{ex}^{(1,2)}$ (q), the results are very close, as it is clear from columns 5 and 4 of tables I and II (see also fig. 1). The change in the functional form of the form factor has the effect of leading to different values of the parameters (and in particular to the value of b, which arises from the correlations), when these are determined by the fitting procedure, with the result that the numerical values of the form factor differ very little from those of the original expression in a wide range of momentum transfer. It is worth-mentioning that an analogous situation appears in comparing the form factor in the two-body approximation with that of the exact expression in all orders[2].

TABLE I.

The values of $F^{(2)}$, $F_{ex}^{(2)}$, $F^{(1,2)}$, $F_{ex}^{(1,2)}$, *(see text), computed with the best fit values corresponding to* $F^{(1,2)}$, *together with the experimental values* F_{exp} *and their errors.*

q^2 (fm^{-2})	$F_{ex}^{(2)}$	$F^{(2)}$	$F_{ex}^{(1,2)}$	$F^{(1,2)}$	F_{exp}	Error
0.50	—0.28189	—0.36795	0.81754	0.81218	0.79600	0.02500
1.00	—0.24359	—0.31987	0.66749	0.65808	0.62600	0.03000
1.50	—0.21082	—0.27863	0.54418	0.53178	0.49400	0.01500
2.00	—0.18274	—0.24319	0.44292	0.42839	0.39100	0.01200
3.00	—0.13792	—0.18633	0.29172	0.27485	0.22500	0.00800
4.00	—0.10472	—0.14384	0.19033	0.17286	0.15850	0.00500
5.00	—0.07996	—0.11183	0.12265	0.10565	0.09650	0.00300
6.00	—0.06138	—0.08752	0.07776	0.06182	0.05950	0.00300
7.00	—0.04735	—0.06892	0.04819	0.03364	0.03190	0.00140
8.00	—0.03671	—0.05457	0.02891	0.01586	0.01840	0.00110
8.10	—0.03579	—0.05333	0.02740	0.01451	0.01750	0.00110
8.50	—0.03237	—0.04865	0.02200	0.00971	0.01180	0.00110
9.00	—0.02857	—0.04343	0.01648	0.00494	0.00690	0.00150
9.50	—0.02525	—0.03880	0.01209	0.00127	0.00550	0.00110
10.00	—0.02233	—0.03471	0.00861	—0.00150	0.00310	0.00190
10.50	—0.01977	—0.03108	0.00587	—0.00356	0.00230	0.00230
11.00	—0.01751	—0.02786	0.00373	—0.00505	0.00430	0.00070
12.00	—0.01378	—0.02243	0.00081	—0.00677	0.00780	0.00110
13.50	—0.00967	—0.01630	—0.00136	—0.00739	0.00940	0.00080
15.00	—0.00682	—0.01191	—0.00209	—0.00684	0.00890	0.00110
16.00	—0.00542	—0.00968	—0.00217	—0.00620	0.00980	0.00080
18.00	—0.00343	—0.00643	—0.00190	—0.00479	0.00820	0.00080
20.00	—0.00219	—0.00429	—0.00147	—0.00352	0.00700	0.00090

TABLE II.

The values of $F^{(2)}$, $F_{ex}^{(2)}$, $F^{(1,2)}$, $F_{ex}^{(1,2)}$, *computed with the best fit values corresponding to* $F_{ex}^{(1,2)}$.

q^2 (fm^{-2})	$F_{ex}^{(2)}$	$F^{(2)}$	$F_{ex}^{(1,2)}$	$F^{(1,2)}$	F_{exp}	Error
0.50	—0.57900	—0.96390	0.81349	0.79300	0.79600	0.02500
1.00	—0.50145	—0.84138	0.66006	0.62410	0.62600	0.03000
1.50	—0.43483	—0.73575	0.53401	0.48664	0.49400	0.01500
2.00	—0.37754	—0.64454	0.43060	0.37508	0.39100	0.01200
3.00	—0.28567	—0.49726	0.27661	0.21215	0.22500	0.00800
4.00	—0.21720	—0.38627	0.17400	0.10732	0.15850	0.00500
5.00	—0.16593	—0.30200	0.10626	0.04142	0.09650	0.00300
6.00	—0.12732	—0.23756	0.06206	0.00138	0.05950	0.00300
7.00	—0.09812	—0.18794	0.03364	0.02169	0.03190	0.00140
8.00	—0.07592	—0.14945	0.01576	—0.03378	0.01840	0.00110
8.10	—0.07401	—0.14610	0.01440	—0.03456	0.01750	0.00110
8.50	—0.06687	—0.13351	0.00960	—0.03703	0.01180	0.00110
9.00	—0.05896	—0.11941	0.00483	—0.03893	0.00690	0.00150
9.50	—0.05202	—0.10691	0.00118	—0.03978	0.00550	0.00110
10.00	—0.04594	—0.09581	—0.00157	—0.03982	0.00310	0.00190
10.50	—0.04061	—0.08595	—0.00359	—0.03924	0.00230	0.00230
11.00	—0.03592	—0.07717	—0.00504	—0.03821	0.00430	0.00070
12.00	—0.02816	—0.06237	—0.00668	—0.03526	0.00780	0.00110
13.50	—0.01965	—0.04555	—0.00718	—0.02982	0.00940	0.00080
15.00	—0.01378	—0.03346	—0.00654	—0.02433	0.00890	0.00110
16.00	—0.01090	—0.02730	—0.00587	—0.02095	0.00980	0.00080
18.00	—0.00686	—0.01827	—0.00442	—0.01519	0.00820	0.00080
20.00	—0.00434	—0.01228	—0.00316	—0.01080	0.00700	0.00090

4. An approximate analytic expression for the charge density distribution and the r.m.s. radius of ^{4}He

It may be pointed out that a feature of the Quarati and Watt expression for the form factor is that it is suitable to obtain from it a rather simple analytic expression for the charge density distribution and the r.m.s. radius of ^{4}He. The integrals in the sine-Fonrier transform:

$$\rho(r') = \frac{1}{2\pi^2 r'} \int_0^\infty F(q) \sin(qr')\, qdq \qquad (9)$$

may be calculated analytically, if in addition a proton form factor of Gaussian form $f_p(q^2) = \exp(-\alpha_p^2 q^2/4)$ is used. The final result is the following:

$$\rho_{ch}(r') = \frac{1}{N} \sum_{j=1}^{4} c_j A_j(r') \qquad (10)$$

where the functions $A_j(r')$ are given by

$$A_j(r') = \frac{1}{(\pi(\alpha_p^2 + \frac{(4-j)}{4j}\alpha^2))^{3/2}} e^{-\frac{r'^2}{\alpha_p^2 + \frac{(4-j)}{4j}\alpha^2}} \qquad (11)$$

and

$$c_1 = 1 - d_{11} + d_{23} - d_{33} = 1 - 3 \cdot 2^{-3/2}\gamma + (3^{-1/2} - \frac{1}{27})\gamma^2$$

$$c_2 = -d_{11} + d_{22} = -3 \cdot 2^{-3/2}\gamma + \frac{3}{8}\gamma^2$$

$$c_3 = d_{21} - d_{32} = (9 \cdot 3^{-3/2} - \frac{1}{9})\gamma^2 \qquad (12)$$

$$c_4 = -d_{31} + d_{41} + d_{42} - d_{51} + d_{61} = (-2 + \frac{1}{2} \cdot 3^{-1/2} + \frac{3}{64} - 6 \cdot (32)^{-3/2} + \frac{1}{512})\gamma^3$$

$$N = c_1 + c_2 + c_3 + c_4$$

The explicit expressions of d_{ij} are

$$
\begin{aligned}
&d_{11} = 3\gamma 2^{-3/2} && d_{21} = 9\gamma^2 3^{-3/2} \\
&d_{22} = \frac{3}{8}\gamma^2 && d_{23} = 3^{-1/2}\gamma^2 \\
&d_{31} = 2\gamma^3 && d_{32} = \frac{1}{9}\gamma^2 \\
&d_{33} = \frac{\gamma^2}{27} && d_{41} = \frac{1}{2}3^{-1/2}\gamma^3 \\
&d_{42} = \frac{3}{64}\gamma^3 && d_{51} = 6\gamma^3(32)^{-3/2} \\
&d_{61} = \frac{\gamma^3}{512}
\end{aligned}
\tag{13}
$$

The first index of the constants d_{ij} denotes «the order of the correlation term» (i.e. that with one f, two f's etc) from which it was derived, while the second is simply used to distinguish the various constants existing in the expression of a correlation term of given order. An analytic expression of the root-mean-square radius R can also be obtained through the well-known formula $R = <r^2>^{1/2}$, where:

$$<r'^2> = 4\pi \int_o^{\infty} \rho(r')\ r'^4 dr' \tag{14}$$

by using expression (10) for $\rho(r')$. The reuslt is the following:

$$<r'^2> = \frac{3}{2}\frac{1}{N}\sum_{j=1}^{4} c_j \left(\alpha_p{}^2 + \frac{(4-j)}{4j}\alpha^2\right) \tag{15}$$

It may be finally noted that analytic expressions can also be easily derived for the charge density distribution and the r.m.s. radius in the two-body approximation. The corresponding Quarati-Watt and exact expressions are

$$\rho^{(1,2)}(r') = \frac{1}{1-6\gamma 2^{-3/2}}\left[\frac{1-3\gamma 2^{-3/2}}{(\pi(\alpha_p^2+\frac{3\alpha^2}{4}))^{3/2}} e^{-\frac{r'^2}{\alpha_p^2+\frac{3\alpha^2}{4}}} - \frac{3\gamma 2^{-3/2}}{(\pi(\alpha_p^2+\frac{\alpha^2}{4}))^{3/2}} e^{-\frac{r'^2}{\alpha_p^2+\frac{\alpha^2}{4}}}\right] \tag{16}$$

and

$$\rho_{ex}^{(1,2)}(r') = \frac{1}{1-6(1+2\gamma^{-2/3})^{-3/2}}\left[\frac{(1-3(1+2\gamma^{-2/3})^{-3/2})}{\pi^{3/2}A^3} e^{-\frac{r'^2}{A^2}} - \frac{3(1+2\gamma^{-2/3})^{-3/2}}{\pi^{3/2}B^3} e^{-\frac{r'^2}{B^2}}\right] \tag{17}$$

$$\text{where } A^2 = \alpha_p^2 + \frac{3\alpha^2}{4} \text{ and } B^2 = \alpha_p^2 + \frac{\alpha^2}{4} + \frac{\alpha^2}{2(1+2\gamma^{-2/3})} \tag{18}$$

For their r.m.s. radii we obtain

$$R^{(1,2)} = [\frac{3}{2}\frac{1}{N^{(1,2)}}\{\alpha_p^2 + \frac{3\alpha^2}{4} - 3\gamma 2^{-3/2}(2\alpha_p^2+\alpha^2)\}]^{1/2} \tag{19}$$

and

$$R_{ex}^{(1,2)} = [\frac{3}{2}\frac{1}{N_{ex}^{(1,2)}}\{\alpha_p^2 + \frac{3\alpha^2}{4} - 3(1+2\gamma^{-2/3})^{-3/2}(2\alpha_p^2+\alpha^2 + \frac{\alpha^2}{2(1+2\gamma^{-2/3})})\}]^{1/2} \tag{20}$$

respectively. In fig. 2, $\rho^{(1,2)}$ and $\rho_{ex}^{(1,2)}$ have been plotted against r'.

It may be noted that the expression for the charge density distribution in the two body approximation is a sum of two Gaussian terms as it has also been found in ref. 6. where the same simple correlation function was used and the Unitary model-operator formalism. The expres-

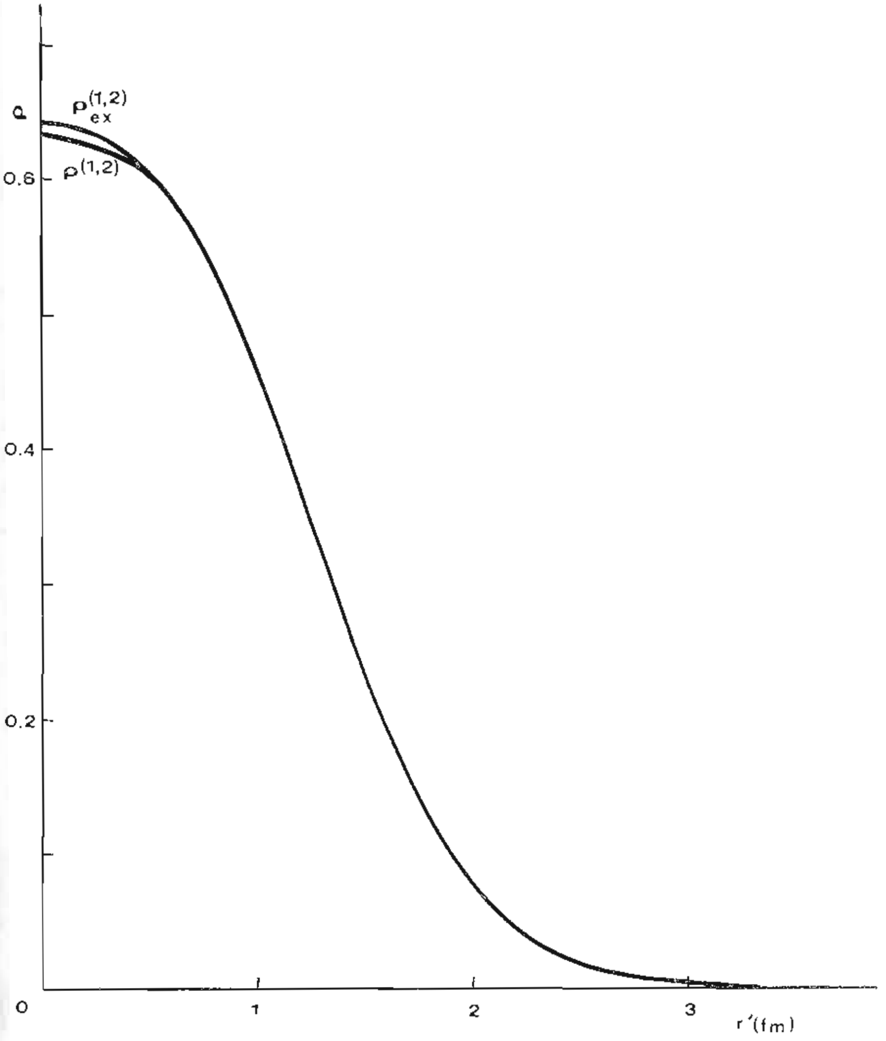

Fig. 2. The charge density distribution in the two-body approximation derived from the exact and the approximate form factor, against r′.

sion of ref. 6 may be written as follows:

$$\rho_{uo}^{(1,2)}(r') = \frac{1}{1-(1+\lambda)^{-3/2}} \left[\{1+2(1+\lambda)^{-3/2}\} \frac{e^{-\frac{r'^2}{A^2}}}{\pi^{3/2}A^3} - 3(1+\lambda)^{-3/2} \frac{e^{-\frac{r'^2}{B^2}}}{\pi^{3/2}B^3} \right] \tag{21}$$

where $\lambda = 2\gamma^{-2/3}$. By comparing with expressions (16) and (17) it is seen that the expressions of the coefficients of the volume-normalized to unity Gaussians are different.

From the previous analysis it is obvious that inclusion of higher order terms in the form factor of ^4He, through the Quarati and Watt approximation, with the correlation function considered, has a rather simple effect on the functional form of the charge density distribution. Apart from modifying the two Gaussian terms existing in the two-body approximation, it leads to the appearance of two additional Gaussian terms.

It is worth mentioning that appropriate sums of Gaussian terms for $\rho_{ch}(r')$ have also been used in analysing the form factor of light nuclei. In these analyses, the form of the density distribution is phenomenological[10].

5. Numerical results and comments

The form factor was computed in the various orders, treating a and b as adjustable parameters. We used a least-squares fit program, which minimizes the quantity

$$\chi^2 = \sum_i \left(\frac{F(q_i) - F_{exp}(q_i)}{\sigma_i} \right)^2 \tag{22}$$

The experimental values of the form factor $F_{exp}(q_i)$ aud their errors σ_i were taken from ref. 5. The «best fit» values of the parameters and the corresponding r.m.s. radii are given in table III. These values correspond to the various expressions of the form factor resulting by including successively higher terms in the cluster expansion:

$F^{(1)}(q) = f_p(q^2) f_{CM} \frac{1}{N^{(1)}} \tilde{F}^{(1)}(q)$, $F^{(1,2)}(q) = f_p(q^2) f_{CM} \frac{1}{N^{(1,2)}} (\tilde{F}^{(1)}(q) + \tilde{F}^{(2)}$ $(q))$, . . . , $F(q)$.

The fitting was obtained with the proton form factor $f_p(q^2) = \exp(-a_p^2 q^2/4)$ $(\alpha_p = 0.653)$ and with the correction for the centre of mass motion $f_{CM} = \exp(a^2 q^2/16)$ [12]

TABLE III.

The best fit values and the r.m.s. radii in the various approximations for the form factor.

Form factor	a (fm)	b (fm)	R (fm)
$F^{(1)}$ (q)	1.43	—	1.45
$F^{(1,2)}$ (q)	1.20	0.62	1.83
$F^{(1,2,3)}$ (q)	1.34	0.83	1.48
$F^{(1,2,..,4)}$ (q)	1.16	0.76	1.57
$F^{(1,2,..,5)}$ (q)	1.14	0.80	1.56
$F^{(1,2,..,6)}$ (q)	1.14	0.79	1.56
$F^{(1,2,..,7)} \equiv F(q)$	1.14	0.80	1.56

It is seen from table III that the changes in the best fit values are extremely small if the correlations of order higher than 4 are included. It should be noted that the smallest value of χ^2 is obtained in the two body approximation.

It is worth mentioning that the best fit values corresponding to the F(q) with the proton form factor of ref. 1, are: $\alpha = 1.21$ fm and $b = 0.79$ fm. It turns out that although the best fit values do not differ very much from their corresponding values obtained with the Gaussian proton form factor, the fitting is considerably improved. A similar improvement also appears for $F^{(1,2)}$ (q) where $\alpha = 1.28$ fm and $b = 0.62$ fm.

In figure 3 the $F^{(1)}$ (q), $F^{(1,2)}$ (q) and F (q) have been plotted as functions of q^2. The experimental points are also indicated.

In figure 4 we plot $F^{(1)}$ (q), $F^{(2)}$ (q), . . ., $F^{(7)}$ (q), the contributions to the form factor arising from the shell model and from integrals con-

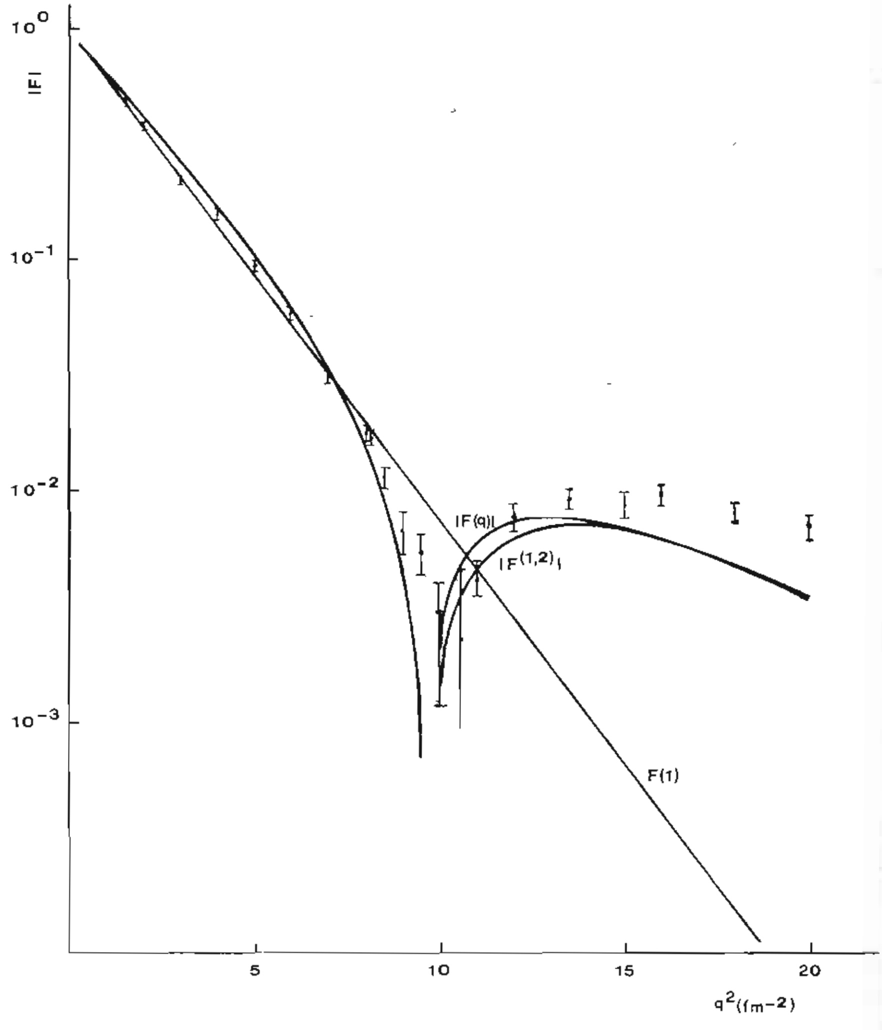

Fig. 3. The charge form factors (absolute values) $F^{(1)}(q)$, $F^{(1,2)}(q)$ and $F^{(1,..,7)} \equiv F(q)$ against q^2.

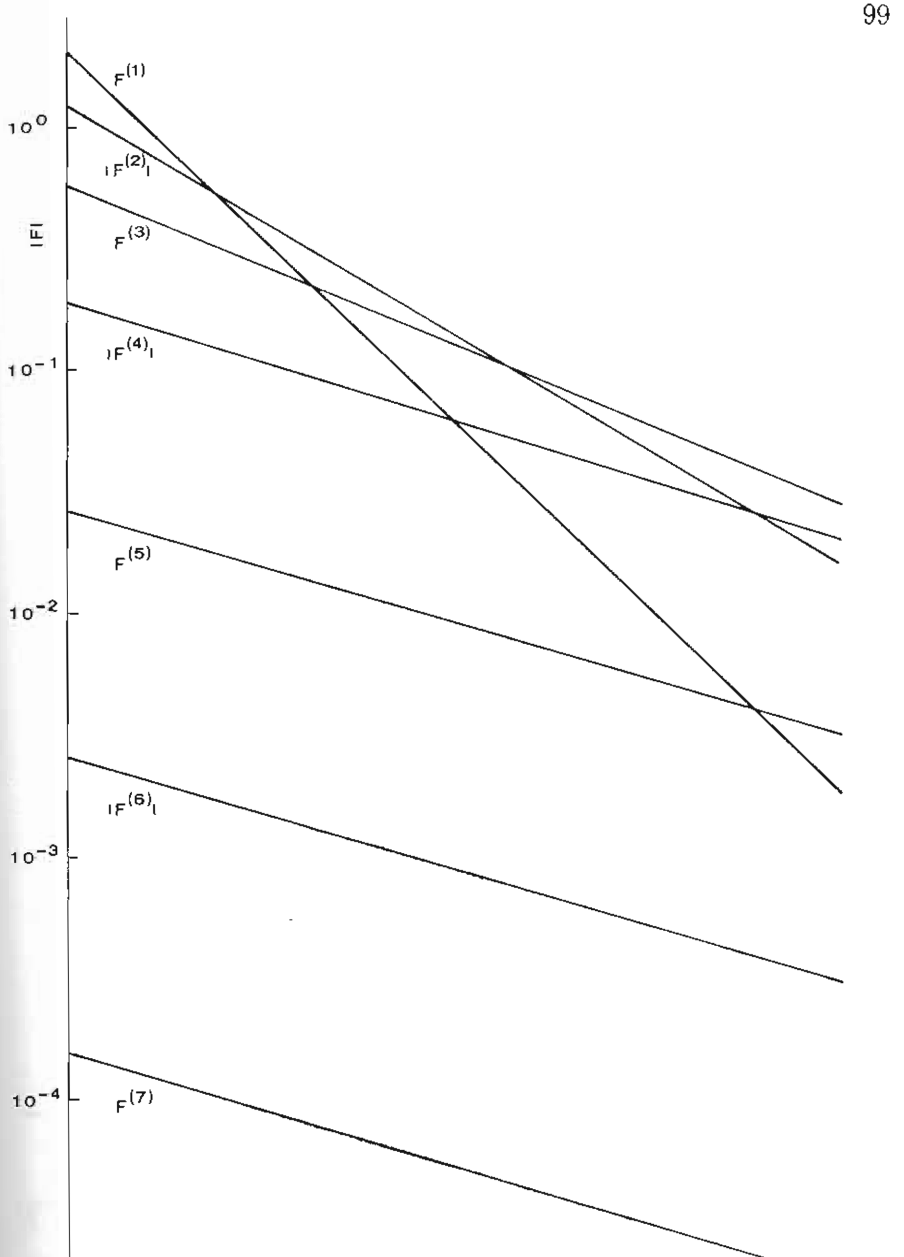

Fig. 4. The contributions to the form factor arising from the shell model and from $F^{(1)}(q)$ $F^{(2)}(q), \ldots, F^{(7)}(q)$ *(absolute values) against* q^2.

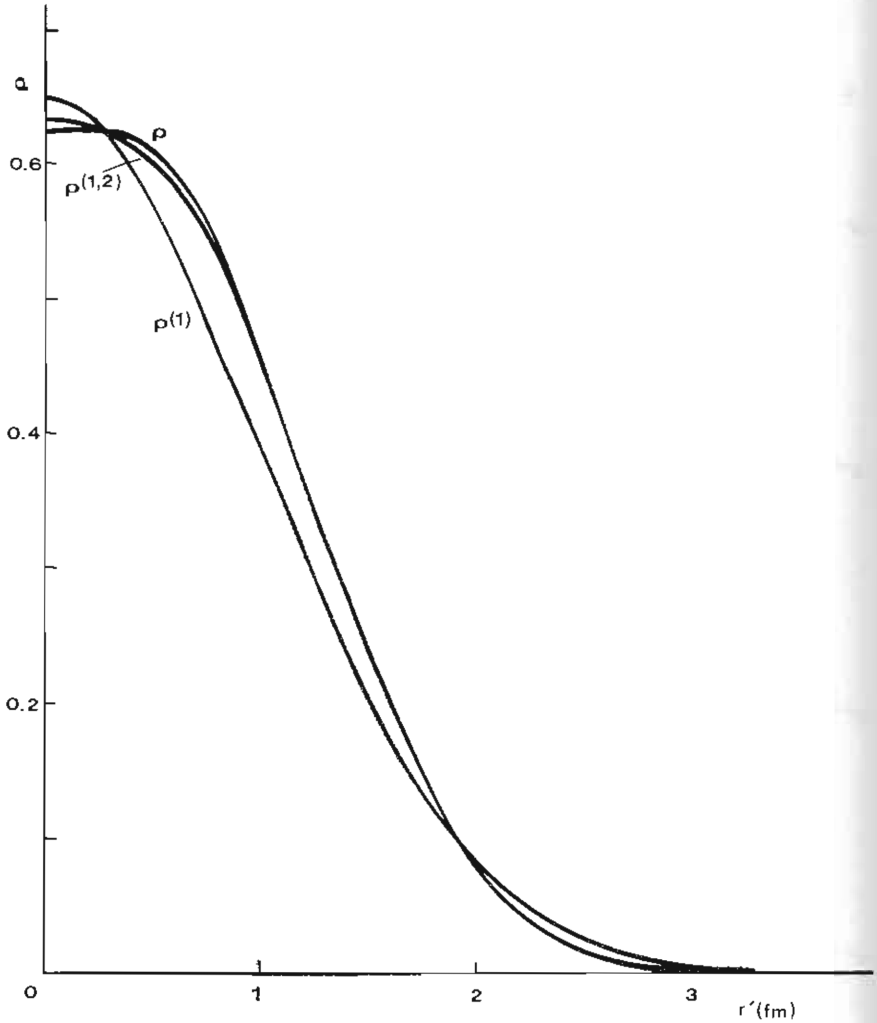

Fig. 5. The charge density distribution corresponding to $F^{(1)}(q)$, $F^{(1,2)}(q)$ and $F(q)$ as a function of r'.

taining products of 1, 2, . . . 6 correlation functions, against q^2. The best fit values corresponding to F(q) have been used.

Finally, the charge density distributions corresponding to $F^{(I)}$,(q), $F^{(1,2)}$ (q) and F(q) are plotted in figure 5 as a function of r′. All the approximations give about the same results for the higher values of r′. The second order and the higher approximations give about similar results. The maximum value of the charge density, in the three approximations of higher order, appears at a distance slightly different from zero. However the difference between this maximum and the value of the density at $r' = 0$ is extremely small. This difference should be larger if the correlation function were appropriate for a hard-core nucleon-nucleon potential.

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ΠΕΡΙΛΗΨΗ

Ο ΣΥΝΤΕΛΕΣΤΗΣ ΣΧΗΜΑΤΟΣ ΚΑΙ Η ΠΥΚΝΟΤΗΤΑ ΦΟΡΤΙΟΥ ΤΟΥ ΠΥΡΗΝΑ ΤΟΥ ^{4}He ΣΤΗΝ ΠΡΟΣΕΓΓΙΣΗ ΤΩΝ QUARATI ΚΑΙ WATT

Ὑπὸ

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Στὴν ἐργασία αὐτὴ θεωροῦμε τὴν ἔκφραση τοῦ συντελεστῆ σχήματος τοῦ πυρήνα ^{4}He (ποὺ προκύπτει μὲ τὴν βοήθεια μιᾶς ἁπλῆς συναρτήσεως ἀλληλοσυσχετίσεως) κατὰ τὴν ἐλαστικὴ σκέδαση ἠλεκτρονίων, ἡ ὁποία δόθηκε ἀρχικὰ ἀπὸ τοὺς P. Quarati καὶ A. Watt. Γίνεται ἕνας ἔλεγχος τῆς ἰσχύος τῆς βασικῆς τους προσεγγίσεως καὶ δίνεται μία προσεγγιστικὴ ἀναλυτικὴ ἔκφραση τῆς κατανομῆς τῆς πυκνότητας φορτίου τοῦ ^{4}He καθὼς καὶ τῆς τετραγωνικῆς ρίζας τῆς μέσης τετραγωνικῆς ἀκτίνας της. Πραγματοποιοῦνται ἐπίσης ἀριθμητικοὶ ὑπολογισμοὶ καὶ διατυπώνονται σχόλια σχετικὰ μὲ τὰ ἀποτελέσματα.