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THE CHARGE FORM FACTOR OF THE ^{4}He NUCLEUS IN THE NON-UNITARY MODEL OPERATOR APPROACH

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Abstract: *The charge form factor of the ^{4}He nucleus is studied in the framework of the non-unitary model operator approach. An approximate expression of the form factor of light closed-shell nuclei is firstly obtained by keeping the one and two-body parts of the factor cluster expansion and using harmonic oscillator wave functions in the initial formula of Clark and Ristig. As correlated relative wave functions are used those derived by means of the variational principle and the separation condition. The results of the calculation are compared to the experimental values of the charge form factor of ^{4}He.*

1. Introduction

The ^{4}He nucleus is interesting for two reasons: firstly it appears in many nuclear reactions due to its great stability and secondly it plays an important role in the construction of nuclear wave functions as a unit of four almost spatially symmetric nucleons. The most valuable information about the ^{4}He nucleus charge form factor has been extracted from experiments on high energy scattering of electrons[7, 1] and nucleons at a ^{4}He target. Recently the elastic electron scattering form factor of ^{4}He has been measured for higher values of the momentum transfer q, namely for values of the momentum transfer square q^2 as high as 46 fm^{-2} [1].

A considerable amount of theoretical work has been done in order to reproduce the characteristics of the ^{4}He charge form factor, especially the pronounced diffraction minimum at $q^2 \simeq 10.5\ fm^{-2}$, which is not predicted by the harmonic oscillator shell model, which gives a ^{4}He form factor of Gaussian shape.

This diffraction ninimum can be reproduced by including short-range dynamical nucleon-nucleon correlations in the ground-state wave function of ^{4}He. This approach has been followed initially by Czyz and Lesniak[6] who used a Jastrow-type wave function and also by others who used the same or other cluster - expansion formalisms in calculations for ^{4}He and heavier nuclei [4 8 11 12 17 20].

In this work the ^{4}He charge form factor is studied in the framework of the non - unitary model operator approach[13]. Firstly, the derivation of a general expression for the charge form factor of a light closed - shell nucleus is outlined.

2. The formalism.

In this analysis the charge form factor of a light, closed-shell nucleus is calculated in Born approximation by including in the nuclear ground state wave function two - nucleon short range correlations by means of the non - unitary model operator: $\hat{F}$. This operator relates an eigenstate Φ of the model system to an eigenstate

$$\Psi = \hat{F}\,\Phi \tag{1}$$

of the true system.

Under a convenient choice of the operator $\hat{F}$, the uncorrelated many - body wave function Φ may be well approximated hy a single Slater determinant, the orbital parts of which are subsequently taken to be harmonic - oscillator wave functions.

On the operator $\hat{F}$ several restrictions are imposed, as for example that it depends only on the spins, isospins and relative coordinates and momenta of the particles in the system, it is a scalar with respect to rotations etc[2]. However, $\hat{F}$ is not considered to be (left) unitary[13]. Because of the various restrictions on $\hat{F}$, Ψ is no longer the true wave function of the system but it may only be considered as a trial wave function.

The correlation operator is determined hy varying the energy expectation value keeping only the one and two-body terms in the factor cluster expansion[13].

The charge form factor in Born approximation, of a closed-shell nucleus consisting of A nucleons, can be written:

$$F_{ch}(q) = f_p(q) f_{CM}(q) \frac{1}{A} \frac{<\Psi|\hat{O}|\Psi>}{<\Psi|\Psi>} \equiv f_p(q) f_{CM}(q) \frac{1}{A} <\hat{O}> \tag{2}$$

where

$$\hat{O} = \sum_{i=1}^{A} \hat{o}(i) = \sum_{i=1}^{A} e^{i\vec{q}\cdot\vec{r}_i} \tag{3}$$

$f_p(q)$ is the proton form factor (the correction for the finite proton size) and $f_{CM}(q)$ the correction for the centre of mass motion.

In order to evaluate the expectation value of $\hat{O}$

$$<\hat{O}> = \frac{<\Psi|\hat{O}|\Psi>}{<\Psi|\Psi>} \tag{4}$$

consider first the generalized normalization intergral[5]:

$$I(\alpha) = <\hat{F}\Phi|e^{\alpha\hat{O}}|\hat{F}\Phi> = <\Phi|\hat{F}^{+}e^{\alpha\hat{O}}\hat{F}|\Phi> \tag{5}$$

corresponding to the operator $\hat{O}$, from which

$$<\hat{O}> = \frac{\partial}{\partial\alpha} \ln I(\alpha)\Big|_{\alpha=0} \tag{6}$$

For the cluster analysis of (5) consider next the definition of the subnormalization integrals $I_i(\alpha)$, $I_{ij}(\alpha)$, ... , for the subsystems of the A-nucleon system and a factor cluster decomposition of these subnormalization integrals. The expectation value of the charge form factor operator may then be written in the form of a factor-cluster or Van Kampen type expansion

$$<\hat{O}> = <\hat{O}>_1 + <\hat{O}>_2 + ... + <\hat{O}>_A \tag{7}$$

where

$$<\hat{O}>_1 = \sum_{i=1}^{A} \frac{\partial}{\partial\alpha} \ln I_i\Big|_{\alpha=0}$$

$$= \sum_{i=1}^{A} \frac{<i|\hat{F}_1^{+}\exp(i\vec{q}\cdot\vec{r}_1)\hat{F}_1|i>}{<i|F_1^{+}F_1|i>} \tag{8}$$

and

$$<\hat{O}>_2 = \sum_{i<j}^{A} \left[\frac{1}{I_{ij}} \frac{\partial I_{ij}}{\partial \alpha} - \frac{1}{I_i} \frac{\partial I_i}{\partial \alpha} - \frac{1}{I_j} \frac{\partial I_j}{\partial \alpha} \right] \Bigg|_{\alpha=0} \tag{9}$$

$$= \sum_{i<j}^{A} \left[\frac{<ij|\hat{F}^{+}_{12}\{\hat{o}(1)+\hat{o}(2)\}\hat{F}_{12}|ij - ji>}{<ij|\hat{F}^{+}_{12}\hat{F}_{12}|ij - ji>} - <i|\hat{o}(1)|i> - <j|\hat{o}(2)|j> \right] \tag{10}$$

and so on.

The cluster expansion (7) establishes a separation of one-body, two-body,...,A - body correlation effects on the form factor. Three and many-body terms shall be neglected in the present analysis assuming that their contribution to the expectation value of the form factor is small. Thus, in the two-body approximation $<\hat{O}>$ may be written

$$<\hat{O}> \simeq <\hat{O}>_1 + <\hat{O}>_2 \tag{11}$$

The second and third matrix elements in (10) may be written in terms of the $<\hat{O}>_1$ which is well known. In order to calculate the numerator N_{ij} of the first term in (10), it is useful to make a transformation to relative and centre of mass coordinates of the two interacting nucleons.

Finally, following a procedure similar to that of ref. 2 and 13, using also the properties of the spherical harmonics we derive the expression

$$N_{ij} = \sum_{nln'l'S} \sum_{\nu=0}^{\infty} \tilde{C}^{ij}_{nln'l'S\nu} \quad <nlS|\hat{F}^{+}_{12} \; j_\nu \; (qr/2) \quad \hat{F}_{12}|n'l'S> \tag{12}$$

where

$$|l' - \nu| \leqq l \leqq l' + \nu \tag{13}$$

and the coefficients $\tilde{C}^{ij}_{nln'l'S\nu}$ involve Clebsch - Gordan and Talmi - Smirnov (or Brody - Moshinsky) coefficients [9 14 16 18] and matrix elements of the wave functions of the centre of mass motion:

$$\tilde{C}^{ij}_{nln'l'S\nu} = [\tfrac{1}{2}\,\delta_{M_s0}\delta_{S0} + (\delta_{M_s1} + \tfrac{1}{2}\,\delta_{M_s0} + \delta_{M_s-1})\;\delta_{S1}]$$

$$\cdot\, i^{\nu}(2\nu+1) \left(\frac{2l'+1}{2l+1}\right)^{1/2} <l'0\nu0|l0> \sum_{\substack{NLM \\ N'L'm}} \sum_{K=0}^{\infty} i^{K}\,(2K+1)$$

$$\cdot \left(\frac{2L'+1}{2L+1}\right)^{1/2} <NL|j_K(qR)|N'L'> <L'MK0|LM> <L'0K0|L0>$$

$$\cdot <l'm\nu 0|lm> \sum_{\lambda\lambda'} <lmLM|\lambda\mu> <l'mL'M|\lambda'\mu> <l_i m_i l_j m_j|\lambda\mu>$$

$$\cdot <l_i m_i l_j m_j|\lambda'\mu> <nlNL\lambda|n_i l_i n_j l_j \lambda> <n'l'N'L'\lambda'|n_i l_i n_j l_j \lambda'>$$

$$\cdot \left[1 + (-1)^{l'+S} \delta_{\tau_i \tau_j} \right] \tag{14}$$

The denominator of the first term in (10) is known[13].

$$D_{ij} = \sum_{nlS} C^{ij}_{nlS} <\psi_{nlS}|\psi_{nlS}> \tag{15}$$

where

$$|\psi_{nlS}> = \hat{F}_{12}|nlS>$$

and

$$C^{ij}_{nlS} = \left[\tfrac{1}{2}\delta_{M_s 0}\delta_{S0} + (\delta_{M_s 1} + \tfrac{1}{2}\delta_{M_s 0} + \delta_{M_s -1})\ \delta_{S1} \right] \left[1 + (-1)^{l+S}\delta_{\tau_i \tau_j} \right]$$

$$\sum_{NL\lambda} <n\,l\,N\,L\lambda|n_i l_i n_j l_j \lambda>^2 <l_i m_i l_j m_j|\lambda\mu>^2 \tag{16}$$

The structure of the expression of N_{ij} is similar to analogous expressions which enter in the charge form factor of light nuclei in the Jastrow[4] or the unitary model operator approach[8].

In the calculations of the form factor the wave functions of the relative motion of two interacting particles ψ_{nlS} are derived by means of the variational method. The variational method is applied to the energy expectation value and the resulting Euler equations are solved numerically for a given potential and harmonic oscillator parameter, b_1, using also a separation condition. The «variational Moszkowski and Scott separation distance» is the one at which the wave function and also its derivative are continuous.

The calculation of the form factor of the ^4He nucleus using the previously described approach is quite simple. The result is the following:

$$F_{ch}(q) = f_p(q) f_{CM}(q) \left\{ e^{-\frac{b_1^2 q^2}{4}} - \left\{ 3 e^{-\frac{b_1^2 q^2}{4}} \right. \right.$$

$$\left. \left. - e^{-\frac{b^2{}_1 q^2}{8}} \left[\left(\frac{1}{N_{000}} + \frac{1}{N_{000}+N_{001}} \right) J_{000} + \left(\frac{1}{N_{001}} + \frac{1}{N_{000}+N_{001}} \right) J_{001} \right] \right\} \right\} \quad (17)$$

where

$$N_{00S} \equiv \langle \psi_{00S} | \psi_{00S} \rangle \quad (18)$$

and

$$J_{00S} \equiv \langle \psi_{00S} | j_0 \frac{(qr)}{2} | \psi_{00S} \rangle \quad (19)$$

3. Preliminary numerical calculations for ^4He.

Numerical calculations of the form factor of the ^4He nucleus have been carried out on the basis of the prescribed procedure, using the first expression for the energy expectation value described in ref. 13 and several types of hard-core potentials. For various values of the harmonic oscillator parameter b_1, the theoretical values of the form factor $F_{ch}(q_i)$ given by (17) are compared to the experimental ones $F_{exp}(q_i)$. The value of b_1 which minimizes the quantity

$$\chi^2 = \left(\frac{F_{ch}(q_i) - F_{exp}(q_i)}{\sigma_i} \right)^2 \quad (20)$$

(σ_i is the experimental error) is the «best fit» value of b_1.

The results of the calculations for the potentials of Kallio - Koltveit[10] (K-K) and Moszkowski - Scott[15] (M-S) using also the proton form factor of ref. 3 and the correction for the centre of mass motion $e^{b_1^2 q^2/16}$ [19], are plotted in fig. 1 and 2 together with the experimental values. The corresponding «best fit» values of the parameter b_1 are 1.34 and 1.27 respectively. In the latter case the results present a small discontinuity in the derivative of the wave function. We are currently looking at a possible elimination of this difficulty. With the Kallio - Koltveit potential, however, such a difficulty does not appear. The fit is very good at low momentum transfers but there are quite large deviations at the higher values of q. It should be kept in mind, however, that there is only one parameter in fitting the data, while in the case of correlation functions there are at least two parameters.

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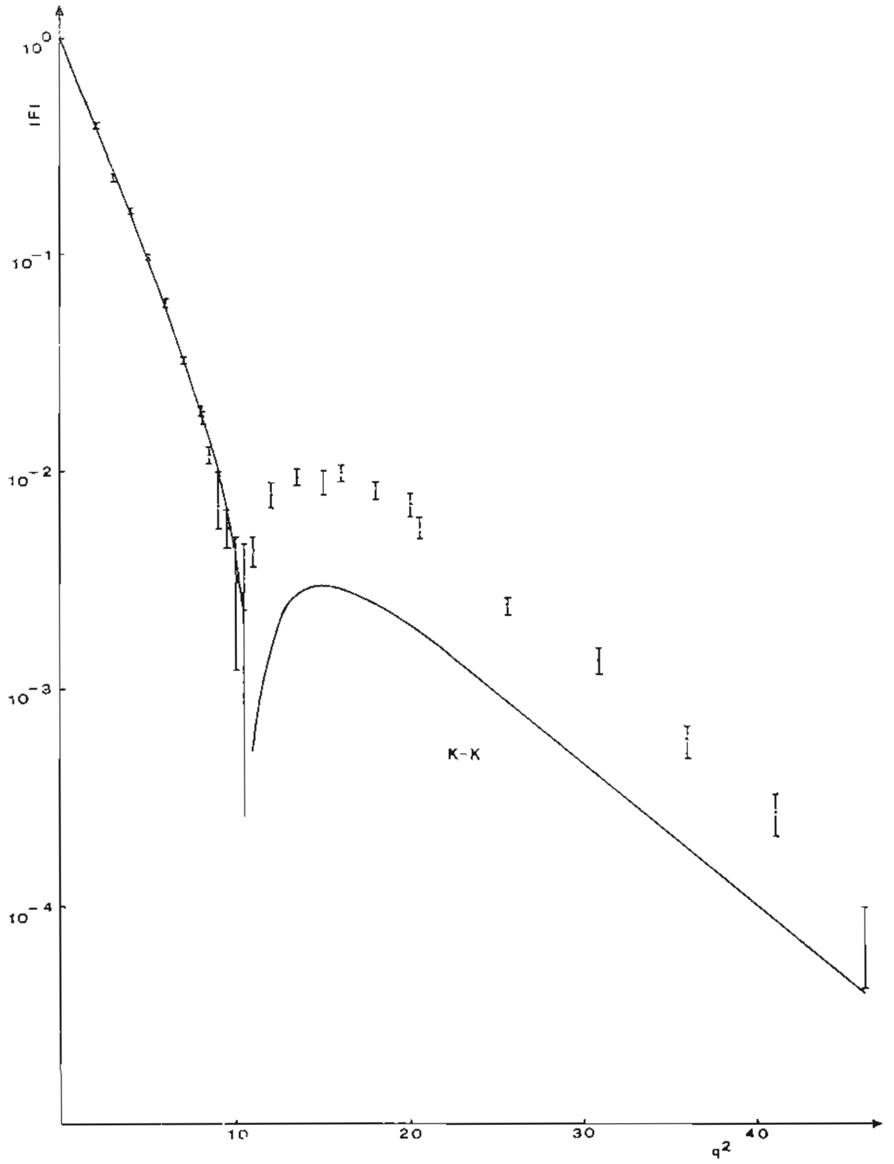

Fig. 1. *The charge form factor of the* ^{4}He *nucleus with the first method and the K-K potential, as a function of* q^2 *(fm^{-2}).*

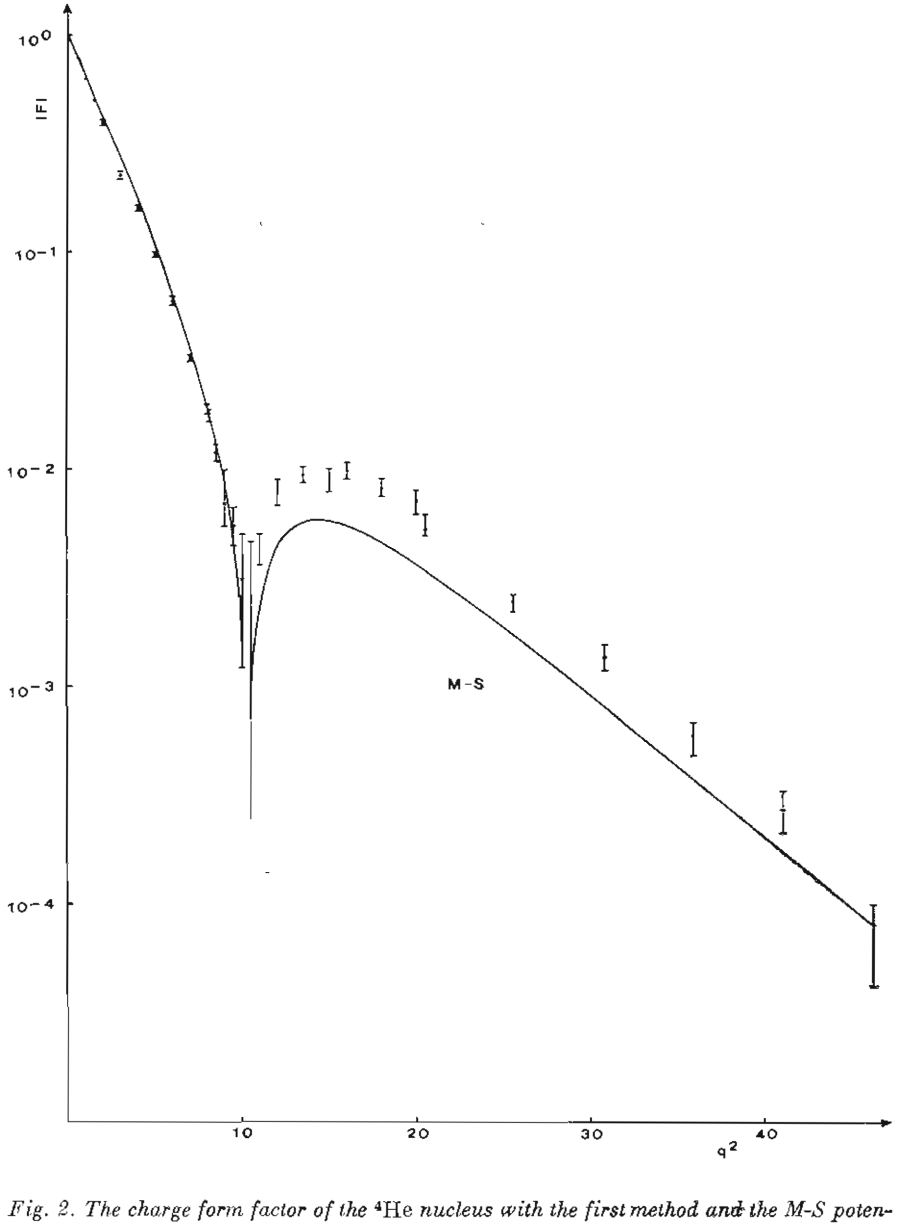

Fig. 2. The charge form factor of the ^{4}He *nucleus with the first method and the M-S potential, against* q^2 *(*fm^{-2}*).*

REFERENCES

1. ARNOLD, R. G. et al (1978): Phys. Rev. Lett., 40, 1429.
2. BRINK, D. M. and GRYPEOS, M. E. (1967): Nucl. Phys., 97A, 81 (1967).
3. CHOU T. T., and YANG C. N. (1968): Phys. Rev. Lett., 20, 1213.
4. CIOFI C., DEGLI ATTI (1969): Nucl. Phys., A 129, 350.
5. CLARK J. W. and RISTIG M. L. (1970): Nuov. Cim., 70A, 3.
6. CZYZ, W. and LESNIAK, L. (1967): Phys. Lett., 25B, 319.
7. FROSCH, R. F., McCARTHY, J. S., RAND, R. E. and YEARIAN, M. R. (1967): Phys. Rev., 160, 874.
8. GRYPEOS M. E. (1969): University of Surrey report.
9. IRVINE J. M. (1972): Nuclear structure Theory.
10. KALLIO A., KOLTVEIT K. (1964): Nucl. Phys. 53, 87.
11. KHANNA F. C. (1968): Phys. Rev. Lett., 20, 871.
12. MALECKI A. and PICCHI P. (1969): Lett. Nuovo Cim., 1, 81.
13. MASSEN S. E. and GRYPEOS M. E. (1975): Nuov. Cim. 30A, 34.
14. MOSHINSKY M. and BRODY T. (1960): Tables of Transformation Brackets, Mografias del Instituto de Fisica (Mexico).
15. MOSZKOWSKI S. A., SCOTT B. L. (1960): Ann. Phys. (N.Y.) 11, 65.
16. SMIRNOV Y. (1961): Nucl. Phys. 27, 177.
17. STOVALL T. and VINCIGUERRA D. (1968): Lett. Nuov. Cim. 1, 100.
18. TALMI I. (1952): Helv. Phys. Acta, 25, 185.
19. TASSIE L. J. and BARKER F. C. (1958): Phys. Rev., 111, 940.
20. WONG S. M. and LIN D. L. (1967): Nucl. Phys. A101, 663.

ΠΕΡΙΛΗΨΗ

Ο ΠΑΡΑΓΟΝΤΑΣ ΜΟΡΦΗΣ ΦΟΡΤΙΟΥ ΤΟΥ ΠΥΡΗΝΑ ΤΟΥ ^{4}He ΣΤΗΝ ΠΡΟΣΕΓΓΙΣΗ ΤΟΥ ΜΗ ΜΟΝΑΔΙΑΙΟΥ ΤΕΛΕΣΤΗ ΠΡΟΤΥΠΟΥ

ὑπὸ

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(Σπουδαστήριο Θεωρητικῆς Φυσικῆς Πανεπιστημίου Θεσσαλονίκης)

Μελετᾶται ὁ παράγοντας μορφῆς φορτίου τοῦ πυρήνα τοῦ ^{4}He μὲ τὴ μέθοδο τοῦ μὴ μοναδιαίου τελεστῆ προτύπου. Δίνεται ἀρχικὰ μιὰ προσεγγιστικὴ ἔκφραση τοῦ παράγοντα μορφῆς ἐλαφρῶν πυρήνων κλειστῶν φλοιῶν βάσει τῶν ὅρων ἑνὸς καὶ δύο σωμάτων τοῦ παραγοντικοῦ ἀναπτύγματος κατὰ συσσωματώματα καὶ μὲ τὴ χρησιμοποίηση κυματοσυναρτήσεων ἁρμονικοῦ ταλαντωτῆ στὸ σχετικὸ τύπο τῶν Clark καὶ Ristig. Ὡς ἀλληλοσυσχετισμένες κυματοσυναρτήσεις τῆς σχετικῆς κινήσεως δύο νουκλεονίων, χρησιμοποιοῦνται αὐτὲς ποὺ προκύπτουν μὲ τὴ βοήθεια τῆς ἀρχῆς τῶν μεταβολῶν καὶ τῆς συνθήκης διαχωρισμοῦ. Τὰ ἀποτελέσματα συγκρίνονται μὲ τὶς πειραματικὲς τιμὲς τοῦ παράγοντα μορφῆς.