Sci. Annals. Fac. Phys. & Mathem., Univ. Thessaloniki, **20b**, 197 (1980)

DIRECT RELATIONS BETWEEN FRESNEL'S AMPLITUDE COEFFICIENTS FOR THE BOUNDARIES OF PLANE PARALLEL ABSORBING PLATE

By

D. B. KUSHEV

Semiconductor Physics and Technology Research Laboratory
University of Sofia

Abstract. *In addition to the direct relations between the Fresnel's complex-amplitude coefficients for the interface of isotropic transparent and absorbing media, similar relations are obtained with all Fresnel's coefficients — for electric and magnetic fields, TE and TK waves, transmission and reflection— for the interface of absorbing and transparent media. The analysis is restricted to the case of plane parallel boundaries system, surrounded by transparent medium. These relations are invariant under the dielectric functions of the two media, which form the interface.*

Recently R. M. A. Azzam [1] has found a remarkable relation between the Fresnel's amplitude reflection coefficients for the parallel and perpendicular polarizations at the same angle of incidence φ:

$$r_p = \frac{r_s(\cos 2\varphi - r_s)}{1 - r_s \cos 2\varphi} \tag{1}$$

The medium of incidence is supposed to be isotropic transparent and the medium of refraction — also isotropic transparent or absorbing. Equation (1) is interesting with its independence on the dielectric functions of the two media which define the interface. As it is shown in [2], such type of invariant relations may be obtained with all Fresnel's amplitude coefficients of the interface under consideration — for electric and magnetic field, TE and TM polarizations, transmission and reflection. Every one of these eight coefficients may be represented by every other without use of material parameters, so the obtained relations are valid for all interfaces of such a type. Several of these relations are simple and widely used, but the remaining seem not to be known earlier.

Following the same idea as in [2] we shall show that similar invariant relations may be derived also for the transmission through and reflection form an interface of absorbing and transparent media if the incident inhomogeneous wave is formed at a boundary which is parallel to the concerned interface.

Let us suppose an absorbing multilayer which is bounded by dielectric medium of refractive index n_0. The wave incident upon the first boundary of this stratified medium will be presumed to be homogeneous one. The angle of incidence is φ. Since the refraction angle after passing the last boundary, according to the Snell's law, is also φ, the Fresnel's amplitude coefficients for the considered interface between the last layer with complex refractive index $N = n + ik$ and the final phase with refractive index n_0 may be written in the following form [3].

TE wave:

$$\rho_s = \frac{E_{rs}}{E_{is}} = \frac{(\epsilon - \sin^2\varphi)^{1/2} - \cos\varphi}{(\epsilon - \sin^2\varphi)^{1/2} + \cos\varphi}\,, \qquad \rho_\pi = \frac{B_{rp}}{B_{ip}} = -\rho_s,$$

$$\tau_s = \frac{E_{ts}}{E_{is}} = \frac{2(\epsilon - \sin^2\varphi)^{1/2}}{(\epsilon - \sin^2\varphi)^{1/2} + \cos\varphi}\,, \qquad \tau_\pi = \frac{B_{tp}}{B_{ip}} = \tau_s/\sqrt{\epsilon};$$

(2)

TM wave:

$$\rho_p = \frac{E_{rp}}{E_{ip}} = \frac{\epsilon\cos\varphi - (\epsilon - \sin\varphi^2)^{1/2}}{\epsilon\cos\varphi + (\epsilon - \sin^2\varphi)^{1/2}}\,, \qquad \rho_\sigma = \frac{B_{rs}}{B_{is}} = \rho_p,$$

$$\tau_p = \frac{E_{tp}}{E_{ip}} = \frac{2\sqrt{\epsilon}(\epsilon - \sin^2\varphi)^{1/2}}{\epsilon\cos\varphi + (\epsilon - \sin^2\varphi)^{1/2}}\,, \qquad \tau_\sigma = \frac{B_{ts}}{B_{is}} = \tau_p/\sqrt{\epsilon},$$

ϵ is the ratio of the dielectric function of the medium of incidence to that of the medium of refraction, i.e. $\epsilon = (N/n_0)^2$. Eliminating ϵ between every two of the above coefficients, an invariant relation will be found. As in [2] we shall give $\sqrt{\epsilon} = \psi$ as a function of all coefficients:

For $\rho_s = -\rho_\pi = \tau_s - 1$

$$\psi = \frac{(1 + \rho_s^2 + 2\rho_s \cos 2\varphi)^{1/2}}{1 - \rho_s} ,$$

$$\text{for} \quad \rho_p = -\rho_\sigma = 1 - \tau_\sigma \tag{3}$$

$$\psi = \frac{\{2(1 + \rho_p)\,(1 + \rho_p + [(1 + \rho_p)^2 - (\rho_p - 1)^2 \sin^2 2\varphi]^{1/2})\}^{1/2}}{2(\rho_p - 1)\cos\varphi}$$

The cases $\psi(\tau_p)$ and $\psi(\tau_\pi)$ lead to fourth order polinomial equations

$$\psi^4 + \frac{1}{1 - \tau_p^2 \cos^2\varphi}[-\tau_p\psi^3 + (0.25\tau_p^2 - \sin^2\varphi)\psi^2 + \tau_p \sin^2\varphi\,\psi - 0.25\tau_p^2 \sin^2\varphi] = 0$$

$$\psi^4 - \frac{4}{\tau_\pi}\psi^3 + (\frac{4}{\tau_\pi} - 1)\psi^2 + \frac{4\sin^2\varphi}{\tau_\pi}\psi - \frac{4\sin^2\varphi}{\tau_\pi^2} = 0,$$

which may be solved by different methods — numerically or by reducing to cubic equations [2]. Substituting ψ from (3) and (4) into (2) we shall find the desired relations. For example, ρ_p, τ_σ, τ_p and τ_π as a function of ρ_s are

$$\rho_p = \frac{\rho_s(\rho_s + \cos 2\varphi)}{1 + \rho_s \cos 2\varphi} , \qquad \tau_\sigma = \frac{1 - \rho_s^2}{1 + \rho_s \cos 2\varphi} ,$$

$$\tau_p = \frac{(1 + \rho_s)(1 + \rho_s^2 + 2\rho_s \cos 2\varphi)^{1/2}}{1 + \rho_s \cos 2\varphi} , \tag{5}$$

$$\tau_\pi = \frac{(1 - \rho_s^2)\cos\varphi}{(1 + \rho_s^2 + 2\partial_s \cos 2\varphi)^{1/2}}$$

If the wave passes not through a set of parallel lamellae of absorbing materials, but only through one plane parallel faced absorbing plate, then some amplitude coefficients of the second boundary (ρ, τ) are simply connected with the amplitude coefficients of the first boundary (r, t[2]), namely

$$r_s = \rho_\pi = -\rho_s = 1 - \tau_s, \tag{6}$$

$$r_\sigma = \rho_p = -\rho_\sigma = 1 - \tau_\sigma$$

Since $\in$ is the same for the first and the second boundary, we are able to calculate every one of the Fresnel's coefficients by any other, regardless the interface, directly from (6) or using similar procedure of eliminating $\in$ in the remaining cases.

REFERENCES

1. R. M. A. Azzam. J. Opt. Soc. Am., **69**, 1007 (1979).
2. D. B. Kushev. Optica Acta, **27**, 875 (1980).
3. E. E. Bell. Handbuch der Physik, **25**/2a, 1 (1967).

ΠΕΡΙΛΗΨΗ

ΑΜΜΕΣΕΣ ΣΧΕΣΕΙΣ ΜΕΤΑΞΥ ΤΩΝ ΠΛΑΤΩΝ ΤΩΝ ΣΥΝΤΕΛΕΣΤΩΝ FRESNEL ΓΙΑ ΤΙΣ ΕΝΔΟΕΠΙΦΑΝΕΙΕΣ ΤΩΝ ΠΑΡΑΛΛΗΛΩΝ ΕΠΙΠΕΔΩΝ ΑΠΟΡΡΟΦΗΣΕΩΣ

Ὑπὸ
D. B. KUSHEV

(Ἐργαστήριο Φυσικῆς Ἡμιαγωγῶν καὶ Τεχνολογικῶν Ἐρευνῶν Πανεπιστήμιο Σόφιας)

Πρόσθετα στὶς ὑπάρχουσες σχέσεις ἀνάμεσα στὰ πλάτη τῶν συντελεστῶν Fresnel γιὰ τὶς ἐνδοεπιφάνειες ἰσοτρόπων μέσων δίνονται καὶ ἄλλες σχέσεις γιὰ τοὺς ἴδιους συντελεστὲς καὶ γιὰ ἠλεκτρικὰ καὶ μαγνητικὰ πεδία μὲ TE καὶ TM κυμάνσεις γιὰ τὴ περίπτωση μεταδόσεως-ἀνακλάσεως καὶ γιὰ ἐνδοεπιφάνειες σὲ ἰσότροπα μέσα διαπερατότητας ἢ ἀπορροφήσεως. Ἡ ἀνάλυση περιορίζεται στὴ περίπτωση συστήματος παραλλήλων ἐπιπέδων ποὺ περιβάλλονται ἀπὸ τὸ διαπερατὸ μέσο. Οἱ σχέσεις αὐτὲς εἶναι ἀνεξάρτητες τῶν διηλεκτρικῶν συναρτήσεων τῶν μέσων ποὺ περιβάλλουν τὴν ἐνδοεπιφάνεια.