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DIFFRACTION TOMOGRAPHY WITH STATISTICAL REGULARIZATION

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ABSTRACT

An important feature of diffraction tomography problems is their incorrectness. Therefore in the solution of inverse problems it is necessary to use regularization methods which are determined by the type of a priori information. In connection with the presence of noise in the data measurement, preference is given to the statistical regularization method. We present the general structure of the statistical algorithm of diffraction tomography and give the results of numerical experiments to demonstrate the resolving power of the observation system.

INTRODUCTION

Traditionally, the application of tomography methods to the retrieval of medium parameters from seismic observations has been based on the model of ray propagation of seismic waves, as well as with various modifications of Radon transform for the inversion process (Dines and Lytle, 1979; Ryzhikov and Troyan, 1985; Nolet, 1987, etc.). In recent years, the interpretation of inverse dynamical linearized problem, such as diffraction tomography, has been widely used (Devanie, 1985; Tarantola, 1984, 1987; Carrion, 1986; Ryzhikov and Troyan, 1988, 1993a,b, 1994). As it is well known, the computational tomography methods make possible the obtention of two- and three-dimensional fields of medium parameters from their integral characteristic-projections. The computational tomography applied to seismic experiments can be interpreted as the method for the solution of the inverse seismic problem. In this sense, *ray tomography* is relevant to an inverse kinematical seismic problem, whereas *diffraction tomography* is relevant to an inverse dynamical problem.

An important feature of the inverse seismic problem is its incorrectness (decision instability). Therefore, in the solution of inverse problems it is necessary the use of regularization methods (Franklin, 1970; Tikhonov, 1963), which are determined by the kind of a priori information. Taking into account the presence of random noise in the seismic data, preference is given to the statistical regularization methods.

The methods in computational tomography have brought new ideas to the solution and interpretation of inverse seismic problems, as for example the notion of tomography functional (Ryzhikov and Troyan, 1988, 1994). As well, two very important procedures in seismic processing such as velocity analysis and migration are involved in the algorithms of diffraction tomography in an

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natural way, which can be interpreted as the functions of the influence of various spatial regions of the medium on a particular sampling of the seismogram.

In this paper we present the general structure of the statistical algorithms of diffraction tomography and give the results of numerical experiments to demonstrate the resolving power of the observation system.

THE ALGORITHM OF DIFFRACTION TOMOGRAPHY

When problems of diffraction tomography are set, a sampled seismogram u_i is used as input, instead of only arrival times (and amplitudes if attenuation is also computed) of the incoming waves as used in ray tomography. The fields of sought-for parameters are elements $\theta(x)$ of functional space: $\theta(x) \in \Theta(R^3)$, for instance, the fields of Lamé's elastic parameters $\lambda(x)$ and $\mu(x)$, density $\rho(x)$ or velocities of P- and S-waves, $V_p(x)$ and $V_s(x)$ respectively. The measurement space is a N-dimensional Euclidean space R^N, where N is the number of samples of the seismogram. The diffraction tomography experiment is determined by the mapping of the functional space of parameters $\Theta(R^3)$ to the measurement space of the seismogram R^N. We can write the model of seismogram as

$$u_i = \wp_i(\theta) + \varepsilon_i, \quad i=1, \cdots N \qquad (1)$$

where $\wp(\theta)$ is the "transition" operator from the functional space $\Theta(R^3)$ to the measurement space and ε_i is the noise component, ε belongs to the normal distribution with mathematical expectation zero and covariance matrix K_ε (Troyan, 1982, 1988). If the sounding seismic impluse field is $\varphi = \varphi(x, t) \in \Theta(R^3 x T)$ and the source field is f, we may assume that the process of propagation of the sounding seismic field is described by the linear equation

$$L_\theta \varphi = f. \qquad (2)$$

The operator L_θ is determined by the medium properties. The problem consists now in the recovery of the operator L_θ from the seismogram. Taking into account the transformation of the sounding seismic signal by the measurement seismic channel H_i, we can write the model (1) as

$$u_i = H_i L_\theta^{-1} f + \varepsilon_i \qquad (3)$$

where H is defined as

$$H_i \varphi = \int d\Omega\, dt\, dx\, h_i(\omega_i, \omega; t_i - t) \,.\, \delta(x_i - x)\, \varphi(\omega, x, t) \qquad (4)$$
$$\omega \in \Omega = \{\omega : |\omega| = 1\}$$

where all sought-for properties of the three dimensional medium are included in the operator L_θ^{-1}. It is not possible to find an accurate solution of the problem of the retrieval of θ from the measured available data. Therefore we use the approximate methods for the description of the propagation of the sounding seismic signal in an inhomogeneous medium. Assume that for a certain value of θ_o we can write

$$\varphi_o = L_o^{-1} f \qquad (5)$$

and consider that the sought for θ is close to θ_o, i.e., $\theta = \theta_o = \delta\theta$, where $\delta\theta \ll \theta_o$. The formal solution in the medium with the field of the parameter θ is determined by

$$\varphi = \varphi_o + L_o^{-1}\delta L_\theta \varphi, \qquad (6)$$

where $\delta L_\theta = L_o - L_\theta$ is the *perturbation operator*. Taking into account (6), the general model (3) can be written as

$$u_i = H_i\left[\varphi_o + L_\theta^{-1}\delta L_\theta \varphi\right] + \varepsilon_i. \qquad (7)$$

In the model (7), the medium properties are included both in δL_θ and in φ. However, if $\delta\theta$ is small enough and if the following condition is satisfied

$$\frac{\left\| H_i L_o^{-1}\delta L_\theta (\varphi - \varphi_o)\right\|^2}{E(\varepsilon_i^2)} \ll 1, \qquad (8)$$

where E is the operator of the mathematical expectation, φ in equation (7) can be replaced by φ. From a physical point of view, the inequality (8) means that the model error is much less than the measurement error. Taking (8) into account the model (7) can be rewritten as

$$u_i = H_i\left[\varphi_o + L_o^{-1}\delta L_\theta \varphi_o\right] + \tilde{\varepsilon}_i. \qquad (9)$$

The error $\tilde{\varepsilon}_i$ includes both the random error ε_i and the error relevant to the determined part of the model. By introducing the scalar product

$$\langle \xi | \eta \rangle_{T,V,\Omega} = \int_\Omega \int_V \xi(\omega, x, t) * \eta(\omega, x, t)\, dx\, d\Omega \qquad (10)$$

the model (9) can be rewritten as

$$u_i = \langle h_i | \varphi_o \rangle_{V,T,\Omega} + \langle h_i | L_o^{-1}\delta L_\theta \varphi_o \rangle_{V,T,\Omega} + \tilde{\varepsilon}_i. \qquad (11)$$

By reducing the experimental data by the known value $u_i^o = \langle h_i | \varphi_o \rangle_{V,T,\Omega}$ we obtain

$$\tilde{u}_i = \langle h_i | L_o^{-1}\delta L_\theta \varphi_o \rangle_{V,T,\Omega} + \tilde{\varepsilon}_i, \qquad (12)$$

where $\tilde{u}_i = u_i - u_i^o$

In the perturbation operator δL_θ we can isolate the monotonous function $v(\theta)$, with respect to which the perturbation operator will be linear. Taking into account the fact that in seismic tomography the operator δL_θ is close to the local one, equation (12) can be rewritten as

$$\begin{aligned} \tilde{u}_i &= \langle L_o^{-1*} h_i | \delta L_\theta \varphi_o \rangle_{V,T,\Omega} + \tilde{\varepsilon}_i \\ &= \left\langle \langle G_o^* h_i | L_v' \varphi_o \rangle_{T,\Omega} | v(\delta\theta) \right\rangle + \tilde{\varepsilon}_i, \end{aligned} \qquad (13)$$

where $G = L_o^{-1}$ is the Green's operator, G^* means conjugated operator and L_v' is defined as

$$L_v' : \frac{\delta u}{\delta v} = \langle G^* h | L_v' \varphi_o \rangle_{T,\Omega}. \qquad (14)$$

The integral kernel of the functional relative to $v(\delta\theta)$ is known as *tomographic functional kernel* p (Ryzhikov and Troyan, 1988) and is defined by

$$p_i = \langle \varphi_{out} | S^v | \varphi_{in} \rangle_{T,\Omega}, \qquad (15)$$

where $\varphi_{in} = \varphi$ is the incoming field in a known reference medium θ_0, φ_0 is defined by $L_0\varphi_0 = f$, φ_{out}, defined by $L_0^*\varphi_{out} = h_i$, is the reversed outgoing field from the receiver and $S^v = L'$ is the operator of the interaction of the fields φ_{in} and φ_{out}.

Taking into account equation (15)m the model (13) can be written

$$\tilde{u} = Pv + \tilde{\varepsilon} \qquad (16)$$

where

$$\tilde{u} = \| \tilde{u}_1, \tilde{u}_2, \cdots, \tilde{u}_N \|^T, \quad \tilde{\varepsilon} = \| \tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \cdots, \tilde{\varepsilon}_N \|^T$$

$$P = \begin{Vmatrix} \langle p_{11}| & \langle p_{12}| & \cdots & \langle p_{1M}| \\ \langle p_{21}| & \langle p_{22}| & \cdots & \langle p_{2M}| \\ \vdots & & & \\ \langle p_{N1}| & \langle p_{N2}| & \cdots & \langle p_{NM}| \end{Vmatrix}, \quad v = \begin{Vmatrix} |v_1\rangle \\ |v_2\rangle \\ \vdots \\ |v_M\rangle \end{Vmatrix}$$

To build up the reconstructing algorithm, taking into account (15) we write the model (16) as

$$u_i = \sum_{\mu} \langle p_{\mu i} | v_\mu(\delta\theta) \rangle + \varepsilon_i, \qquad (17)$$

For an arbitrary linear functional $l(v)$ we shall seek for a solution in the form of a linear combination of measurements

$$l(v) = \sum_i \alpha_i u_i = (\alpha, u) \qquad (18)$$

where

$$l(v) = \langle l | v \rangle.$$

For example, the functional $l(v)$ can have the physical sense of the mean value of the sought for field in the vicinity of the point **X**

$$l = \frac{1}{(abc)^{-1}} H\left(\frac{a}{2} - |x_1 - X_1|\right) H\left(\frac{b}{2} - |x_2 - X_2|\right) H\left(\frac{c}{2} - |x_3 - X_3|\right), \qquad (19)$$

where H(x) is the Heaviside step function

$$H(x) = \begin{matrix} 1, & x \geq 0 \\ 0, & x < 0 \end{matrix}.$$

The error of the solution can be obtained in the form

$$\begin{aligned} \eta_\mu^l &= l(v_\mu) - \sum_{\mu'} (\alpha, \langle p_{\mu'} | v_{\mu'} \rangle) - (\alpha, \varepsilon) \qquad (20) \\ &= \langle l - (\alpha, p_\mu) | v_\mu \rangle - \sum_{\mu' \neq \mu} \langle (\alpha, \rho_{\mu'}) | v_{\mu'} \rangle - (\alpha, \varepsilon). \end{aligned}$$

We consider as a solution the function $\hat{I}(\nu)=(\hat{\alpha},u)$ which minimizes the square of the linear form of the error

$$\hat{\alpha} = arg\ inf\{(\lambda - Q^{*}\alpha)^{*}K(\lambda - Q^{*}\alpha)\} \ . \qquad (21)$$

where Q = P I , Is the identity operator with dimension of the number of measurements and

$$(Q^{*}\alpha)^{*} = [\alpha_1 \cdots \alpha_N]\begin{vmatrix} \langle p_{11}(x)| & \cdots & \langle p_{M1}(x)| & 1 & 0 & \cdots & 0 \\ & \vdots & & & & & \\ \langle p_{1N}(x)| & \cdots & \langle p_{MN}(x)| & 0 & 0 & \cdots & 1 \end{vmatrix} \qquad (22)$$

$$\lambda^{*} = \| \underset{1}{\langle 0(x)|} \cdots \langle I(x)| \cdots \underset{\mu}{\langle 0(x)|} \quad \underset{M}{0} \quad \cdots \quad \underset{M+1}{0} \quad \| \ . \quad M+N \qquad (23)$$

The operator K must be positive. An optimal estimation of $\hat{\alpha}$ is given by

$$\hat{\alpha} = (QKQ^{*})^{-1}QK\lambda \qquad (24)$$

The statistical interpretation of the operator K and the estimation $\hat{\alpha}$ is the following. Let the set of fields $\{\nu_{\mu}(\delta\theta)\}$ be characterized by the gaussian measure and the random component by a normal distribution. Then the solution is obtained by the minimization of the mean square of the retrieval error $E(\eta^{x2}_{\mu})$, and the integral operator **K** has the meaning of the covariance operator

$$K = \begin{vmatrix} k_{\nu_1\nu_1} & \cdots & k_{\nu_1\nu_N} & k_{\nu_1\varepsilon} \\ \cdots & & & \\ k_{\nu_\mu\nu_1} & \cdots & k_{\nu_\mu\nu_N} & k_{\nu_\mu\varepsilon} \\ \cdots & & & \\ k_{\varepsilon\nu_1} & \cdots & k_{\varepsilon\nu_N} & k_{\varepsilon\varepsilon} \end{vmatrix} .$$

NUMERICAL EXPERIMENTS

Some numerical experiments have been carried out to investigate the resolving power of the observation system formed by a regular net of 3-component geophones. To simplify the computation the following assumptions have been made: the reference medium is homogeneous and boundless, the incident wave field is a plane P-wave with a given shape of signal and normal to wave front. To scan the resolving power we placed the testing mask into the concrete domain of the recovering medium. This testing mark was formed by regular net of point diffractors, which were used for computing the synthetic seismogram. The geometry of these experiments is illustrated in figure 1. All results are presented in the scale induced by the characteristic wave length λ of the incident P-wave in the reference medium (V_p = 4000 m/s, V_s = 2000 m/s). The P-wave velocity inside of scatters is equal to 3300 m/s. Synthetic seismograms are constructed in accordance with modified Born's approximation. The image of the test mask is recovered by the treatment of several experiments differed with normals to wave fronts and probably with shapes. In our experiments the incident plane waves are characterized with normals n_z, n_1 and n_2, such that $(n_z, e_z) = 1$, $(n_1, e_z) = \sqrt{2}/2$, $(n_2, e_x) = -\sqrt{2}/2$, where (e_1, e_2, e_3) are unit coordinate vectors. For the imitation of a real seismogram noise ε is added, with

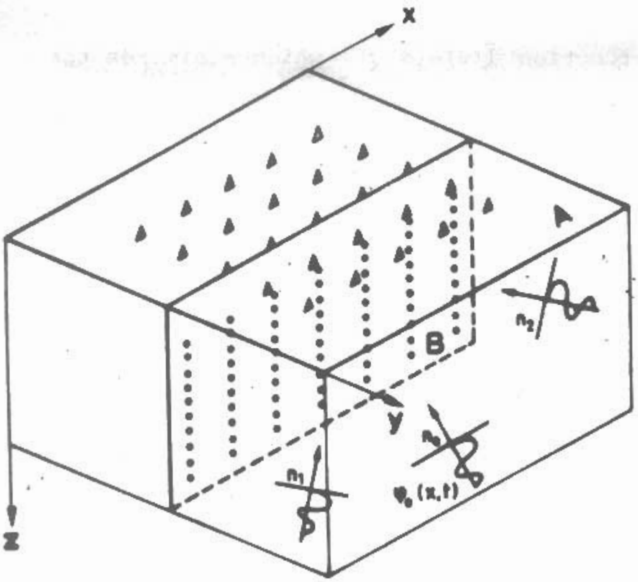

Fig. 1: Geometry of the numerical experiments: A is the plane (z = 0) of regular net of geophones (d is a step of the net), B is the plane (y = constant) of the test mask. The step between scatters along the z-axis is 1.5λ and the step along the x-axis is 1.0λ. n_1, n_2 and n_3 are the normals, lying in the B plane, to the incident plane P-wave: $(n_1, e_z) = 1$, $(n_2, e_x) = \sqrt{2}/2$, $(n_3, e_x) = -\sqrt{2}/2$

mathematical expectation $E\varepsilon = 0$ and covariance $E\varepsilon\varepsilon^T = \sigma^2_\varepsilon I$.

We deal with the resolving power of diffraction tomography controlled by parameters such as the distance between geophones in our observation system. Figures 2 to 4 represent the series of numerical experiments distinguished by distance d between geophone in regular net (0.5λ, 3λ and 5λ respectively), whereas the level of noise is 5% and the two incident plane waves $(n_2, e_x) = \sqrt{2}/2$, $(n_3, e_x) = -\sqrt{2}/2$) are used for the generation of synthetic seismograms. The best recovery of testing mask is reached for d = 3λ, see figure 3.

DISCUSSION AND CONCLUSIONS

The following points summarize the main features of the diffraction tomography:

1. In diffraction tomography there is not any need to

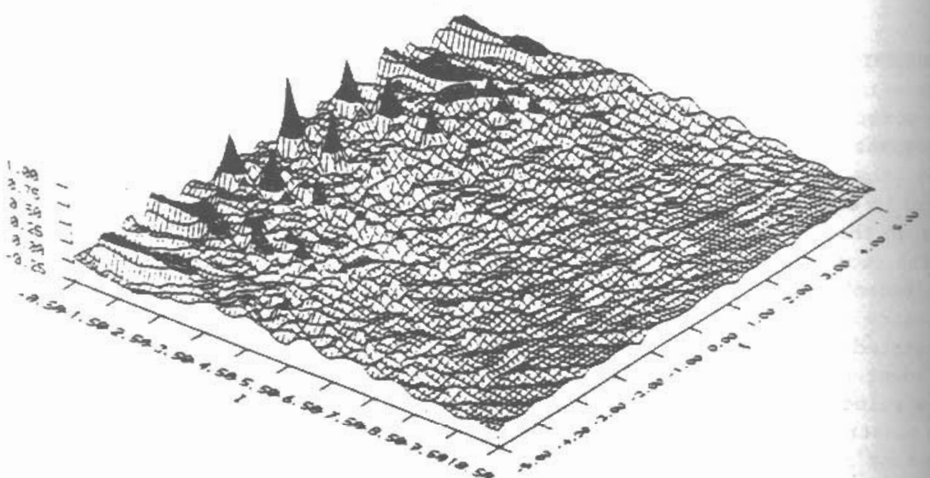

Fig. 2: 3-component diffraction seismogram computed for the geophone profile y = 2d (d = 0.5λ). The seismogram contains 5% of noise.

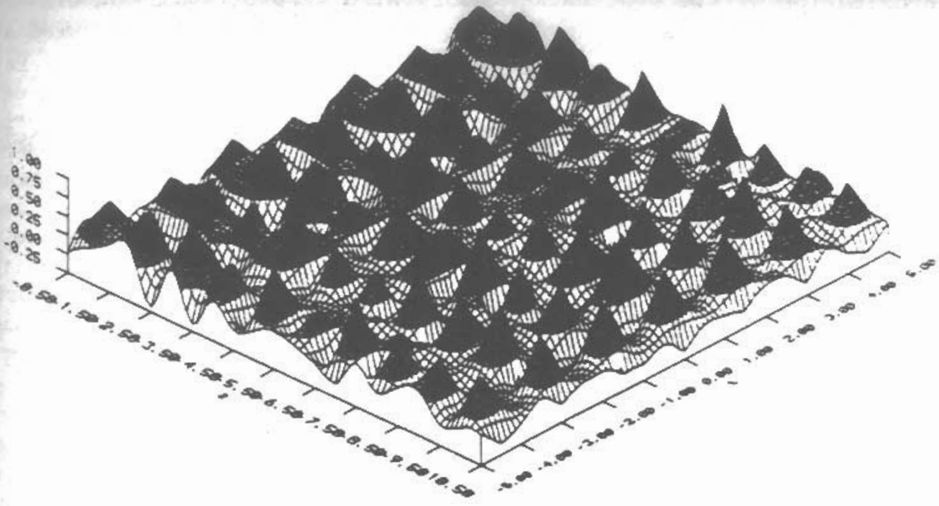

Fig. 3: Test mask image generated by two incident plane waves with normals n_1 and n_2. A 5% of noise is added to the synthetic sismogram. The step d in the net of geophones is set to 0.5λ.

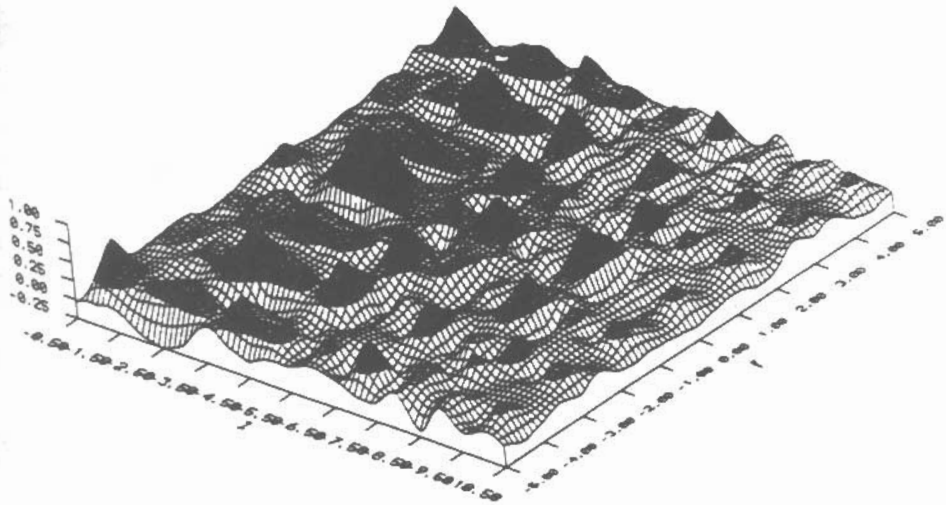

Fig. 4: Same as figure 2 expect for d = 3λ.

distinguish the different phases of the wave for the interpretation as well as the parameters of the waves such as the arrival times or amplitudes, as necessary in ray tomography. In the general case of diffraction tomography one uses all set of sampled seismograms without any preliminary processing. This point is very important for the practical application of the method of tomography.

2. In diffraction tomography use is made of a more adequate physical model of the process of propagation of seismic waves, based on the linearized

approximation of the scattering theory with respect to ray tomography.

3. In ray tomography the tomography functional is singular, localized along the ray connecting source and receiver, and the weight along the ray is constant. At the contrary, in diffraction tomography every element of the spatial region has its own weight. Analysis of the tomography functional makes it possible to understand distinctly how a concrete measurement is connected with a possible variation of the parameters of the medium at different spatial points, which permits to estimate the spatial region with respect to which a given system of observation is informative.
4. The results of numerical experiments show that the suggested modifications of the diffraction tomography method allows us to recover the medium imaging in the rather wide space domain. This effect is due to gathering of the seismic information from set of experiments. The numerical experiments can be used for he design of seismic real experiments.

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