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ESTIMATE OF THE REGIONAL STRESS FIELD USING JOINT SYSTEMS

M. CAPUTO* R. CAPUTO**

Summary

We present the first attempt to estimate the stress ellipsoid using joint systems. We consider only tensional joints and we study the relationships between the stress field and the formation of joints as an expression of the strain.

We suppose also that all the joints belong to the same stress field.

The method allows to compute the three principal directions of the stress ellipsoid and the ratio R.

The complete mathematical procedure is given as well as examples.

ΣΥΝΟΨΗ

Παρουσιάζεται μία πρώτη προσπάθεια να εκτιμηθεί το ελλειψοειδές τάσεων με την χρησιμοποίηση των συστημάτων διακλάσεων. Αξιολογούνται μόνο εφελκυστικές διακλάσεις και μελετώνται οι σχέσεις ανάμεσα στο εντατικό πεδίο και τον σχηματισμό των διακλάσεων σαν μία έκφραση της παραμόρφωσης. Επίσης υποτίθεται ότι όλες οι διακλάσεις ανήκουν στο ίδιο εντατικό πεδίο. Η μέθοδος επιτρέπει τον υπολογισμό των τριών κυρίων διευθύνσεων του ελλειψοειδούς τάσεων και τον συντελεστή R. Δίνεται πλήρης μαθηματική διαδικασία καθώς και ορισμένα παραδείγματα.

M. CAPUTO and R. CAPUTO. Εκτίμηση του γενικού εντατικού πεδίου με τη χρησιμοποίηση συστημάτων διακλάσεων.

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* Dipartimento di Fisica, Università degli Studi "La Sapienza" di Roma, p.le A. Moro, 00100 Rome, Italy.

** Dipartimento [illegible] degli Studi di Firenze, v. G. La Pira 4, 50121 Florence, Italy.

Introduction

One of the most important problems in geology is the determination of the elastic stress field on the Earth's surface and its interior. As examples of geologic problems in which the knowledge of this field is crucial are:

1) the study of the nature and the size of the forces driving the plates,
2) why is the stress tensional in the back-arc areas?
3) why do arcs rise?
4) why is the outer arc seismicity shallow?
5) what is the nature of deep earthquakes beneath island arcs?

But the knowledge of the stress field is very important also in geophysics; examples of problems are:

1) study at very low strain rate the rheological properties of the anelastic medium in the Earth's interior;
2) study of the causes of earthquakes, which may lead to earthquake prediction;
3) the construction of accurate elastic models of the Earth based on its free modes which are influenced by the state of stress of the Earth (CAPUTO, 1984a,b);
4) the determination of the correct stress-strain relation for the Earth's interior (CAPUTO, 1979a);

but also in Earth's science problems of applied research such as:

1) deformation of the rocks in sites to be used as radioactive or chemical waste disposals (CARTER, 1976; CAPUTO, 1983b);
2) the estimation of the seismic risk by determining regions with large shear stress and where there is a seismic gap (CAPUTO, 1983);
3) the study of crack openings in geothermal energy reservoirs;
4) the safety measures in mining due to the interaction of the regional stress field with that induced by man (STILLER *et al.*, 1983);
5) the estimate of the ground accelerations caused by earthquakes (CAPUTO, 1981).

The stress field in the interior of the Earth may be estimated with many methods. The stress state at depth in the lithosphere and the association with topography and gravity have received great attention in recent times. A recent issue of the Journal of Geophysical Research (vol. 85, B11, Nov. 1980) and a volume on Earth rheology (MÖRNER, 1980) are examples of the many interesting studies made using different methods and hypotheses.

In many studies the surface load causing the stress field and its isostatic compensation are assumed known, then the corresponding field is calculated in different rheological models.

Some authors (JEFFREYS, 1943; KAULA, 1963; CAPUTO, 1965) have searched for a minimum estimate of the strength of the crust or the mantle. Others abandoned the linear stress-strain relation to search for the minimum strength.

Other studies (JEFFREYS, 1976; CAPUTO *et al.*,

1984) consider the crust as a thin shell overlying a fluid sphere that cannot support shear stress; in which case the maximum shear stress may be as much as three times the applied load (CAPUTO *et al.*, 1984) while in the sphere has the rigidity of the crust (CAPUTO *et al.*, 1985) the stress is generally at most one third of the load.

Artyushkov (1973; 1974) considered that isostasy equilibrates only require vertical forces while a true equilibrium also requires the balance of horizontal forces and moments; therefore a locally compensated crust will depart from true equilibrium proportional to the scale of topography or horizontal density anomalies. Artyushkov (1973) estimates the average values of deviatoric stress due to the density anomalies of the lithosphere. However, all these solutions are not of interest for most problems of seismology and tectonophysics where one needs detailed descriptions of the stress field.

Studies have been made on a global scale using gravity and geoid anomalies and wave length. McKenzie (1967), excludes lithospheric anomalies that require a strength above a given threshold, and assumes that gravity anomalies are supported by the lithosphere. He calculates a minimum stress of 830 bar to support plate flexure near the Tonga and Puerto Rico with a 50 km thick lithosphere, or 220 bar for a 100 km lithosphere.

Lambeck and Nakiboglu (1980) give the average maximum shear stress associated with gravity anomalies supported by a 100 km thick lithosphere.

O'Connell and Hager (1983) used global flow models for their estimation of stresses in collisional boundaries. These were found to be around 70 bar distributed over 100 km thick lithosphere. Since the stress generated by topographic loads and their isostatic masses are often much larger, they certainly play an important role in generating equilibrium on a large scale or local shear stress and therefore seismogenetic volumes.

Other workers, looking for more sophisticated phenomena have analysed the thermal and mechanical effects of the addition or removal of overburden (VOIGT and SAINT PIERRE, 1974; HAXBY and TURCOTTE, 1976).

Many authors (e.g. MCKENZIE *et al.*, 1974) assumed a Newtonian convection mantle to predict surface elevation and gravity anomalies. However, it is doubtful whether these studies, mostly undertaken for purposes other than the estimation of the stress field of the lithosphere, give relevant information on this subject. Also one might question the validity of the rheological model considered (CAPUTO, 1979a; 1984a).

Knowledge of the stress field in these regions could contribute to the solution of many geodynamic problems such as the mapping of the maximum horizontal tectonic stress which causes plate motion. For this purpose Nakamura and Uyeda (1980) analyzed the stress gradient in back-arc regions and plate subduction compiling lines (called trajectories) connecting the orientation of the maximum horizontal tectonic stress in five regions. The orientations are inferred from dike swarms, faults and earthquake source mechanism.

However, one should note that the results are subjected to considerable uncertainty concerning the relationships between the following factors: the forces presently driving the plates, the source mechanism of earthquakes, the trajectories of maximum horizontal stress, and the stress field. In fact in the stress field of the lithosphere a component due to the topographic load is present (JEFFREYS, 1976; CAPUTO *et al.*, 1984) which may generate maximum shear stress of several hundred bars; unless one can eliminate this component of stress, it will be impossible to distinguish the components of force arising from other sources.

Another uncertain factor is the effect of preexisting weak planes which may be preferred for the release of elastic energy instead of the direction of the maximum shear stress as has been indicated by many authors (STEIN, 1979; BERGMAN and SOLOMON, 1980). Many authors rely on the consistency and smoothness of the results of their observations ignoring that the above mentioned bias could give a result drastically different but still smooth and consistent with a phisical interpretation which could however be far from reality.

If one wants the stress tensor due to the tectonic forces then the stress tensor due to topography and isostatic compensation should be subtracted from the total stress tensor observed.

The estimate of the stress field could also contribute to the assessment and perhaps the reduction of seismic risk (expecially in the areas where earthquakes are tsunami-generating) as was the case in the study of the Apennines (CAPUTO *et al.*, 1984) which determines regions where there is a large shear stress but where earthquakes have not occurred in historic time (CAPUTO, 1983a), therefore indicating a set of gaps in the space-time domain.

The stress field near the surface of the crust is known with more resolution and realism than in the case of the interior. We shall be concerned here with the study of the elastic stress field in this region of the Earth.

The deepest point where this field has been observed seems to be in the Michigan basin at 5110 m where 135 MPa have been measured (MCGARR and GAY, 1978). Very few measurements have been made below two kilometres.

The estimate of the complete stress tensor on the surface of the Earth has been made in the Phlaegrean Fields (CAPUTO, 1979b; CAPUTO M. and CAPUTO R., 1988a) using surface geodetic measurements of deformation.

More commonly the stress field on the Earth's surface is studied by analyzing the directions of the slicken-sides observed in the field (*e.g.* CAPUTO R., 1984; 1987). The regional stress field, however, may be estimated more rigorously using also an analitic method of data analysis and adjustement (CAPUTO M. and CAPUTO R., 1988b).

The direction of the joints observed in the field being strictly related to that of the slicken-side allows also to determine the direction of the stress field as we shall show here

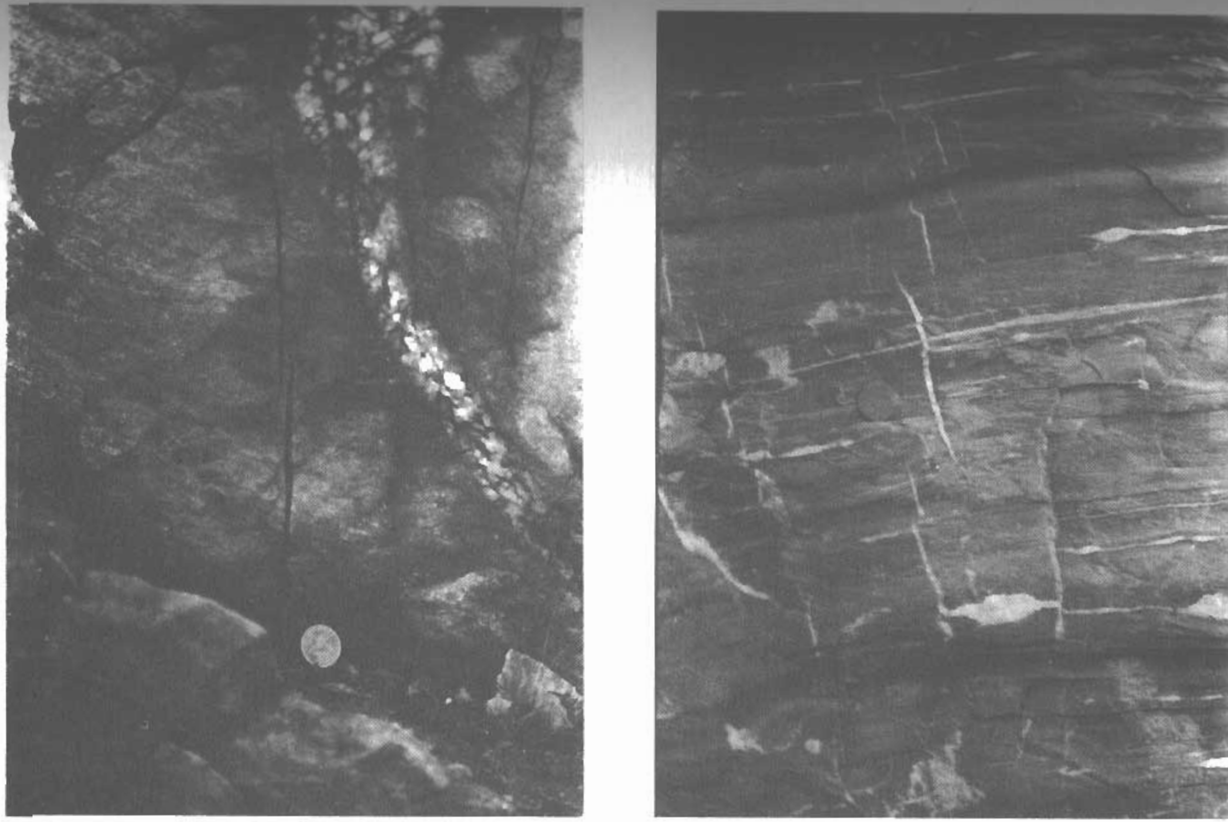

Fig. 1 - Examples of tensile joints open (a) and filled (b). In both cases is clear a prevailing normal relative movement with a measurable displacement. The coin in the figures is that of two drachmas (photo by R. Caputo).

Hypothesis

In the literature the most common definition of joint is a fracture in a rock between the sides of which there is no observable relative movement (WHITTEN and BROOKS, 1972; BLES, 1981; SUPPE, 1985). Joints are usually grouped in sets, while parallel, and in groups of intersecting sets (*i.e.* systems of joints).

Joints, sets of joints and systems of joints develop because a stress field is applied to the rock; we may have different origins of such a field but the one in which we are more interested is the tectonic one because of its more lateral continuity and uniformity instead of those related, for example, to cooling or those related to the unloading of the rock mass when the cover is eroded.

In this paper we refer to the joints due to a tensional stress; in a second paper we will describe those due to a compressive stress such as stylolitic joints.

Because of their nature, joints related to a tensional regime are often open and, depending on many physical and chemical conditions of the rock mass, they could be filled by different kind of materials (fig. 1a and 1b).

Herein we consider a joint as a fracture in a rock mass between the sides of which there has been normal relative movement and no tangential relative movement. This definition is more limited but fits better the use of "joint" that it is done on field. In fact, either the pure compressive joints (*i.e.* stylolites) or the pure extensional joints (*i.e.* tension gashes) denote a normal relative movement, it may be micrometric, but always a normal relative movement which is observable in many cases.

It is not possible to define a fracture with "no observable relative movement" (WHITTEN and BROOKS, 1972; BLES, 1981; *etc.*) because it is intrinsic in the nature of every kind of fracture, related to a fragile strain field, to have a relative movement between the blocks separated by these "planes".

Theoretically, we could find all the intermediate kinds of relative movements (*i.e.* with either a normal component and a tangential one) and the pure normal and pure tangential may be very rare statistically, but in nature the most common kind of fractures develop close to one of the two extreme cases so that we almost always have a prevailing relative movement qualifying us to talk only about "joints" or faults.

Considering only tensional joints we study the relationships between the formation of joints, as an expression of the strain, and the stress field. We suppose also that all the joints belong to the same stress field.

This hypothesis is fundamental and it must be checked directly in the field using the common criteria for the relative chronology of jointing: somewhere one set cuts the other, somewhere else the second cuts the first: it means that somewhere one set is locally younger, somewhere else is locally older but from a geological point of view, in a geological time-scale they are coeval. A comparison could be made with faulting. Let us consider the simple and very well known Anderson model of two conjugate faults: two intersecting faults cannot never move together: one is younger than the other, but mechanically they belong to the same stress field and geologically coeval. Exactly the same as for jointing.

Let us consider a stress ellipsoid where the σ_1 is the compressive axis and the σ_3 is the tensional one ($\sigma_1 ! \sigma_2 ! \sigma_3$). Obviously the σ_3 is perpendicular to the joint and the σ_1 could be everywhere in its plane.

If only one joint exists we cannot define the stress ellipsoid but only the direction of the σ_3.

If two intersecting joints exist we can find, using the intersection, the σ_1 because it must stay on both planes. If the two joints intersect at a right angle we are in a particular condition because the stress ellipsoid is an ellipsoid of revolution with the $\sigma_1 > \sigma_2 = \sigma_3$ and the three principal directions are easily found (see next section).

Let us consider now a population of joints grouped in one or two sets. The conclusions just found with one or two planes remain valid using the statistical approach described in the next section of this paper.

Following the formalism of Caputo M. and Caputo R. (1988a) and the fact that on the surface of the earth the stress is nul we may define the deformation matrix and deduce the principal directions. One of them is perpendicular to the surface of the earth while the other two are parallel to the surface and perpendicular to each other.

If the body is homogeneous the just above defined directions correspond to the principal directions of the stress. For this reason the joints related to extension, like those which are the subject of this paper, must be at a right angle. The normal to these planes should be the principal directions of the stress as we shall see better in the next section.

In particular, if two roughly orthogonal sets exist we can find the direction of the σ_1 as the direction closest to all the planes; in this case if the two sets are equivalent (see next sections) we are in the condition of an ellipsoid of revolution with $\sigma_1 > \sigma_2 \approx \sigma_3$. Otherwise if one of the two sets is more "important" it means that, also if both the σ_2 and σ_3 are tensional, it results $\sigma_2 > \sigma_3$ and the "average" direction perpendicular to this set corresponds to the σ_3 (see next section). When we have a couple of roughly orthogonal sets of joints the stress ellipsoid is completely defined either in its space orientation either in its shape.

Method

Let l_i, m_i, n_i, be the normalized components $(l^2+m^2+n^2=1)$ of the $N \geq 2$ joints; the components x, y, z of the vector v parallel to these planes must satisfy the condition

$$l_i x + m_i y + n_i z = 0, \quad i=1-N \tag{1}$$

This system of N equations in the unknowns x, y, z has the solution x=y=z=0 which is not of geologic interest.

In order to find a solution of geologic interest the vector v must have non zero norm and its normalized components must satisfy the condition

$$x^2 + y^2 + z^2 - 1 = 0 \tag{2}$$

The problem may then be reduced to the solution with the least square method with the condition (2). Using a multiplier of Lagrange the function to minimize is

$$\sum_i^N (l_i x + m_i y + n_i z)^2 - K(x^2 + y^2 + z^2 - 1) = \min \tag{3}$$

The solution of the problem is found differentiating the function (3) and solving the obtained system

$$\begin{aligned} &\sum_i^N l_i(l_i x + m_i y + n_i z) - Kx = 0 \\ &\sum_i^N m_i(l_i x + m_i y + n_i z) - Ky = 0 \\ &\sum_i^N n_i(l_i x + m_i y + n_i z) - Kz = 0 \end{aligned} \tag{4}$$

which may be written

$$\begin{aligned} &(\sum_i^N l_i^2 - K)x + (\sum_i^N l_i m_i)y + (\sum_i^N l_i n_i)z = 0 \\ &(\sum_i^N l_i m_i)x + (\sum_i^N m_i^2 - K)y + (\sum_i^N m_i n_i)z = 0 \\ &(\sum_i^N l_i n_i)x + (\sum_i^N m_i n_i)y + (\sum_i^N n_i^2 - K)z = 0 \end{aligned} \tag{5}$$

The system (5) is homogeneous and in order to have solutions its matrix must have rank smaller than 3. To satisfy this condition we shall use the factor K; it must be solution of the equation

$$\begin{aligned} &(\sum_i^N l_i^2 - K)(\sum_i^N m_i^2 - K)(\sum_i^N n_i^2 - K) + 2\sum_i^N l_i m_i \sum_i^N m_i n_i \sum_i^N l_i n_i + \\ &- (\sum_i^N l_i m_i)^2 (\sum_i^N n_i^2 - K) - (\sum_i^N l_i n_i)^2 (\sum_i^N m_i^2 - K) + \\ &- (\sum_i^N m_i n_i)^2 (\sum_i^N l_i^2 - K) = 0 \end{aligned} \tag{6}$$

obtained equating to zero the determinant of the symmetric matrix of the system (5). The secular equation (6) which has three real roots, is

$$\begin{aligned} &- K^3 + K^2(\sum_i^N l_i^2 + \sum_i^N m_i^2 + \sum_i^N n_i^2) - K[\sum_i^N l_i^2 \sum_i^N m_i^2 + \sum_i^N l_i^2 \sum_i^N n_i^2 \\ &+ \sum_i^N m_i^2 \sum_i^N n_i^2 - (\sum_i^N l_i m_i)^2 - (\sum_i^N l_i n_i)^2 - (\sum_i^N m_i n_i)^2] + \\ &+ 2\sum_i^N l_i m_i \sum_i^N l_i n_i \sum_i^N m_i n_i + \sum_i^N l_i^2 \sum_i^N m_i^2 \sum_i^N n_i^2 + \\ &+ (\sum_i^N l_i m_i)^2 \sum_i^N n_i^2 - (\sum_i^N l_i n_i)^2 \sum_i^N m_i^2 - (\sum_i^N m_i n_i)^2 \sum_i^N l_i^2 = 0 \end{aligned} \tag{7}$$

Noting that the components l_i, m_i, n_i are normalized and that the known form of equation (7) is the determinant Δ of the matrix of (5) with K=0 which in turn is the determinant of the quadratic form (3) with K=0 which is positive defined, equation (7) may be written

$$\begin{aligned} &- K^3 + NK^2 - K[\sum_i^N l_i^2 \sum_i^N m_i^2 + \sum_i^N l_i^2 \sum_i^N n_i^2 + \sum_i^N m_i^2 \sum_i^N n_i^2 + \\ &-(\sum_i^N l_i m_i)^2 - (\sum_i^N l_i n_i)^2 - (\sum_i^N m_i n_i)^2] + \Delta = 0 \end{aligned} \tag{8}$$

For any N we have

$$\sum_i^N l_i^2 \sum_i^N m_i^2 - (\sum_i^N l_i m_i)^2 = \tfrac{1}{2}\sum_i^N (l_i m_i - l_i m_i)^2 > 0$$

$$\sum_1^N l_i^2 \sum_1^N n_i^2 - (\sum_1^N l_i n_i)^2 = ½\sum_1^N (l_i n_i - l_i n_i)^2 > 0$$

$$\sum_1^N m_i^2 \sum_1^N n_i^2 - (\sum_1^N m_i n_i)^2 = ½\sum_1^N (m_i n_i - m_i n_i)^2 > 0 \qquad (9)$$

The functions (9) are the determinants of the principal minors of the matrix of (5) with K=0, which should be positive because the quadratic form (3) with K=0, whose matrix is the same that of (5) for K=0, is positive defined.

Also for N=2 one may verify that

$$\Delta \equiv 2(\sum_1^N l_i m_i)(\sum_1^N l_i n_i)(\sum_1^N m_i n_i) + \sum_1^N l_i^2 \sum_1^N m_i^2 \sum_1^N n_i^2 +$$

$$- (\sum_1^N l_i m_i)^2 \sum_1^N n_i^2 - (\sum_1^N l_i n_i)^2 \sum_1^N m_i^2 - (\sum_1^N m_i n_i)^2 \sum_1^N l_i^2 = 0 \qquad (10)$$

For N=2, the equation (8) has the solution K=0, which it nullifies (3); equation (8) is then reduced to a second order equation whose real roots could be readily computed, but are not necessary because K gives the minimum of (3). In fact in this case the least square method is not really needed because the direction perpendicular to (l_1, m_1, n_1) and to (l_2, m_2, n_2) is readily computed analytically, it is unique if the two given vectors are not parallel and gives $\sum (l_i x + m_i y + n_i z)^2 = 0$.

For N>2 the equation (8) does not have the solution K=0 because the known term Δ is the determinant of the matrix of (5), with K=0, which is necessarily different from zero if the vectors (l_i, m_i, n_i) are not parallel nor split in two groups parallel to two different directions. Equation (8) must then be solved directly obtaining the three solutions K_r (r=1-3).

Using a computer we can easily obtain the roots with one of the several available approximation methods and with the desired approximation. In the appendix we show how the solutions are easily found in this particular case.

With the three values of K (K_1, K_2, K_3), solutions of (8), we obtain the following three systems (r=1-3):

$$(\sum_1^N l_i^2 - K_r)x + (\sum_1^N l_i m_i)y + (\sum_1^N l_i n_i)z = 0$$

$$(\sum_1^N l_i m_i)x + (\sum_1^N m_i^2 - K_r)y + (\sum_1^N m_i n_i d)z = 0 \qquad (11)$$

and the three solutions (r=1-3):

$$x_r = \sum_1^N l_i m_i \sum_1^N m_i n_i - (\sum_1^N m_i^2 - K_r)\sum_1^N l_i n_i$$

$$y_r = \sum_1^N l_i m_i \sum_1^N l_i n_i - (\sum_1^N l_i^2 - K_r)\sum_1^N m_i n_i \qquad (12)$$

$$z_r = (\sum_1^N l_i^2 - K_r)(\sum_1^N m_i^2 - K_r) - (\sum_1^N l_i m_i)^2$$

which represent three mutually orthogonal vectors. These three directions, corresponding to the extremals of the function (3), could be physically interpreted as the principal directions of the stress ellipsoid: the minimum corresponds to the σ_1, the maximum to the σ_3 while the third extremal obviously represents the σ_2.

Moreover, calculating the values of the function in the three extremals, and normalizing them, we may compute the ratio between the values and it gives us the shape of the ellipsoid.

Examples and discussion

Hereafter we show some regional example of joint systems elaborated with the method exposed in the previous sections.

The data come from Central Greece and more exactly from the Larissa Basin that is one of the Aegean areas undergoing to a recent extension. For a more complete neotectonic analysis using either analytical methods for the treatment of faults (*e.g.* CAPUTO M. and CAPUTO R., 1988b), either the previously described method for the analysis of joints see CAPUTO and PAVLIDES (in preperation).

The data were collected preferentially in anphitheatric-shaped quarries to diminish the error due to the orientation of the outcrop's surface, because is well known that joints could develop parallel to erosional or anthropic surfaces ignoring the regional state of stress and so creating a bias error, undetectable with any statistical analysis.

Not in all the directions, joints could develop easily; sometimes there are preferential directions where the formation of a new joint is easy; on the contrary, in some other direction the strength of the rock is so high that new tension can act only on old joints. In such a way, with a theoretical stress ellipsoid of revolution we could have, in one direction, a numerous set of short spaced joints, while, in the orthogonal direction, few spaced joints but well developed.

To diminish this error, the normal relative movement on each joint have been collected too. Then, during the statistical analysis we considered the normal relative movement as the weight of each observation; that is, each joint has been computed M times, in equation (3), depending on the value of the weight.

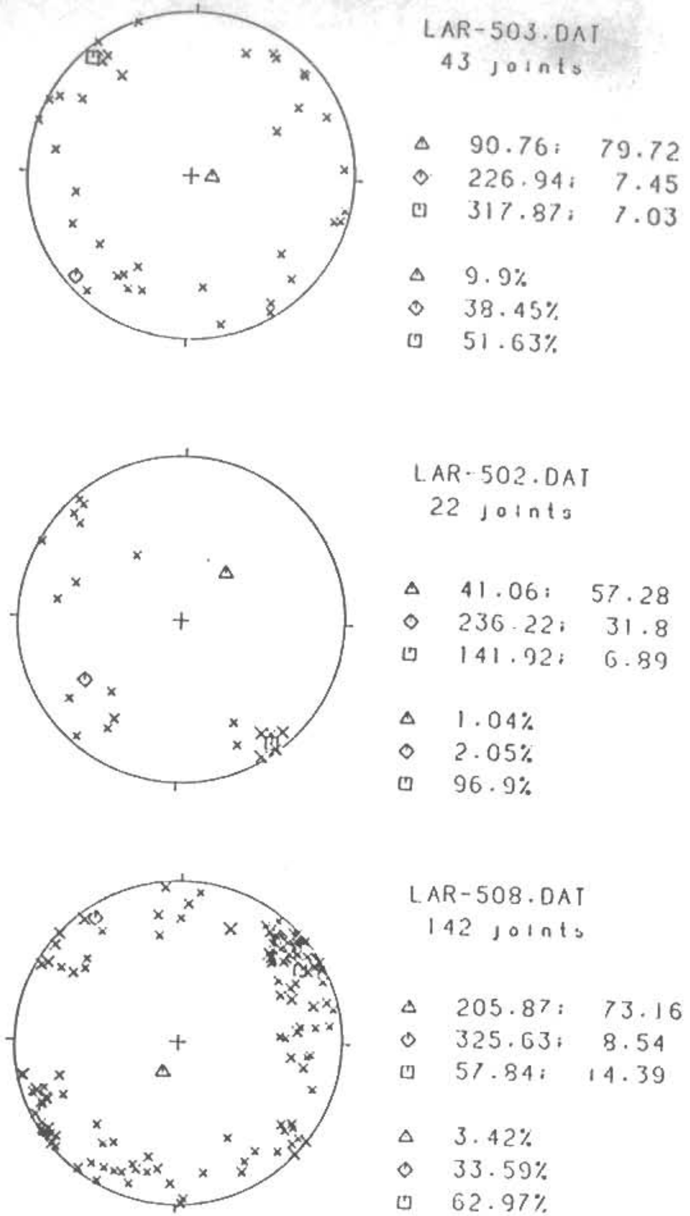

Fig. 2 - Examples of elaboration of three joint systems. The crosses represent the poles of the joints; the different sizes refer to different normal relative movement (see text). The triangles, rhombs and squares represent the σ_1, σ_2 and σ_3, respectively; the data following the triangles, rhombs and squares above give the azimut and dip angle of the σ_1, σ_2 and σ_3, respectively. The data following the triangles, rhombs and squares below give the relative mean standard deviations normalized in percent. All the three examples have been computed with the weight M_1 corresponding to the normal relative movement collected on field and calculated in millimetres $(M_1 \geq 1)$ (see text).

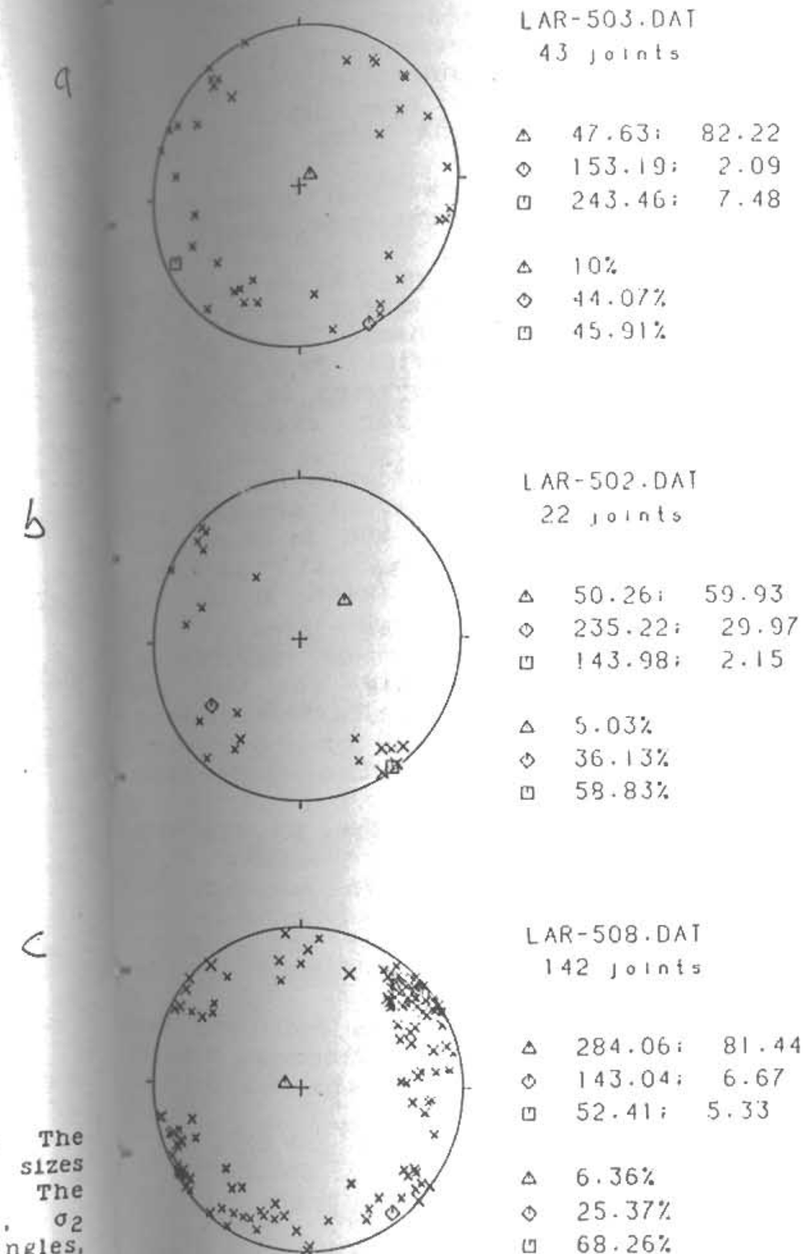

Fig. 3 - The same three examples of joint systems of figure 2 but computed with a constant weight for every joint $(M_1=1)$. The symbols are the same of those in figure 2.

In figures 2 and 3 are shown some stereo-nets in which a plotted the poles of the joints (crosses) and the thr principal axes of the stress ellipsoid (a triangle for th σ_1; a rhomb for the σ_2 and a square for th σ_3).

The dimension of the crosses is proportional to the norm relative movement calculated on field for each joint. Because graphic problems the crosses are grouped in four classes: up 1mm; between 1mm and 10mm; between 10mm and 10^2mm; more tha 10^2mm.

The examples were chosen either with two well grouped an roughly orthogonal sets, either with more scattered joints; th difference is visible on the stereo-nets.

Due to the uncertainty of the choice of the multiplyin factor we elaboreted the 3 joint systems twice. As a firs attempt of multiplying factor M_1, we choose the norma relative movement calculated in millimetres (fig. 2a,b,c). As a check we have analyzed the same three joint systems of figure with a constant multiplying factor (M_1=1 for every joint (fig. 3a,b,c).

As one may see, from the examples, the orientation of the three principal axes obtained with the two different analysis are quite similar. Only in the example of figures 2a and 3a there has been an exchange between σ_2 and σ_3; it is not suprising, because, as shown in figure 3a (with M_1=1), the statistical weight of the σ_2 and σ_3 is almost the same so that the stress ellipsoid is almost of revolution However, if we consider the weights (M_1≠1), (fig. 2a) the two axes, still remaining in the same position, are differentiated The former analysis clearly shows an ellipsoid of revolution while, in the latter one, it has been stressed the three-axial nature of the ellipsoid.

In the second case (fig. 2b and 3b) we have an opposite phenomenon. From the analysis with constant weights (M_1=1 (fig. 3b) we obtain a three-axial ellipsoid. As we consider the weights resulting from the observed opening (M_1≠1) (fig. 2b), we obtain the same three principal directions but the relative standard deviation of the σ_2 becomes close to that of the σ_3 showing us an uniaxial extension.

In the third case (fig. 2c and 3c) the analysis with the two different methods does not show any remarcable difference, either for the directions of the three axes, either for the statistical results.

The first two examples (fig. 2a,b and 3a,b) were intentionally chosen with relatively few joints to enlarge the statistical weight of the single weights and, consequently, of the two different methods of analysis.

Furthermore the difference between the results of the two methods of computation could be due to the just mentioned scarce number of joints considered.

Appendix

In order to find the solutions of

$$y(K) = -K^3 + NK^2 - AK + \Delta = 0 \qquad \text{(A1)}$$

let us assume to have diagonalized the matrix a_{ij} and be K_i its diagonal elements; then, since N, A and Δ are invariant we have

$$N = K_1 + K_2 + K_3 > 0$$

$$A = K_1K_2 + K_2K_3 + K_3K_1 > 0 \qquad \text{(A2)}$$

$$\Delta = K_1K_2K_3 > 0$$

The roots K_1, K_2, K_3 of (A1) may now be found as follows. K_1 is found in the interval $0,[N-\sqrt{(N^2-3A)}]/3$ because $y(0)=\Delta>0$ and $y(K_m)<0$; furthermore $y(K)$ has a relative maximum in $K_M=[N+\sqrt{(N^2-3A)}]/3$.

N^2-3A is positive, in fact assuming $K_3>K_2>K_1$ we have from (A2)

$$N^2 - 3A = (K_1 + K_2 + K_3)^2 - 3(K_1K_2 + K_2K_3 + K_3K_1) =$$

$$= K_1^2 + K_2^2 + K_3^2 - K_3K_2 - K_1K_3 - K_2K_1$$

$$= (K_3 - K_2)(K_3 - K_1) + (K_2 - K_1)^2 > 0$$

The other roots are then found considering the first and the third equations (A2) which give (s=2,3)

$$K_s^2 + (K_1 - N)K_s + \frac{\Delta}{K_1} = 0$$

and

$$K_s = \tfrac{1}{2}\left[\!\left[N - K_1 \pm \sqrt{\left[(N - K_1)^2 - \frac{4\Delta}{K_1}\right]}\right]\!\right]$$

which are real since

$$(N - K_1)^2 - \frac{4\Delta}{K_1} = (K_1 + K_3)^2 - 4K_2K_3 =$$

$$= (K_2 - K_3)^2 > 0$$

Since $y(K)$ is continuous, has a maximum in $K_M=[N+\sqrt{(N^2+3A)}]/3$, a minimum in $K_m=[N-\sqrt{(N^2-3A)}]/3<K_M$, with $y(K_m)<0$, it follows

$$K_2 = \tfrac{1}{2}\left[\!\left[N - K_1 - \sqrt{\left[(N - K_1)^2 - \frac{4\Delta}{K_1}\right]}\right]\!\right] > 0$$

$$K_3 = \tfrac{1}{2}\left[\!\left[N - K_1 + \sqrt{\left[(N - K_1)^2 - \frac{4\Delta}{K_1}\right]}\right]\!\right] > K_2 > K_1 > 0$$

as it should be because the form (5) is positive defined.

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