

A NON-EXTENSIVE STATISTICAL PHYSICS APPROACH TO THE CHARACTERIZATION OF THE PYROCLASTIC DEPOSITS OF THE KOS VOLCANIC CENTER

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Abstract

In this paper we present a Non-Extensive Statistical Physics (NESP) approach in order to investigate the grain size distribution of pyroclastic deposits from the Kos volcanic center that cover parts of the main Kos island as well as the surrounding areas. The Kos Plateau Tuff (KPT) of 161 Ka represents the largest Quaternary volcanic explosion of eastern Mediterranean. Pyroclastic deposits are formed by the fragmentation of magma and rocks, the driving mechanism of explosive volcanic activity. Rocks fragmentation can be considered in general as a non-linear process, where long-range interactions and scaling are important. It is exactly these processes that NESP was formed for to describe. The results of the analysis indicate that NESP is an appropriate framework for the statistical physics interpretation of volcanic rock fragmentation processes phenomena.

Key words: Tsallis Entropy, south Aegean volcanism, volcanic rocks.

Περίληψη

Στην εργασία αυτή χρησιμοποιούμε την γενικευμένη θεωρία της μη-εκτατικής στατιστικής φυσικής για την διερεύνηση της κατανομής του μεγέθους των κόκκων των πυροκλαστικών αποθέσεων, προερχομένων από το ηφαιστειακό κέντρο της νήσου Κω. Οι αποθέσεις αυτές καλύπτουν κυρίως τμήματα της νήσου Κω και παρακείμενων περιοχών. Οι πυροκλαστικές αποθέσεις σχηματίζονται από τον κατακερματισμό του μάγματος και των πετρωμάτων κατά την διάρκεια της εκρηκτικής φάσης της ηφαιστειακής δραστηριότητας. Η διεργασία του κατακερματισμού των πετρωμάτων είναι μη γραμμική διαδικασία και ανήκει ακριβώς σε αυτές τις διαδικασίες που αναφέρεται η γενικευμένη θεωρία της μη-εκτατικής στατιστικής φυσικής. Τα αποτελέσματα της ανάλυσης δείχνουν ότι η μη-εκτατική στατιστική φυσική είναι ένα κατάλληλο εργαλείο για την ερμηνεία της διαδικασίας θρυμματοποίησης των ηφαιστειακών σχηματισμών.

Λέξεις κλειδιά: Εντροπία Tsallis, ηφαιστειότητα νοτίου Αιγαίου, ηφαιστειακά πετρώματα.

1. Introduction

Pyroclastic deposits are those formed by the fragmentation of magma melt and rocks, which is the main mechanism regarding the explosive volcanic activity. Mechanical and granulometric analyses are used extensively in volcanology and volcanic hazard assessment, as they can provide valuable information for the eruption dynamics (Walker, 1971; Pyle, 1989). Moreover, the mass, internal stratigraphy, textural and lithofacies characteristics and componentry of the pyroclastic deposits are useful qualitative indicators of temporal variations in eruption intensity (Allen, 2001).

Since the Sparks's (1978) pivotal work, magmatic fragmentation has been regarded as a consequence of magma vesiculation, i.e., with decreasing external pressure a gas-saturated magma body evolves through a bubbly-foamy stage to erupting gas-pyroclast dispersion due to the collapse of bubble walls. An alternative mechanism for magma fragmentation is by conduit shear stress, as has been proposed for the pyroclastic deposits of Kos volcanic center (Palladino et al., 2008).

Fragmentation process can be defined as the structural failure of the earth's brittle material by the propagation of multiple fractures at different length scales (Perfect, 1997). Recently, the statistical properties of the fracturing process have attracted a wide interest in the scientific community (Herrmann and Roux, 1990; Chakrabarti and Benguigui, 1997). In this context, fracture can be seen as the outcome of the irreversible dynamics of a long-range interacting, disordered system (Krajcinovic and Van Mier, 2000). Several experimental observations have revealed that fracture is a complex phenomenon, described by scale invariant laws and fractality (Turcotte, 1986; Sornette et al., 1990).

The main motivation of our work is to investigate the statistical physics properties of the fracturing process that takes place during explosive volcanism. Non-extensivity represents one of the most intriguing characteristics of systems that experience long-range spatial correlations or long-range memory effects. Since disorder and fractality are two of the key components of the fracturing process, we use a current generalization of Boltzmann-Gibbs (BG) statistical physics due to Tsallis, referred as Non-Extensive Statistical Physics (NESP) (Tsallis, 1988; 2009), to explore the grain size distribution of pyroclastic deposits from the Kos volcanic center. NESP is a suitable framework for studying complex dynamic systems where fractality and long-range interactions are important and has been successfully applied in geophysics (see Vallianatos and Telesca, 2012 and references therein).

The applicability of this approach in earth sciences has been demonstrated in a series of recent publications on seismicity (Abe and Suzuki, 2003; 2005; Silva et al., 2006; Telesca, 2010; 2012; Vilar et al., 2007), natural hazards (Vallianatos, 2009; 2013), plate tectonics (Vallianatos and Sammonds, 2010), geomagnetic reversals (Vallianatos, 2011) and rock physics (Vallianatos et al., 2011; 2012; Vallianatos and Triantis, 2012).

2. The KPT Eruption

The volcanic Kos-Nisyros area forms the easternmost end of the Hellenic active volcanic arc (Figure 1). The volcanic activity in this area initiated in Pliocene time (3.4 – 1.0 Ma) and continued during Pleistocene and Quaternary time (Bellon and Jarrige, 1979; Dalabakis and Vougioukalakis, 1993). The Kos Plateau Tuff (KPT) eruption took place at 161 ka (Smith et al., 1996) and was probably the largest eruption in the Pliocene–Quaternary South Aegean arc (Allen, 2001). The source is placed between Kos and Nisyros and resulted into a caldera collapse (Allen, 2001; Pe-Piper et al., 2005).

Pyroclastic deposits from this eruption are spread in the island of Kos, the surrounding islands of Kalymnos, Pserimos, Tilos and Pachia, as well as on the Turkish peninsula of Bodrum and Datça region (Allen and Cas, 1998). The eruption of KPT produced a sequence of a well preserved,

rhyolitic fallout, stratified pyroclastic-density-current deposits and massive ignimbrites (Allen, 2001). Six major stratigraphic units can be identified that represent a change in the eruptive conditions from initial and final phreatomagmatic activity depositing fallout and internally stratified pyroclastic density current deposits to dry explosive, during the more intense phases of the eruption that produced the formation of ignimbrites (Allen et al., 1999).

In the present work 12 samples of pyroclastic deposits have been collected, 9 samples from the island of Kos and 3 samples from the island of Kalymnos. The grain size distribution for each of the samples has been obtained through granulometric analysis and is being used further on in their analysis.

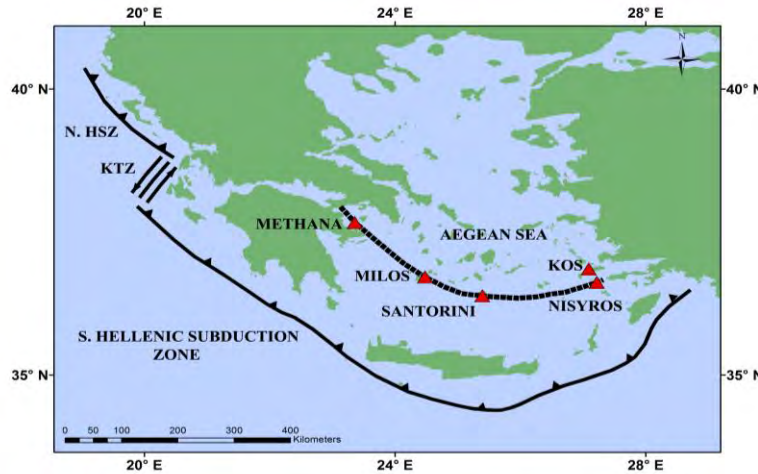


Figure 1- The location of the volcanic Kos-Nisyros area along the Hellenic volcanic arc. The modern active trenches (thick dark lines with solid barbs) for the Hellenic Subduction Zone (HSZ) are also presented, as Royden and Papanikolaou (2011) indicate them. The junction of the north and the south part of the HSZ, coincides with the Kephalonia Transform Zone (KTZ).

3. Non Extensive Statistical Physics Analysis

Non-Extensive Statistical Physics (NESP) refers to the non-additive entropy S_q (Tsallis, 1988; 2009), which is a generalization of Boltzmann–Gibbs entropy S_{BG} and has been frequently used to characterize complex dynamical systems that exhibit scale-invariance, (multi)fractality and long-range interactions (e.g. Gell-Mann and Tsallis, 2004; Tsallis, 2009). Tsallis entropy S_q is non-additive in the sense that is not proportional to the number of the system's elements as S_{BG} does. The Tsallis entropy S_q reads as:

Equation 1- Tsallis entropy

$$S_q = k_B \frac{1 - \sum_{i=1}^W p_i^q}{q-1}, \quad q \in \mathbb{R}$$

and in integral formulation as:

Equation 2- Integral formulation of Tsallis entropy

$$S_q = k \frac{1 - \int p^q(x) dx}{q-1}$$

where k_B is Boltzmann's constant, p_i is a set of probabilities, W is the total number of microscopic configurations and q the entropic index. The latter index is a measure of the non-extensivity of the system and for the particular case $q=1$, the Boltzmann-Gibbs entropy S_{BG} is obtained:

Equation 3 -The Boltzmann-Gibbs entropy

$$S_{BG} = -k_B \sum_{i=1}^W p_i \ln p_i$$

The cases $q>1$ and $q<1$ correspond to subadditivity and superadditivity respectively. Although Tsallis entropy shares a lot of common properties with the Boltzmann- Gibbs entropy, S_{BG} is additive, whereas S_q ($q \neq 1$) is nonadditive (Tsallis, 2009). According to this property S_{BG} exhibits only short-range correlations and the total entropy depends on the size of the systems' elements. On the other hand, S_q allows all-length scale correlations and seems more adequate for complex dynamical systems when long-range correlations between the elements of the system are present.

The process of rock fragmentation, especially when high energies are involved, leads to the existence of long-range interactions between all parts of the object being fragmented. This type of entropy is nonadditive and depends on the object as a whole. Thus, the use of the non-extensive approach seems adequate to analyze the complex mechanism of fragments frequency-size distribution. Sotolongo-Costa and Posadas (2004), proposed a model for fragmentation considering the interaction of two rough profiles and the fragments filling the gaps between them, where the fragments are produced by the local breakage due to externally applied stress. In addition, Silva et al. (2006) have revisited the fragment asperity model for earthquakes, as it is introduced by Sotolongo-Costa and Posadas (2004), leading to a modified expression of frequency – magnitude distribution of earthquakes.

Following their approach, in order to estimate the probability distribution $p(\sigma)$ of a parameter σ , the surface of a fragment in our case, we maximized the non-extensive entropy under appropriate constraints, using the Lagrange multipliers method. The first constraint used refers to the normalization condition that reads as:

Equation 4 - Normalized condition

$$\int_0^{\infty} p(\sigma) d\sigma = 1$$

Introducing a generalized expectation value (q -expectation value), σ_q is defined as:

Equation 5- Definition of q-expectation value

$$\sigma_q = \langle \sigma \rangle_q = \int_0^{\infty} \sigma P_q(\sigma) d\sigma$$

where the escort probability is given by (Tsallis, 2009) as follows:

Equation 6 – The escort probability

$$P_q(\sigma) = \frac{p q(\sigma)}{\int_0^{\infty} p q(\sigma) d\sigma}$$

The extremization of S_q with constraints (4) and (5) yields

Equation 7- Probability distribution of fragments' surface

$$p(\sigma) = \left[1 - \frac{1-q}{2-q} (\sigma - \sigma_q) \right]^{\frac{1}{1-q}}$$

where the q -exponential function is defined as:

Equation 8- Definition of q-exponential function

$$\exp_q(X) = \begin{cases} [1 + (1-q)X]^{1/(1-q)} & (1 + (1-q)X \geq 0) \\ 0 & (1 + (1-q)X < 0) \end{cases},$$

whose inverse is the q -logarithmic function:

Equation 9 – The q -logarithmic function

$$\ln_q(X) = \frac{1}{1-q} (X^{1-q} - 1).$$

Following Sotolongo-Costa and Posadas (2004) and Silva et al. (2006), the proportionality between the fracture surface σ and grain size r becomes:

Equation 10

$$\sigma - \sigma_q = \left(\frac{r}{r_0}\right)^2,$$

where r_0 is a geometric scaling factor.

Thus, the size distribution of fragments is obtained in the following manner: using equations (7) and (10) we lead to:

Equation 11 – The fragment size distribution function

$$p(r) = \frac{\frac{2r}{r_0^2} dr}{\left[1 - \frac{1-q}{2-q} \left(\frac{r}{r_0}\right)^2\right]^{\frac{1}{1-q}}}.$$

The normalized cumulative number of fragments can be obtained by integrating equation (11) as:

Equation 12 – The normalized cumulative number of fragments

$$\frac{N(>r)}{N_0} = \int_r^\infty \frac{\frac{2r}{r_0^2} dr}{\left[1 - \frac{1-q}{2-q} \left(\frac{r}{r_0}\right)^2\right]^{\frac{1}{1-q}}},$$

where $N(>r)$ is the number of fragments with size larger than r , and thus :

Equation 13

$$\frac{N(>r)}{N_0} = \left[1 + \left(\frac{q-1}{2-q}\right) \left(\frac{r}{r_0}\right)^2\right]^{\frac{q-2}{q-1}}.$$

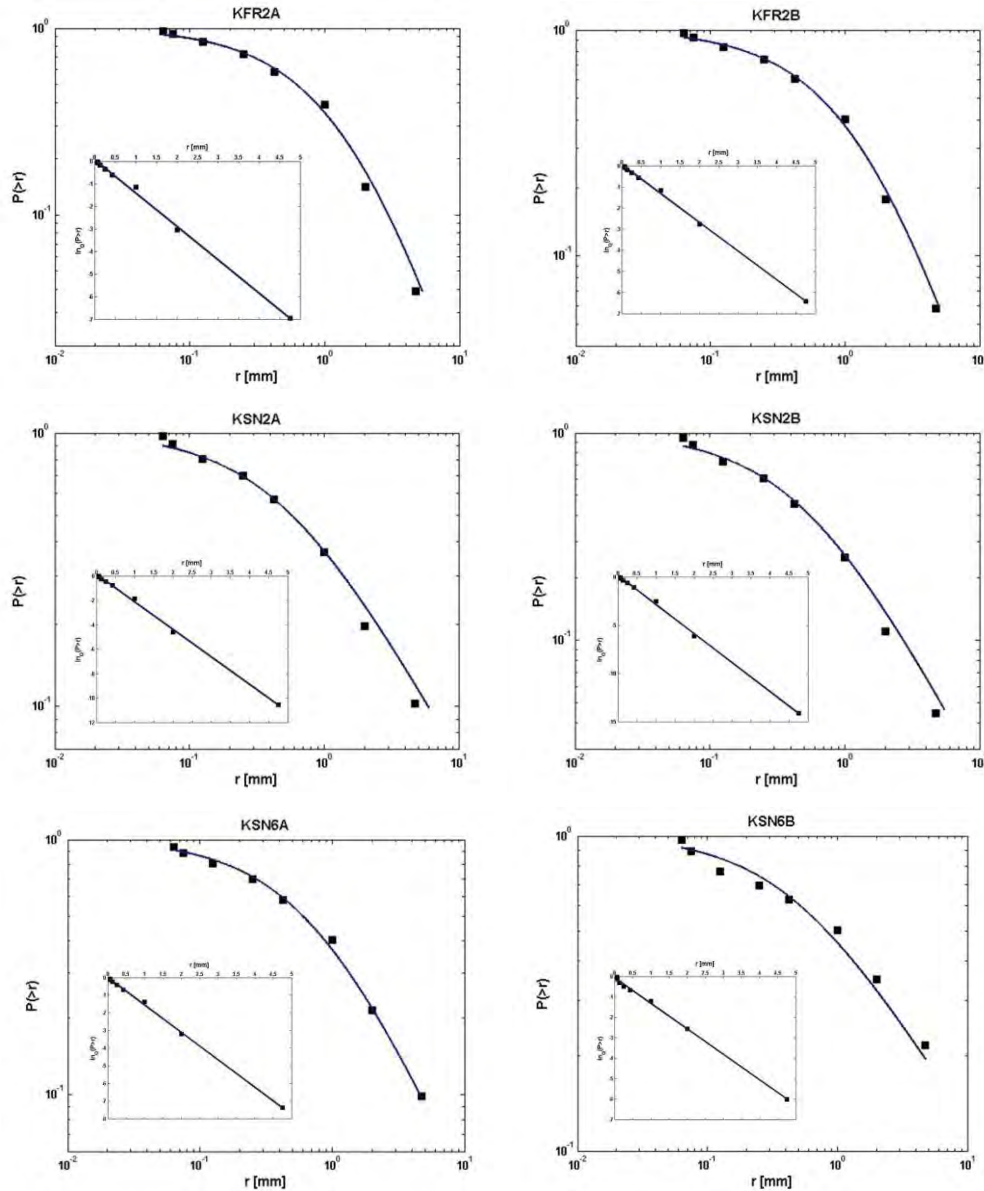
The latter expression has the form $P(>r) = \left[1 + \left(\frac{q-1}{2-q}\right) \left(\frac{r}{r_0}\right)^2\right]^{\frac{q-2}{q-1}}$ which if we define $q = 2 - \frac{1}{Q}$ leads to:

Equation 14

$$P(>r) = \exp_Q(-r/r_0) = \left[1 + (Q-1)\left(\frac{r}{r_0}\right)\right]^{-\frac{1}{Q-1}},$$

having a Q -exponential form. The inverse of Eq. (14) is the Q -logarithmic distribution $\ln_Q(P(>r))$. After the estimation of the appropriate Q that describes the observed distribution $P(>r)$, the Q -

logarithmic distribution $\ln_Q(P(>r))$, where $\ln_Q(x) = \frac{1}{1-Q} (x^{1-Q} - 1)$, is linear with r (Abe and Suzuki, 2005). In the limit $Q \rightarrow 1$, the ordinary exponential and logarithmic functions are obtained from the $e_Q(x)$ and $\ln_Q(x)$ functions, respectively.



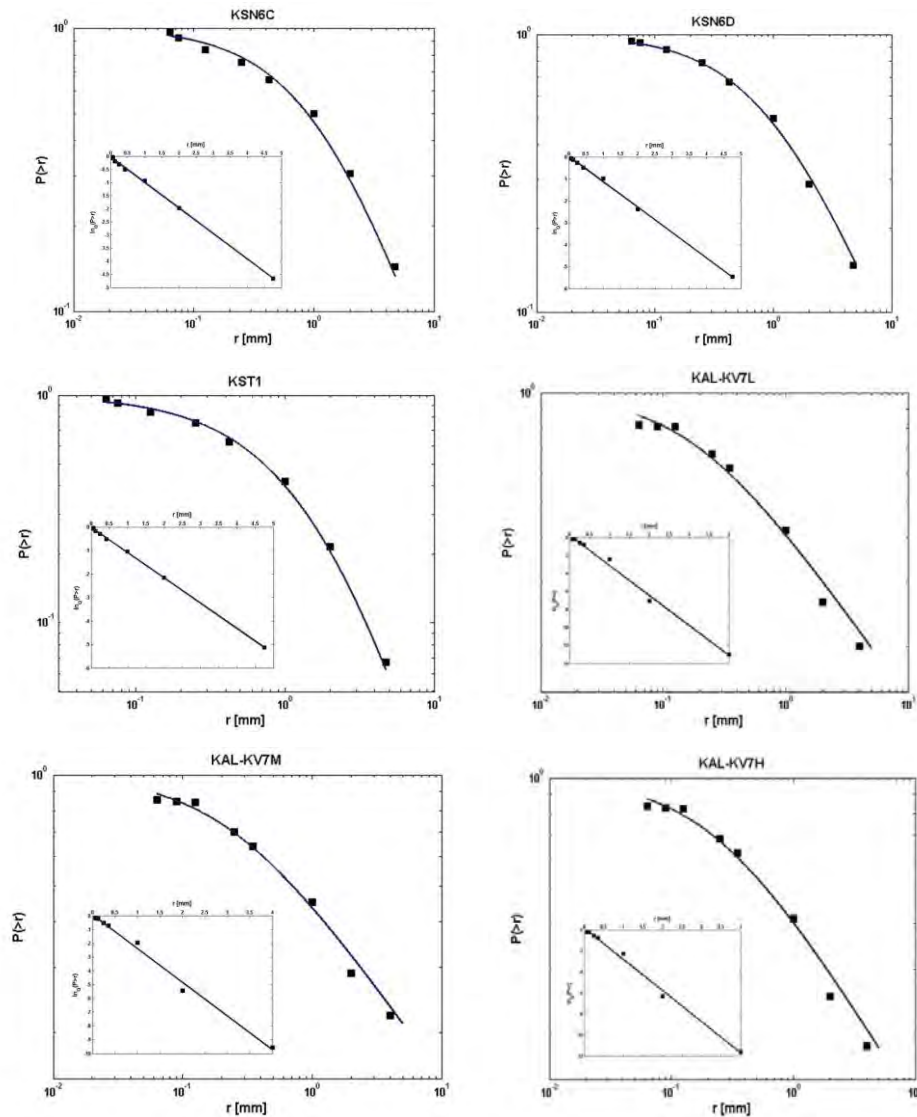


Figure 2 - Cumulative grain size distribution (squares) and the equivalent Q -exponential distribution according to Eq.(14) for each sample from the pyroclastic deposits collected from the islands of Kos and Kalymnos. In the inset of each figure the Q -logarithmic distribution is also plotted.

We have applied this approach to the cumulative grain size distributions $P(>r)$ of the pyroclastic deposits collected from the islands of Kos and Kalymnos. In Figure 2 the Q -exponential distribution according to Eq.(14) is plotted for each sample. We can see that for the appropriate values of q and r_0 , the Q -exponential distribution can describe quite well the observed grain size distributions $P(>r)$. In the insets of each figure (Figure 2), the Q -logarithmic distribution is also plotted for each sample that approaches linearity with high correlation coefficients. The q and r_0 that resulted from the analysis, as well as the equivalent correlation coefficients are referred in Table 1.

Table 1 - The q and r_0 values that resulted from the regression analysis according to Eq.(14) and the equivalent correlation coefficients (ρ).

Sample	q-value	r_0 (mm)	ρ
KFR2 (A)	1.3	0.762	-0.9988
KFR2 (B)	1.34	0.786	-0.9995
KST1	1.302	0.885	-0.9998
KSN2 (A)	1.528	0.548	-0.9993
KSN2 (B)	1.448	0.405	-0.9994
KSN6 (A)	1.463	0.636	-0.9995
KSN6 (B)	1.6	0.671	-0.9991
KSN6 (C)	1.444	0.948	-0.9996
KSN6 (D)	1.489	0.925	-0.9994
KAL-KV7H	1.648	0.423	-0.9972
KAL-KV7M	1.677	0.469	-0.9963
KAL-KV7L	1.674	0.378	-0.9968

4. Conclusions

In the present work the statistical physics of one of the largest volcanic eruptions in the Pliocene–Quaternary age in South Aegean volcanic arc is being investigated (KPT eruption), by analyzing the grain size distribution of the pyroclasts that deposited in the islands of Kos and Kalymnos during the different stages of the eruption. For this purpose we used Non-Extensive Statistical Physics (NESP), a recent generalization of Boltzmann-Gibbs statistical physics that has been frequently used to describe complex dynamical systems. The analysis that was performed to the cumulative grain size distribution for each of the 12 samples according to the non-extensive model of Eq.(14) indicated that these distributions can be successfully described using NESP. These results imply that non-linear dynamics control the fragmentation process during explosive volcanism and that NESP is an appropriate framework for the statistical physics interpretation of complex Earth dynamic systems.

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