RECURSIVE INVERSION : SOME ASPECTS OF ITS APPLICATION TO SYNTHETIC AND REAL SEISMIC DATA

KONSTANDINOS KALENDEROGLU

DEP-EKY -
Department of seismic interpretation
199 Kifissias Ave.
151 24 Athens

ABSTRACT

This paper describes the application of the recursive inversion technique to synthetic and real seismic data. The geoseismic modelling studies are carried out on borehole data measured at three oil-gas productive areas in Hungary (Kisujszallas, Fegyvernek and Battonya). In a synthetic study, input data are sonic logs.

A simple seismic modelling technique is applied to obtain synthetic seismograms. The so computed model traces are then transformed into synthetic sonic logs by using an algorithm which fits the seismic trace to the input sonic log. The real seismic traces nearest to the well logs are also inverted and the derived seislogs are compared with the corresponding sonic logs. Random and correlated noise is added to the synthetic seismogram and its effect on the computed sonic log is also included in this study. Two amplitude adjustment procedures used to scale the approximate reflection coefficient series are checked. Finally the pseudo velocity log computed by our computing program is compared to that obtained as the output at the pseudo velocity program package used by CGG.

INTRODUCTION

One of the major objective of the reflection seismology is to recover the acoustic impedance as a function of depth from observed seismic data. The recursive inversion is considered to be one of the most widely used to achive this goal.

The basic idea of the method is simply the reverse procedure of that used to obtain synthetic seismogram from acoustic logs. It is well known that the pressure amplitude reflection coefficient for vertically travelling waves is given by

$$C_i = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i} \qquad i = 1,2,3,\ldots \qquad (1)$$

where C_i is the reflection coefficient of the i-th interface $Z_i = Q_i \cdot V_i$ is the acoustic impedance of the i-th layer, Q_i is the density and V_i is the propagational velocity of compressional waves. By rearranging the terms of eq. 1 we obtain

$$Z_{i+1} = Z_i \cdot \frac{1 + C_i}{1 - C_i} \qquad (2)$$

By succesive application of the above equation we obtain

$$Z_i = Z_1 \cdot \prod_{k=1}^{i-1} \frac{1 + C_k}{1 - C_k} \qquad (3)$$

Further manipulation of the equation 3 leads to the following exponential formula

$$Z_i = Z_1 \cdot e^{2 \cdot \sum_{k=1}^{i-1} C_k} \qquad (4)$$

(3) and (4) are nonlinear formulae and can be used for the inversion procedure. For values $|C_k| \leq 0.2$ eq 4 is satisfactory. In the computer program the exponential formula 4 was used.

COMPUTER MODELLING

Two sets of well-log data were utilised. The first set includes both sonic and density logs measured in a productive gas field near Kisujszallas. One line of seismic reflection data is located in the immediate vicinity of the well logs. The second set of borehole data includes only sonic logs which come from two areas Battonya and Fegyvernek. Density information was not available.

The computer program includes two processing stages. In the first stage input sonic logs are converted to synthetic seismic traces. The reflection coefficient series computed from the logs is convolved with Ricker type zero-phase wavelet of dominant frequency 30, 40 and 50 Hz. In the second stage the seismic model is entered as input to the pseudo velocity program package, where it re-appears at the output as reconstructed sonic log. Finally the input velocity and the computed pseudo velocity are plotted together to indicate the quality of the latter and the mean square deviation (MSD) is calculated.

Some results are shown in figures 1,2, and 3. Comparison of the three synthetic traces clearly illustrates the increased resolving power of the seismic trace for higher dominant frequency. It is aparent that finer details of the sonic log can be resolved by increasing the band width of the synthetic trace.

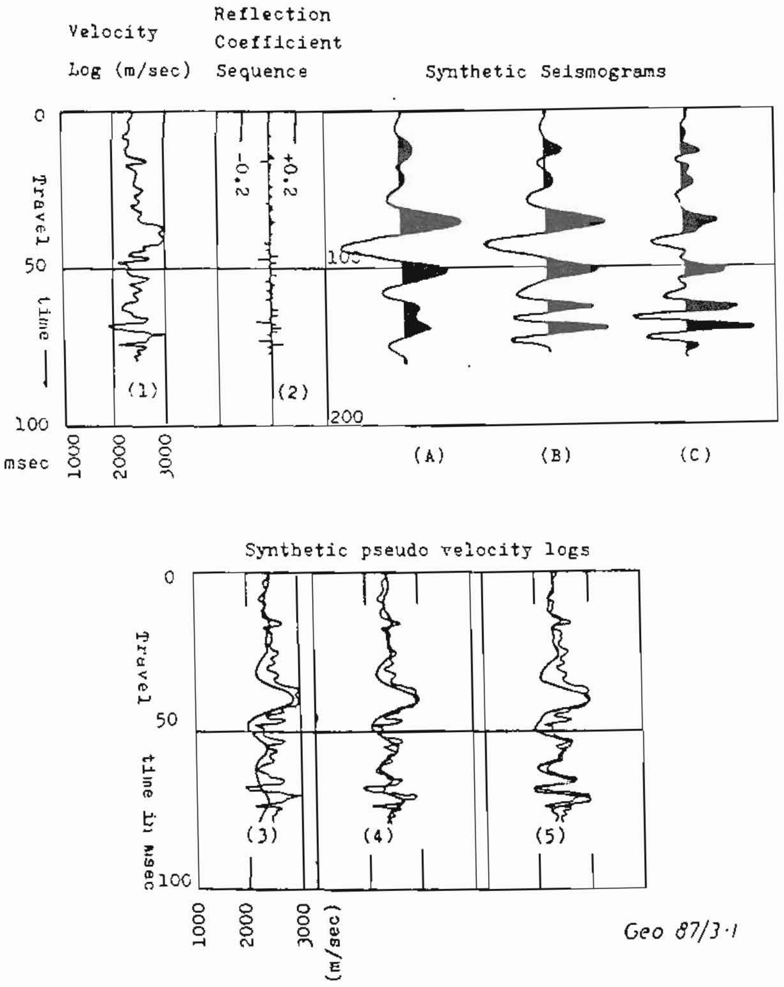

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Fig. 1 Synthetic pseudo velocity logs obtained from kisujszallas 15 borehole data.

Table 1 gives the mean square errors δ of the pseudo velocity values. It can be seen that the mean square errors decrease with increasing dominant frequency of the Ricker wavelet. Some differences between the pseudo velocity log and the input Sonic log can be seen. For instance, in figure 3 at the time of 200 ms there is a sharp velocity decrease. Inversion does not yield the sharp step but the velocity shift to higher velocity immediately after. In general this is the result of the deficiency of seismic data in low frequencies across major acoustic interfaces. Figure 4 shows the power spectra of the reflection coefficient series from one sonic log, the 50 Hz dominant frequency Ricker wavelet, the model trace and the pseudo velocity log.

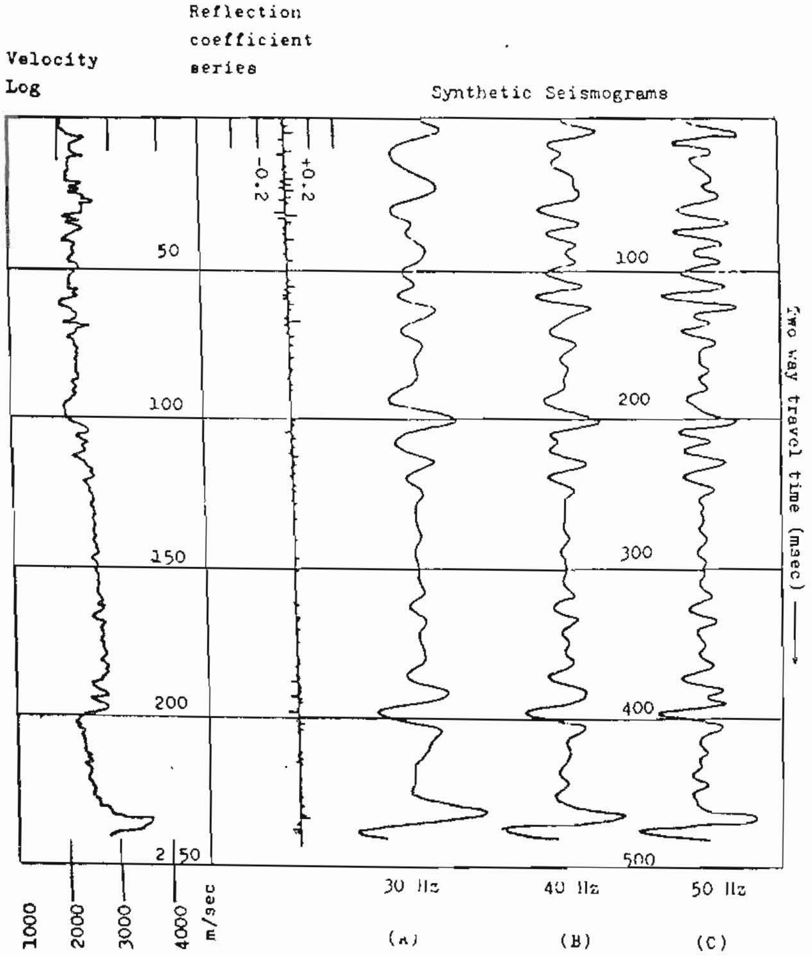

Fig. 2 Ricker wavelet synthetic seismograms optained from Batonya E_4 borehole data.

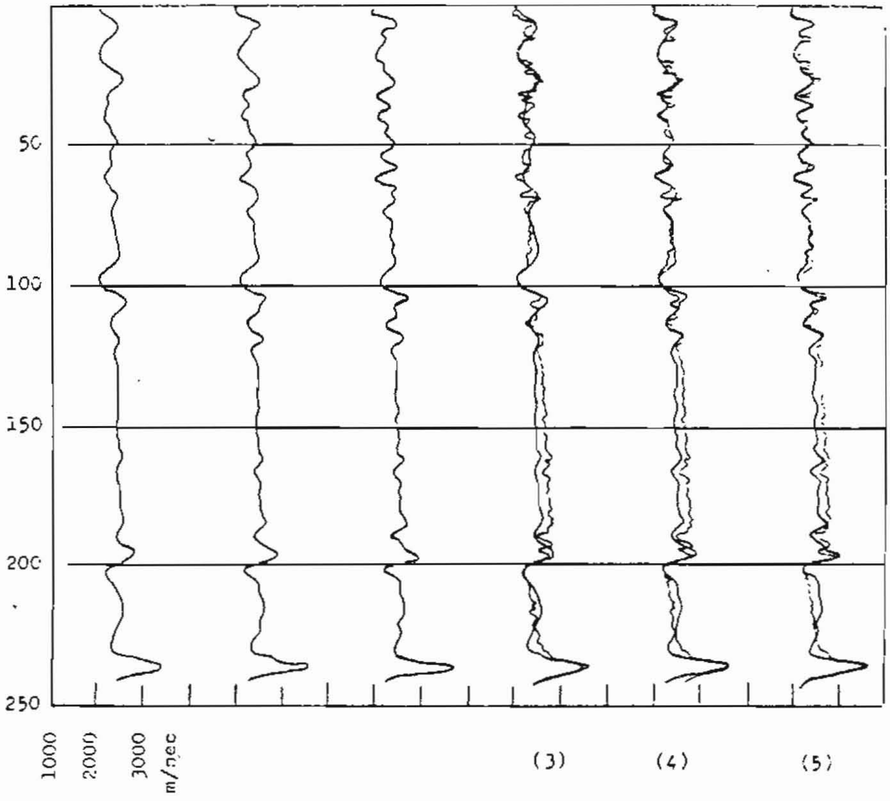

Fig. 3 Synthetic pseudo velocity logs obtained from Battonya E_4 borehole data.

The spectra show that the reflection coefficient though far from being white, but it is rich in high frequencies. The spectrum of the model trace is characterized by band limitation with frequencies confined between 25 and 80 Hz. It is also apparent that the spectrum of pseudo velocity log is somehow shifted toward lower frequencies with respect to the spectrum of the synthetic trace. This fact proves that the recursive inversion increases further the amplitudes of the low frequency components of data. The six real seismic traces (fig.5) nearest to the well logs were inverted in an attempt to check the reliability of seislog traces by using the well log data. On the basis of the provided check shot data, it was attempted to fit the corresponding sonic log to the seislog trace. In tying the seislog to sonic logs it was found that the intergrated sonic log times did not agree with the check-shot-derived times and some time shift was necessary.

Figure 6 shows the four seismic traces after deconvolution and the relative seislog traces together with the corresponding sonic logs. The poor resolution of the seislog traces as compared to the corresponding sonic log is evident in the figure. Comparison between the details of seislog traces and the sonic logs shows some correspondence between the negative anomalies but this correspondence is not so good. A number of well log synthetics were constructed by using the Kisujszallas borehole data and then compared with the seismic traces of the well site. An example is shown in figure 7 where a series of convolutional models of the same reflection coefficient series is constructed. The dominant frequency of the Ricker wavelet is 20, 25 and 30 Hz. The synthetic trace constructed by using a 30 Hz dominant frequency Ricker wavelet and the real seismic trace match fairly good. On the basis of the check shot data from Kisujszallas 38 borehole, it was attempted to fit the sonic log obtained by inverting the 30 Hz dominant frequency model trace to the corresponding relative seislog. Figure 8 shows the three traces plotted together.

The same procedure was applied to the other sets of borehole data from the Kisujszallas area but a good match was found in a few cases only. This failure in matching can be attributed to the fact that the seismic trace is not exactly a convolutional model of the log reflection coefficients. It is probable that some disturbing effects such as multiples, which were not properly attenuated during seismic processing are present and cause these discrepancies.

COMPARATIVE STUDIES
THE EFFECT OF CORRELATED AND RANDOM NOISE

It is difficult to analyse the effect of the additive noise on the computed pseudo velocity because, as can be seen in eq.3 the transformation of the reflection coefficient series to pseudo velocity is not linear. Nevertheless some synthetic examples are given in this section to demonstrate the effect of noise.

Two type of noise are examined, the correlated noise which refers to errors in the reflection coefficient and random noise which is added to the amplitudes of the

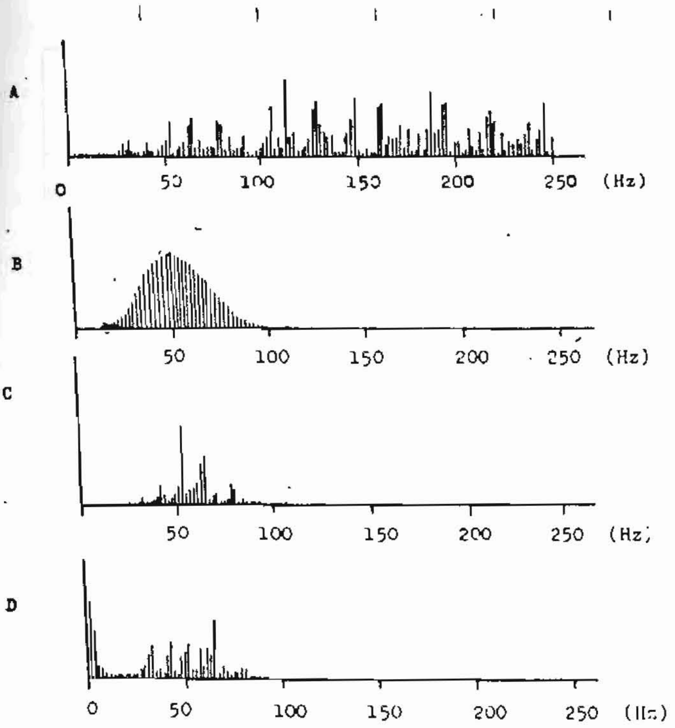

Fig. 4 The power spectrum of the reflection coefficient series (A), of the Ricker wavelet (B), of the model trace (C) and of the pseudo velocity (D)

model trace. The correlated noise is considered as an aproximate model of the multiple background. Four different realizations of correlated noise were generated ($n = 0.1$, 0.2, 0.3 and 0.4) and this noise was added to the reflection coefficient series. That is, we have

$$C'_k = C_k + Q_k$$

Where C_k is the primary reflection coefficient and Q_k the correlated noise.

In the second case, again four different realizations of a white noise process were generated and then added to the amplitudes of the constructed synthetic trace. The noise levels were $\sigma = 5, 7, 10, 12$.

That is, we have

$$\hat{C}_k = C_k * W_k + P_k = \sum_j C_{k-j} \cdot W_j + P_k$$

where C_k is the primary reflection coefficient sequence

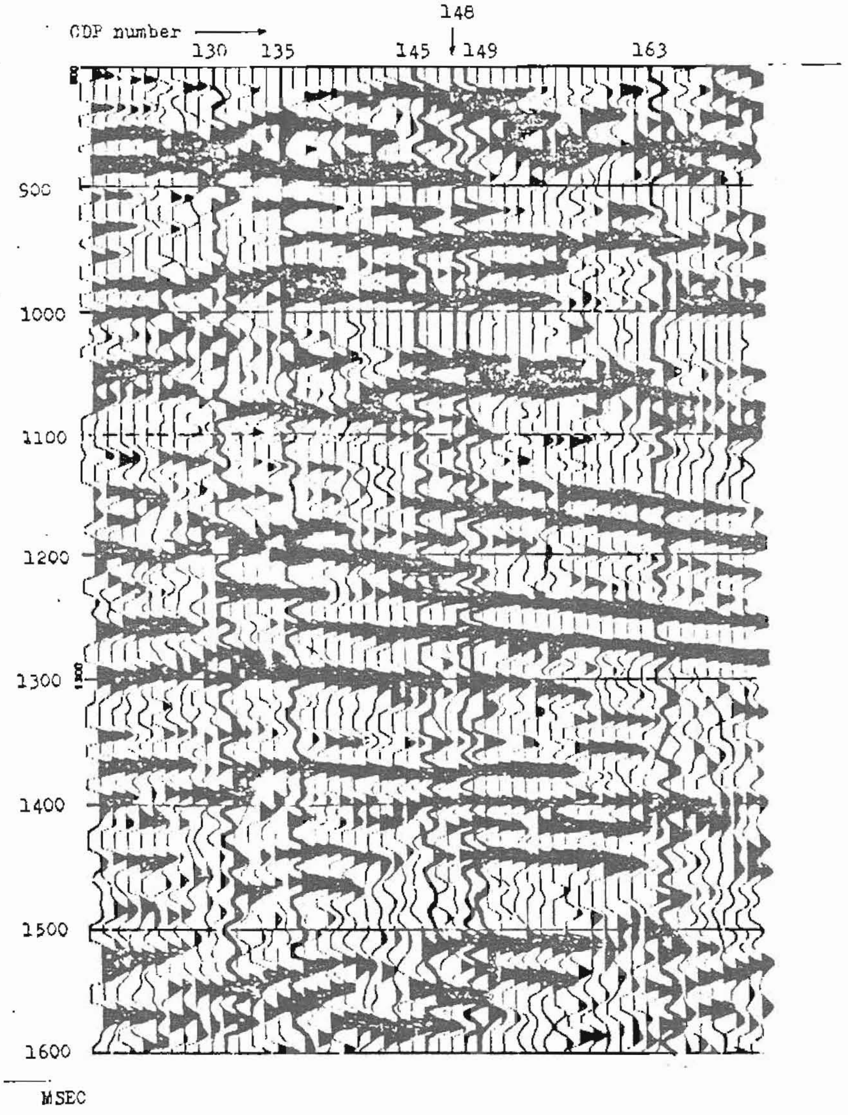

Fig. 5 The seismic window

W_k the Ricker wavelet and P_k the added zero-mean white noise with variance σ^2.

Figure 9 shows the sonic log measured in Fegyvernek 6 borehole with the pseudo velocity computed from a model trace with correlated noise added. Table 2 shows the mean square deviations (MSD) of the computed pseudo velocities from the input velocity functions for the two cases of correlated and random noise. The MSD values are indicative

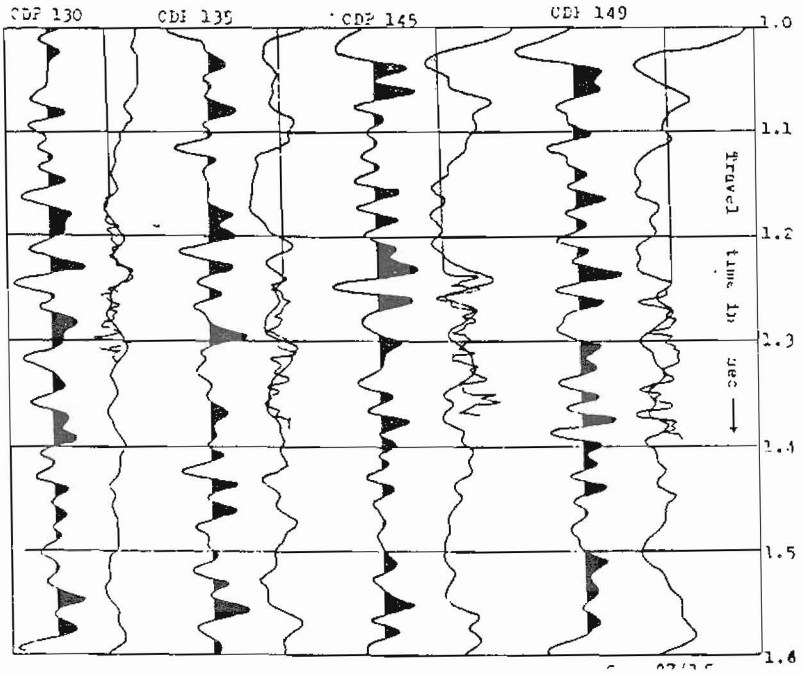

Fig. 6 Comparison of the sonic logs in Kisujsrallas -15, -36, -37, -38 with the pseudo velocity logs computed from seismic traces recorded in the vicinity of the borehole

of the errors produced in the pseudo velocity especially the two extreme values **$n = 0.4$** and **$\alpha = 12$** compared to the noise-free case. It is certain that higher values of correlated and random noise will produce even larger errors.

THE EFFECT OF MUTING THE REFLECTION COEFFICIENT SEQUENCE

The effect of removing progressively larger reflection coefficients from a series was also investigated. The sonic log measured at the Fegyvernek 6 borehole was chosen for this study. This sequence was muted progressively by 10%, 20% and 30% of its maximum absolute value.
Then the model trace constructed by convolving the muted reflection coefficient series with a 40Hz Ricker wavelet was inverted. The resulting pseudo velocity log was then compared directly with the original sonic log. The MSD values of the derived pseudo velocity are given in table 3. An examination of the power spectra of the model traces

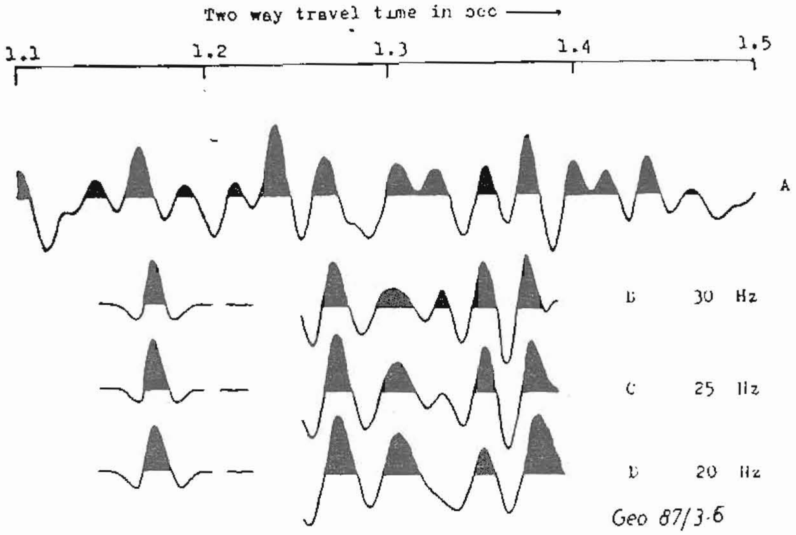

Fig. 7 Matching the synthetic seismic trace (B,C,D) to the real seismic trace (A). B, C, D are synthetic traces constracted by using Ricker wavelet of frequency 30, 25 and 20 Hz respectively.

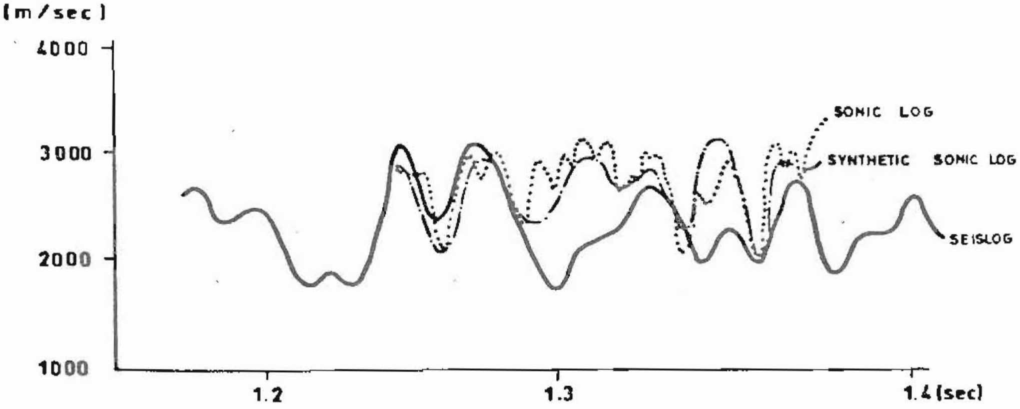

Fig. 8 The three traces plotted together, the sonic log, the synthetic sonic log and the relative seislog.

TABLE 1

The mean square errors (σ) of the pseudo velocity values

Dominant frequency of the convolving Ricker wavelet Hz	Mean square deviation (MSD)σ(V) m/sec				
	Bat.E5A	BatE5B	BatE6	BatE4	Feg 6
30	88.06	253.98	176.25	170.41	140.23
40	83.71	248.27	171.47	166.96	
50	85.47	240.03	171.34	167.58	129.30
70					124.39

TABLE 2

a. Correlated noise

n	Mean square deviation
0.	112.53
0.1	142.61
0.2	128.19
0.3	131.86
0.4	156.22

b. Random noise

σ	Mean square deviation
0.	112.53
5	140.77
7	125.72
10	136.06
12	146.73

TABLE 3

Muting factor (MF)	Mean square deviation
0%	112.53
10%	111.47
20%	118.74
30%	140.50

shows that they are being shifted toward lower frequencies as the value of MF increases. Therefore there is an associated loss of high frequencies in the computed pseudo velocities.

AMPLITUDE SCALING OF THE SEISMIC TRACE

We have seen that before applying the exponential formula (4) the amplitudes of the seismic trace must be multiplied by a scaling factor. In our previous examples this scaling factor K was computed from the formula

$$K = \ln\left|\frac{V_L}{V_O}\right| \frac{1}{\sum_{s=0}^{t} a_s} \qquad (5)$$

Where V_L is the final velocity, V_O the initial velocity and a_s the seismic amplitude.

Two amplitude scaling procedures were tested. The first method is based on computing a scaler K which scales the seismic trace in a way that the computed pseudo velocity is closest to the linear trend of the input sonic log in the least-squares error sense.
The following formula gives the appropriate scaler

$$K = \frac{\sum_{t=0}^{T} \sum_{s=0}^{S} a_s \ln(1+m\,t)}{\sum_{t=0}^{T} \left(\sum_{s=0}^{t} a_s\right)^2} \qquad (6)$$

where

$$m = \frac{1}{V_O} \, \frac{V_L - V_O}{T}$$

As a test of this method, the program was run several times with different sets of borehole data and by using the scaling factor K determined from the above equation. In all the cases it was found that the computed pseudo velocity was the linear trend of the input velocity function with some minor deviations. It seems that the so computed scaler K is very small and the amplitudes are not properly scaled.
In the second procedure, the rms amplitude value at the seismic channel is set equal to a given value λ and the required scaler K is computed by the following formula

$$K=\frac{\lambda}{\sqrt{\frac{1}{N}\sum_{j=1}^{N} g_i^2}} \qquad (7)$$

Where g_i is the seismic trace.
Our program was applied for different values of λ.
The results are shown in fig. 10 and summarized in table 4.
It can be seen that within the range between 0.007 and 0.03 the value of λ is not important. It gives a synthetic log which more or less approximates the input sonic log.
As the value of λ increases beyond the value of 0.05 these deviations become larger and finally the results become unstable. At the other end of this range the derived synthetic log tend to approximate the linear trend of the sonic log.

An attempt was made to compare the pseudo velocity log computed by our computing program to that obtained as the output of the pseudo velocity program package used by CGG.
The applied formual is given by

$$V_i = V_{i-1} - V_L + V_{L_{i-1}}\left(\frac{1 + C_{i-1}}{1 - C_{i-1}}\right) \qquad (8)$$

where V_i is the computed pseudo velocity

V_{L_i} is the low frequency information

and C_i the approximate reflection coefficient series.
The linear trend of the original sonic log was used as the low frequency information in the above formula.
The mean square errors of the derived pseudo velocity are given in table 5 and compared to those computed by our present program.
It can be seen from the table that the CGG method gave values a little higher than those computed by the present method. An examination of the comparison plots also shows that the method gives approximately the same results.

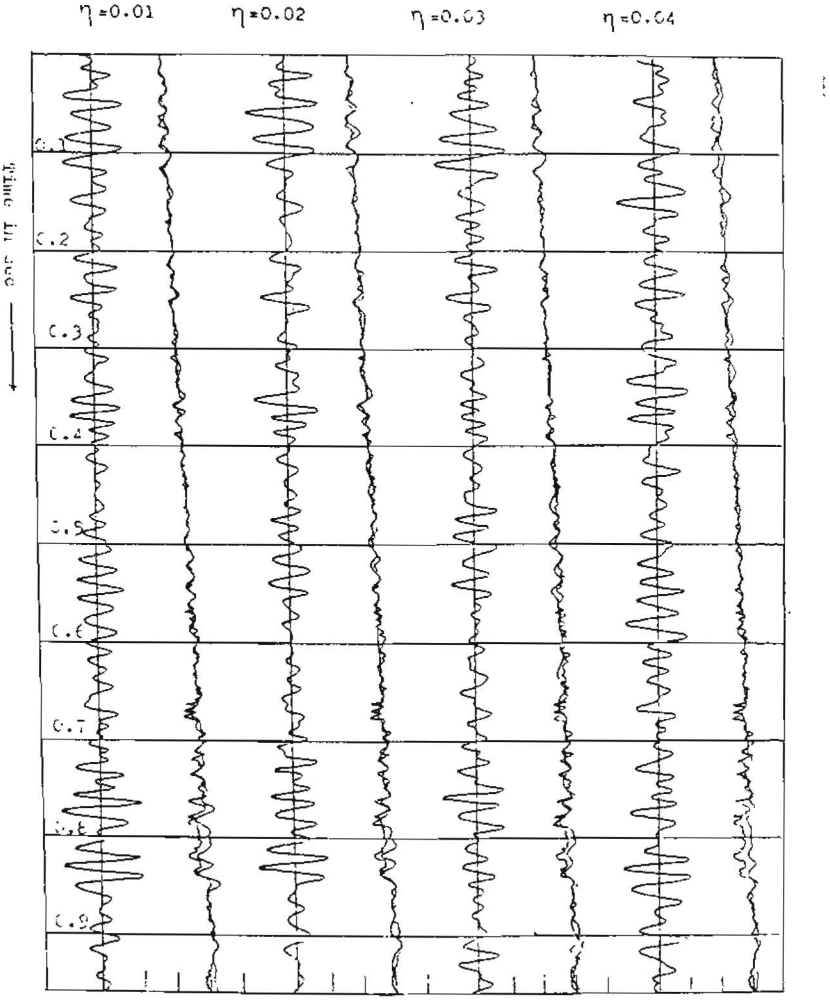

Fig. 9 Comparison of the sonic log measured in Fegyvernek 6 with the pseudo velocity log computed from a model trace with correlated noise added.

CONCLUSIONS

A computer program has been given for the computation of pseudo velocity from seismic data. We have tested the program extensively with available well log and seismic data and have found that it yields good results.
Six real seismic traces were inverted and the computed seislogs were compared with the sonic logs measured in their vicinity. Comparison of the seislog traces reveals

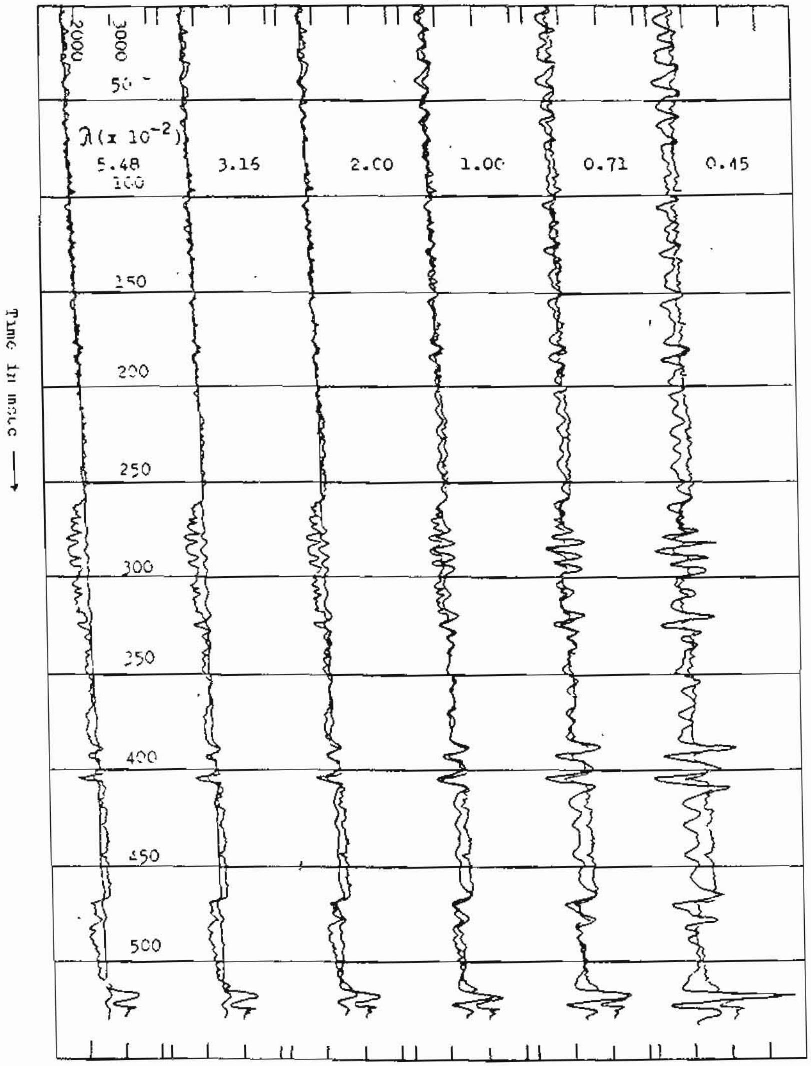

Fig. 10 Comparison of the sonic log with the pseudo velocity computed for a synthetic trace scaled by using several values of **λ**.

TABLE 4

λ ($x10^{-2}$)	K($x10^{-4}$)	mean square deviation
5.48	51.29	360.40
3.16	29.61	227.76
2	18.72	181.08
1	9.36	172.41
0.71	6.62	177.44
0.45	4.18	184.80

TABLE 5

Mean Square deviation (MSD) $\sigma(v)$

Dominant frequency of R.W.	Present method	CGG method
30	176.25	167.48
40	171.47	193.31
50	171.34	225.70

that some agreement between the three over many hundreds of meters of depth can be obtained by the seismic inversion method.

An effort has been made to analyse, which effect the additive noise, correlated and random, would have on the computation of pseudo velocity. The results prove that the contamination of seismic signal with noise produces errors. The estimated MSD values show that these errors become larger with increasing values of **n** and **σ**.

It has been shown that the effect of muting of the reflection coefficient series has similar results. It introduces errors to the computed pseudo velocity. A large number of individually insignificant interfaces are equally important as a few major ones.

Two amplitude scaling procedures were tested. Our results proved that the first method does not compute the proper scaler. In the second method, it has been shown that rms values should be in the range between 0.03 and 0.007 while a value equal to 0.01 gives the best match of pseudo velocity to the original sonic log.

Finally, it has been shown that our program and the CGG pseudo velocity program package gave similar results.

Some references relevant to this paper are given below.

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