A NEW SECOND DERIVATIVE OPERATOR FOR USE IN THE SPACE DOMAIN

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Summary:

Following the solution of the Laplace equation in terms of a Fourier-Bessel expansion, a new operator for the estimation of the second derivative in the space domain has been deduced. The operator applies to gridded data and uses the average value of the potential field along three rings around each grid point. The radii have been chosen to offer more grid points along their corresponding rings, thus increasing the validity of the average value along them. The operator is applied to a set of data artificially constructed from a realistic model with the addition of a steep regional trend and a strong random noise. The results are compared to the equivalent ones from other second derivative operators applied also in the space domain.

1. Introduction

Following the description of Henderson and Zietz (1949), the value of the second vertical derivative at each point of gridded data is given by the equation :

$$\partial^2 \Delta T / \partial z^2 = \sum_{k=1}^{K} \cdot \mu_k \cdot A_k \qquad (1)$$

where, μ_k are the positive roots of the n degree Bessel function of the first order : $J_n(\mu_k \cdot \alpha) = 0$ (α is a large distance from the point at which the anomaly is assumed to effectively become zero). A are coefficiants calculated from the equation :

$$\overline{\Delta T}(r) = \sum_{k=1}^{k} A_k \cdot J_0 (\mu_k \cdot r) \qquad (2)$$

$\overline{\Delta T}(r)$ is the average value of the anomaly along a ring of radius r. The data are assumed to lie on a plane at z = 0. In practice $\overline{\Delta T}(r)$ is calculated as the mean of the anomaly values at the grid points which lie on each ring. For K different radii (including the radius r = 0) equation (2) results in a system of K simultaneous equations of K unknowns, namely the coefficiants A_k. The values of A_k are calculated as :

$$A_k = (1/D) \cdot \sum_{k=1}^{K} (-1)^{i+k} \cdot \overline{\Delta T}(r_i) \cdot M_{ik} \qquad (3)$$

where D is the value of the determinant $|J_0(\mu_k \cdot r_i)|$, k,i=1,...,K and M_{ik} is the value of the minor determinant, which comes from the above one by the elimination of the row i and column k. Substituting equation (3) in (1), one gets the second derivative value at a grid point :

$$\partial^2 \Delta T / \partial z^2 = \sum_{i=1}^{k} W_i \cdot \overline{\Delta T}(r_i) \qquad (4)$$

where,

$$W_i = (1/D) \sum_{k=1}^{K} (-1)^{i+k} \cdot \mu_k \cdot M_{ik} \qquad (5)$$

From equation (5) the weighting factors W_i for different radii configurations (the " vanishing" distance a can also be changed) are calculated.

2. Discussion

The second vertical derivative of potential fields has been used as a tool for pointing out disturbancies (anomalies) of the field for forty years. In the literature there are several versions of equation (4) proposed by different authors, which differ in the way they conform the data. Discussions of the different formulae have been given by Nettleton (1971), Fuller (1967) and Mesko (1966). The problems in the calculation of the second derivative through the above described method arise from the fact, that a continuous potential field is represented by discrete values at the grid points. The finer the grid becomes, the more accurate this representation is. The reduction of the grid spacing though, tends to emphasize the small details of the potential field , when a second derivative operator is applied to the data, since these operators use generally radii up to two grid intervals (the degree to which different field features are emphasized depends on the relation of their dimensions to the diameter of the more heavily weighted rings). The grid interval is therefore kept to a size commensurate to the expected (or sought) anomalies.

In the present work it was tried to increase the radius of the outer ring to about four grid units (the inner radii are also increased), and to study the consequences on an artificial set of data. The increase of the radii allowss more grid points to lie on the equivalent rings; thus their distribution becomes more even, and the operator is less influenced by directionality. The increase of the number of points along each ring is also expected to reduce the effect of random errors (noise) in the data. The radii of the rings (also the square of the radii), the corresponding weighting factors, and the total number of points along each of them are as follows :

	$r_1=0$	$r_2=2.24$	$r_3=3.06$	$r_4=4.18$
W_i	1.50	-2.55	1.18	-0.13
N_i	1	8	12	12
r_i	0	5	(9+10)/2	(17+18)/2

The "vanishing" distance α was also increased to twenty grid units from ten assumed in Henderson and Zietz (1949).

The artificial set of data constructed simulates the magnetic effect of a buried ancient road's corner. The road is 1m thick and 3m wide. The outer sides of the corner are 8 and 10 meters long. The road is buried under 1m of soil, and has a susceptibility contrast of -0.0025 (SI system) with the surrounding soil. Inclination and declination of the magnetic vector were assumed to be 54 and 2 degrees respectively, in accordance with the present situation in Greece. The model's plan view and its magnetic effect are shown in fig.1a. The produced anomaly has minimum and maximum values of -12 and +6 nT respectively. To the calculated model values were added : (a) a second degree" regional" change of 5 nT/m, ranging from 100 to 300 nT, and (b) a noise ranging from -10 to +10 nT. Such data, if original, would represent a difficult not very probable case. The resulting magnetic map is seen in fig.1b.

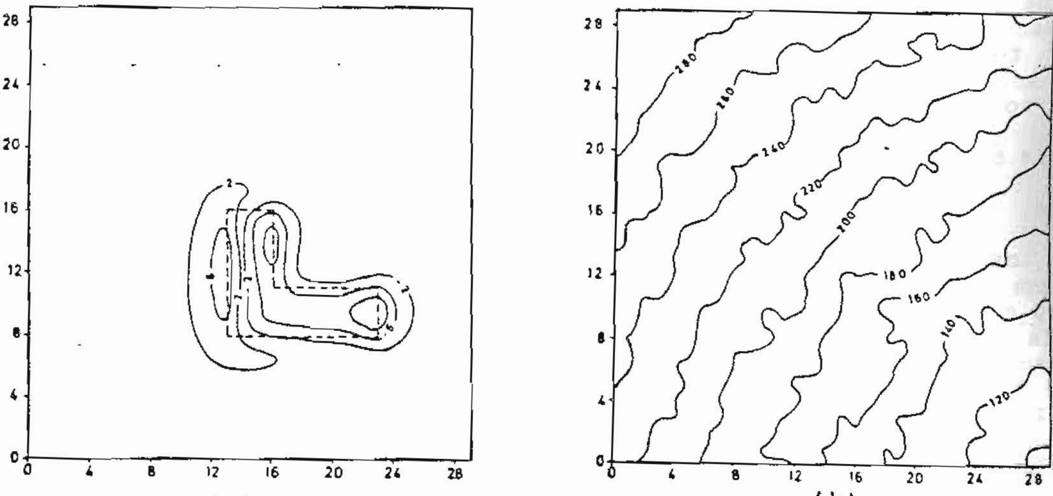

Fig. 1. (a) Plan view of a vertical sided prism simulating a buried road corner and its magnetic effect.
(b) The magnetic anomaly map after the addition of regional trend and random noise.

Four different second derivative space domain operators were applied to the constructed data. These are: (a) the Henderson and Zietz (1949) equation 13 operator, (b) the Rosenbach (1953) equation 16 operator, (c) the Elkins (1951) equation 15 operator, and (d) the Peters (1949) equation 27 operator. In order to examine the degree to which the noise affects the operators, they were also applied to a smoothed version of the data produced by a five point running average filter. The resulting second derivative maps are shown in fig. 2 to 5.

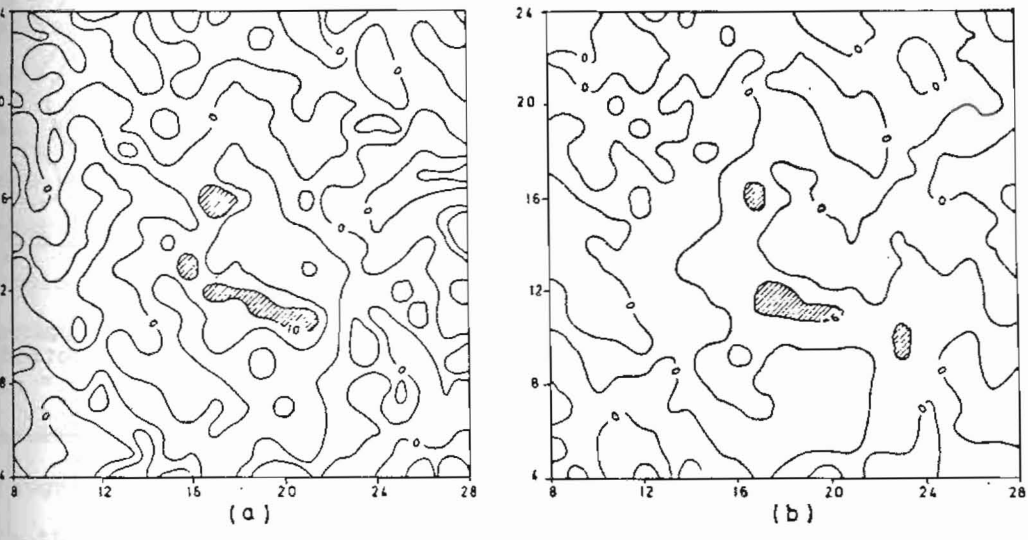

Fig. 2. Second derivative map produced by the Henderson operator applied: (a) on the "original" set of data, (b) on the smoothed set of data.

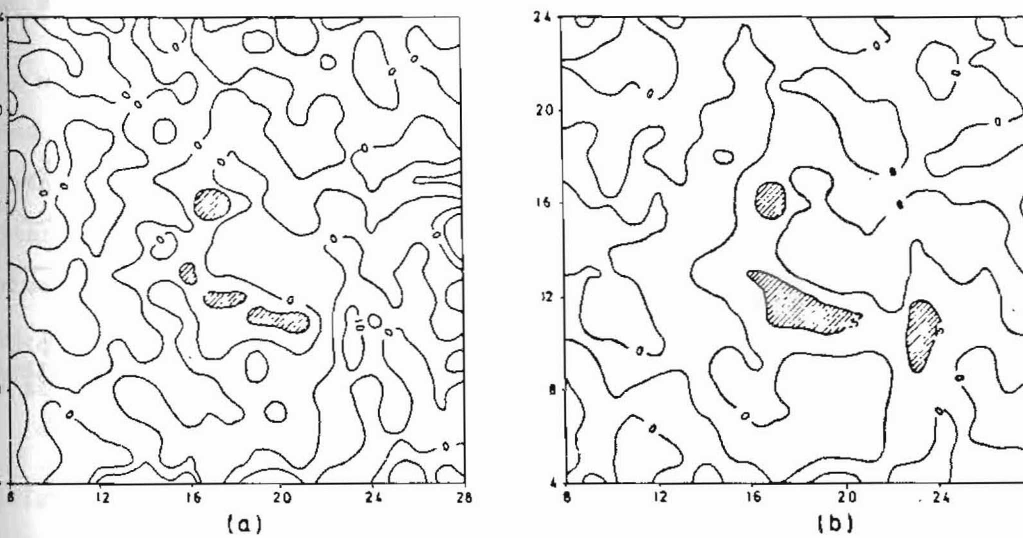

Fig. 3. Second derivative map produced by the Rosenbach operator applied: (a) on the "original" set of data, (b) on the smoothed set of data.

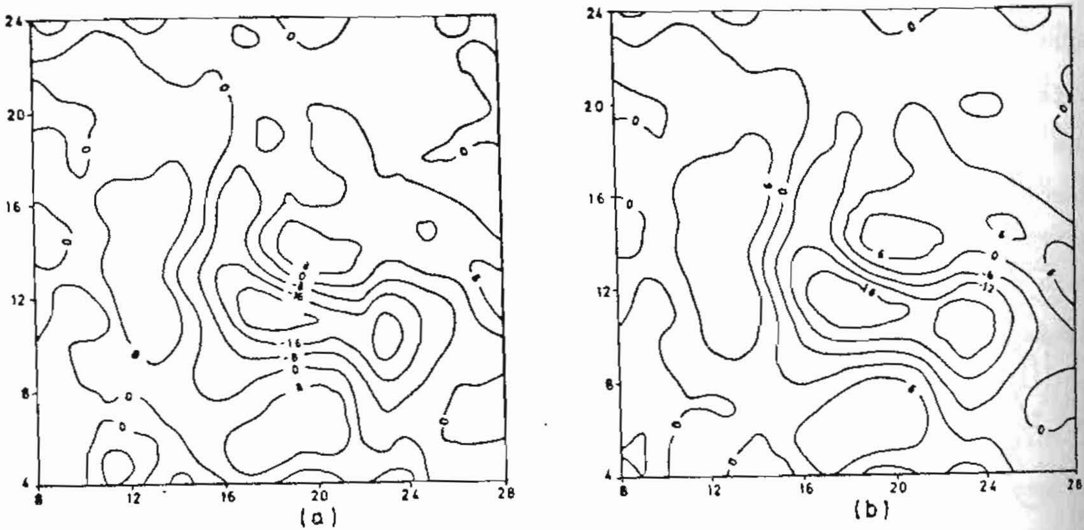

Fig. 4. Second derivative map produced by the Elkins operator applied: (a) on the "original" set of data, (b) on the smoothed set of data.

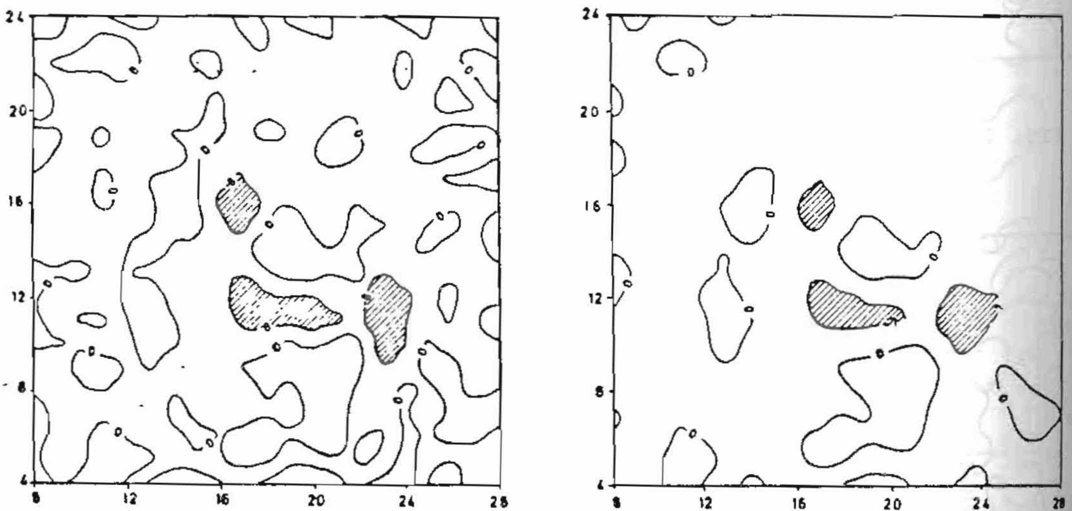

Fig. 5. Second derivative map produced by the Peters operator applied: (a) on the "original" set of data, (b) on the smoothed set of data.

The Henderson and Zietz, and Rosenbach operators gave similar results. This was expected since their frequency responce is similar and close to the theoretical one (Fuller 1967, Mesko 1966). The maps produced from the "original" data are rich in short wavelengths. The anomaly produced by the road's corner is indicated on the maps by the stippled lows. These lows form a right angle, which shows a clockwise rotation of 20-30 degrees in relation to the anomaly produced by the model (fig. 1a). The results from the smoothed data are obviously better. The maps are more clear and the anomaly's orientation is close to the real one, although the angle looks wider than 90 degrees. The Elkins and the Peters operators act as band pass filters, (Fuller 1967). In the case of the Elkins operator, the right angle on the anomaly of the model is indicated by the turn of the -8 units (-6 for the smoothed version) contour line. This turn is about 90 degrees, but again its angle orientation angle shows a clockwise rotation of about 20 degrees. The " original" and smoothed versions of the data produce very similar results by using the Elkins and Peters operators. The latter are thought to be the best of the four known operators examined up to now. The three stippled lows form an angle quite close to 90 degrees and its orientation is better than the previous ones, although it is evident a slight rotation again.

The way, in which the new introduced operator transformed the data, is seen in figures 6a and 6b. The three lows form a right angle and the orientation is almost ideal. The output from the smoothed data is very clear and almost the only feature on it is the "organized" (negative values surrounded by positive ones) second derivative anomaly, indicating the presence of a real anomaly in the data. The frequency response of the new operator is shown in figure 7.

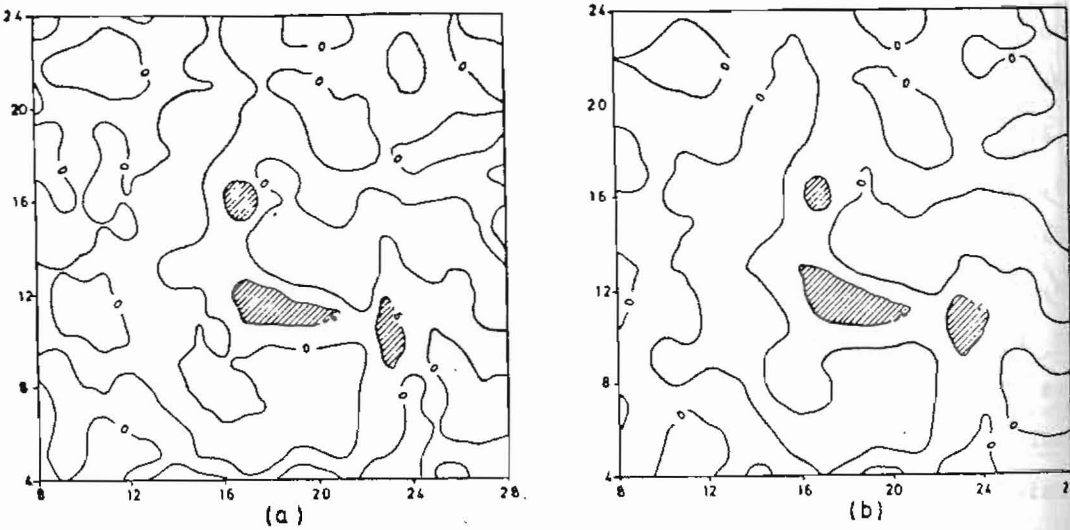

Fig. 6. Second derivative map produced by the new derived operator applied: (a) on the "original" set of data, (b) on the smoothed set of data.

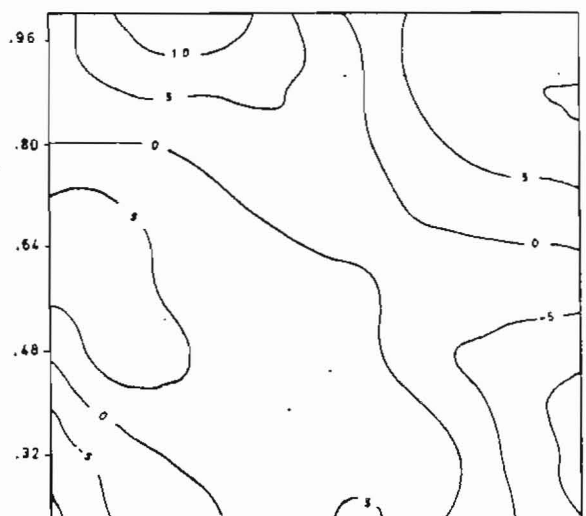

Fig. 7. Frequency response of the new second derivative operator.

It is believed, that the performance of the new operator on the artificially constructed set of data is encouraging, and that it can be succesfully employed in the search of archaeological sites, where orientation is important, but in other cases as well.

R E F E R E M C E S

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