

GLOBAL STUDY OF THE DISTRIBUTION OF EARTHQUAKES IN SPACE AND IN TIME BY THE FRACTAL METHOD

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A B S T R A C T

Two fractal techniques are used to study the spatial and temporal distributions of shallow earthquakes in sixteen seismic zones all over the world. For each zone, we determine the spatial and temporal fractal dimensions, D_s and D_t , which characterize the strength of event clustering: the smaller the dimension, the stronger the clustering. We use two global homogeneous and complete catalogues with the following time spans and magnitude cutoffs: 1930-1985 ($M_s \geq 6.5$) and 1966-1985 ($M_s \geq 5.5$). We interpret the regional variations of D_s in terms of an asperity model and inspect for possible correlations between D_s and D_t . We found: 1) a considerable temporal clustering of events in both catalogues; 2) a significant correlation ($r=0.65$) between the values of D_s determined using the $M_s \geq 5.5$ catalogue, and those of D_t for the first catalogue. Our results imply that the probability of a large event is highest immediately after a previous large event and decays with time. This pattern is generally more pronounced in zones with stronger temporal and spatial clustering of events.

ΜΕΛΕΤΗ ΤΗΣ ΧΩΡΙΚΗΣ ΚΑΙ ΧΡΟΝΙΚΗΣ ΚΑΤΑΝΟΜΗΣ ΤΩΝ ΣΕΙΣΜΩΝ
ΣΕ ΣΕΙΣΜΟΓΕΝΕΙΣ ΠΕΡΙΟΧΕΣ ΤΗΣ ΓΗΣ ΜΕ ΤΗ ΜΕΘΟΔΟ
ΤΗΣ ΚΛΑΣΜΑΤΙΚΗΣ ΑΝΑΛΥΣΗΣ.

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Π Ε Ρ Ι Λ Η Ψ Η

Εφαρμόζονται δύο τεχνικές κλασματικής ανάλυσης για τη μελέτη επιφανειακών σεισμών σε 16 σειсмоγενείς ζώνες. Για κάθε ζώνη υπολογίζονται η χωρική και η χρονική κλασματική διάσταση, D_s και D_t , που χαρακτηρίζουν το μέγεθος της συγκέντρωσης των σεισμών. Χρησιμοποιούνται δύο παγκόσμιοι κατάλογοι με μεγέθη $M_s \geq 6.5$ και $M_s \geq 5.5$, αντίστοιχα. Οι χωρικές μεταβολές της D_s ερμηνεύονται με βάση το "μοντέλο εμποδίου" και ελέγχεται η πιθανή ύπαρξη συσχέτισης ανάμεσα στη D_s και στη D_t . Βρέθηκε: 1) αξιοσημείωτη χρονική συσχέτιση των σεισμών και στους δύο καταλόγους, 2) σημαντική συσχέτιση ($r=0.65$) ανάμεσα στις τιμές του D_s , που υπολογίστηκαν με βάση τον κατάλογο με μεγέθη $M_s \geq 5.5$ και εκείνες του D_t που βρέθηκαν με βάση τον κατάλογο με μεγέθη $M_s \geq 6.5$. Τα

αποτελέσματα δείχνουν ότι η πιθανότητα ενός μεγάλου σεισμού είναι μέγιστη αμέσως μετά από προηγούμενο μεγάλο σεισμό και ελαττώνεται με το χρόνο. Αυτή η συμπεριφορά είναι περισσότερο εμφανής στις ζώνες με ισχυρότερη χρονική και χωρική συγκέντρωση των σεισμών.

INTRODUCTION

Clustering in space and in time seems to be a fundamental feature of seismicity regardless of the scale of observation. The Earth's seismicity map shows that the spatial distribution of earthquakes is strongly nonuniform on all scales of observation: patches or clusters of comparatively high seismic activity alternate with zones of comparatively low activity. At the same time it is well known that on a short-term time scale seismicity is strongly clustered (foreshock-aftershock sequences). Moreover, a number of recent statistical studies imply that even the long-term evolution of seismicity, particularly shallow, is characterized by clustering (e.g. see Kagan and Jackson, 1991, and references therein). Scale-invariant properties such as earthquake clustering in space and in time can be conveniently studied by means of the so-called fractal approach.

The concept of fractals was introduced and developed by Mandelbrot (1983, and references therein). In a very general sense, fractal (or scale-invariant, or self-similar) is called an object, process or phenomenon that preserves its characteristics regardless of the scale of observation, i.e., if a power-law dependence is found between at least one characteristic and scale (the definition of scale invariance).

In geosciences one very often encounters the situation when the number of objects, N , with size larger than r is related to r by

$$N \sim r^{-D} \quad (1)$$

For the range of scales of r where the above relation holds with D a constant it is said to define a fractal distribution with fractal dimension D . The frequency-size distribution of islands, craters, earthquakes, rock and mineral fragments, ore deposits and oil fields often satisfy relation (1) (Turcotte, 1989, and references therein).

Aki (1981) was the first to introduce the fractal approach in seismology. He showed that, under certain assumptions, the Gutenberg-Richter empirical law, $\log N = -bM + a$, can be put in the form of the fractal distribution (1) with $r=A^{1/2}$, where A is the fault-break area and $D \approx b$ is the fractal dimension of the fault-length distribution.

Since Aki's (1981) pioneering work, the fractal approach has been successfully applied to a wide spectrum of earthquake-related problems, from fracture mechanics to tectonics (e.g., see Main, 1988; Turcotte, 1986; Kagan and Jackson, 1991; Hirata, 1989).

Seismicity studies can be divided into two categories: those dealing with the distribution of seismicity in space and those focusing on its evolution in time. Because scale invariance means absence of a characteristic scale, the fractal method is readily

applicable to the study of earthquake occurrence in time (indeed, only a few cases are known where major events occur at more or less regular time intervals.) Kagan and Jackson (1991) used declustered catalogues to show that fractal clustering and not Poissonian or periodic behaviour is the major feature of shallow earthquakes on both the long-term and short-term time scales. On the other hand, fractal distributions of earthquakes in space have been observed on various scales (e.g., Kagan and Knopoff, 1980; Hirata, 1989; Dimitriu and Papadimitriou, 1990; DeRubeis et al., 1993). Two quantities are therefore needed to describe the distribution of earthquakes in space and in time: the spatial and temporal fractal dimensions, D_s and D_t . The two quantities provide a measure of the strength of event clustering: the smaller the fractal dimension, the stronger the clustering.

In our present study we evaluate D_s and D_t for sixteen of the Earth's most active seismic zones, using two complete and homogeneous global catalogues of shallow earthquakes. We try to establish whether the obtained values reflect regional seismotectonic features and check for a possible correlation between the two quantities.

EARTHQUAKE CATALOGUES AND SEISMIC ZONATION

For a seismic catalogue to be suited for fractal studies it must be homogeneous, complete and as accurate as possible. Tsapanos et al. (1988) used some of the best available catalogues and ISC Bulletin data to compile a homogeneous global catalogue for the period 1897-1985. The following are the time spans and the magnitude cutoffs for which the catalogue was found to be complete: 1898-1985 $M_s \geq 7.0$; 1930-1985 $M_s \geq 6.5$; 1952-1985 $M_s \geq 6.0$; 1966-1985 $M_s \geq 5.5$. We found the 1930-1985 and 1966-1985 catalogues to suit our purposes best.

Gutenberg and Richter (1954) were the first to regionalize global seismicity using seismotectonic, geologic and seismicity criteria. Duda (1965) added two more criteria, namely that a continuous spatial distribution of shocks defines a seismic region (e.g., the circum-Pacific and Eurasian belts) and that a change in the trend of epicentre distribution marks a region's limit. Tsapanos (1985) relied on the above criteria and the available seismicity data to draw a refined map of seismic zones.

Here we use the seismic zonation of Tsapanos (1985), with only two modifications, suggested by the work of Tajima and Kanamori (1985) (see Fig.1). First, we shifted the northern border of zone 1 (Chile) to include the 1974 Peru earthquake, which produced an aftershock expansion pattern typical of the large Chilean earthquakes rather than the Peruvian ones. Second, we divided zone 17 (Papua-Solomon islands -New Hebrides islands) into three smaller zones, to take account of the observed differences in aftershock expansion patterns.

FRACTAL TECHNIQUES

Several fractal techniques are known that permit to quantify the distribution of seismicity in space and in time. The methods

adopted in this study are formal and universal (independent of arbitrary regionalization of seismicity and any a priori model of earthquake interaction), robust (capable of treating relatively small amounts of data), simple and reliable.

For studying the spatial distribution of earthquakes we use the fractal technique based on the correlation integral (Grassberger and Procaccia, 1983):

$$C(r) = 2N(R < r) / N(N-1) \quad (2)$$

where $N(R < r)$ is the number of pairs of events separated by a distance less than r and N is the total number of events. If the distribution is fractal, then

$$C(r) \sim r^{D_s} \quad (3)$$

where D_s is the (spatial) fractal dimension (or, more strictly, the correlation dimension; Grassberger, 1983). D_s can be evaluated from the slope of the best-fitting straight line in the log-log plot of $C(r)$ versus r : the smaller its value, the stronger the spatial clustering of events.

Geometrically, the values of D_s are limited to the range from 0 (point) to 3 (volume). If epicentres rather than hypocentres are used (a 2-D distribution), then the highest limit is 2 (an area completely filled with epicentres).

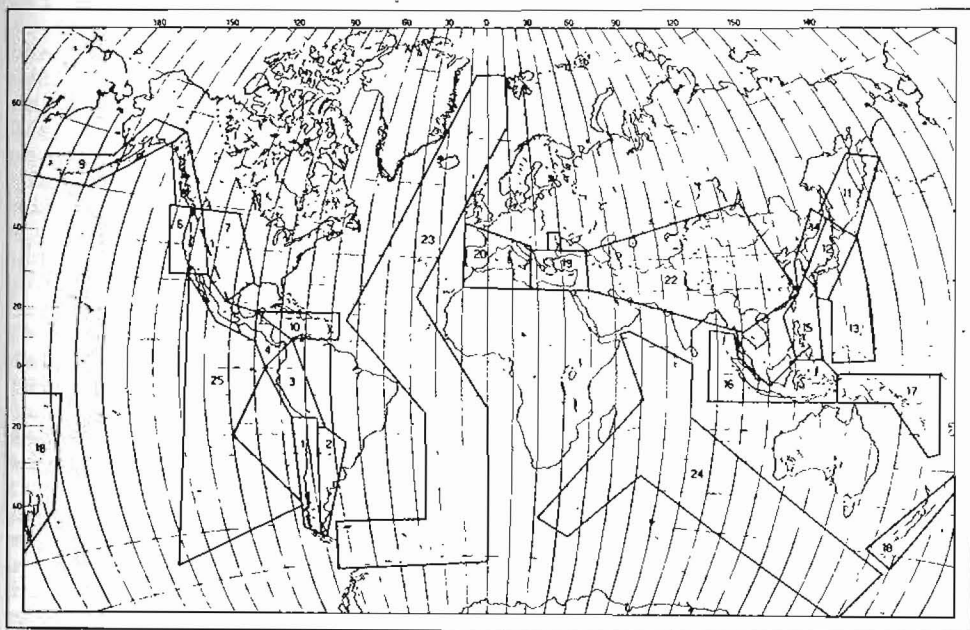


Fig.1. Zonation of global seismicity (after Tsapanos, 1985).

A crucial issue in determining the fractal dimension is the optimum number of events required. According to Smith (1988), the number of events should not be less than 42^M , where M is the

integer part of the fractal dimension. Here, since we use epicentres rather than hypocentres, we expect D_s to be less than 2. In the catalogue we use to evaluate D_s , the $M_s \geq 5.5$ catalogue, zone 4 (central America) has the fewest events, 39.

For studying the temporal distribution of events we adopt the method elaborated by Kagan and Jackson (1991). The method uses a quantity they termed 'time-distance statistic moment':

$$m(t) = N(T < t, r) / n_p \quad (4)$$

here $N(T < t, r)$ is the number of pairs of events separated in time by less than t and in space by less than r (fixed) and $n_p = (N_T / T_c) [t(2T_c - t)]$ is the expected number of event pairs if the events followed a Poissonian distribution (N_T is the total number of pairs of events separated in space by less than r and T_c is the catalogue span). A temporal distribution of events is fractal if

$$m(t) \sim t^{D_t - 1} \quad (5)$$

In (5), D_t is the (temporal) fractal dimension. It can be evaluated from the slope of the best-fitting straight line in the log-log plot of $m(t)$ versus t . $D_t = 1$ corresponds to a Poissonian distribution, and $D_t = 0$ to all events concentrated in one point in time. For intermediate values of D_t the process is scale invariant.

The effect of the (fixed) distance interval r in (4) on the value of D_t is essential. Thus, if a small r is chosen, only a few events will be located close to the reference event, and these events most likely will occur near the time of the reference event. $N(T < t, r)$ will be constant for large time intervals, and $m(t)$ will decay with time as $1/t$ for small intervals, meaning a D_t close to 0. On the other hand, for very large r , the events will be independent and distributed uniformly in time. This situation corresponds to a Poissonian distribution ($D_t = 1$).

RESULTS

Here we present the results of applying the above fractal methods to the shallow seismicity of the sixteen major seismic zones shown in Figure 1. In computing of the spatial and temporal fractal dimensions, D_s and D_t , we used earthquake epicentres rather than hypocentres, because the depth determination is incomplete and unreliable.

As discussed previously, to evaluate D_s and D_t of an earthquake distribution it is essential to use an appropriate interevent distance range. A fundamental requirement is that the earthquake spatial distribution should satisfy relation (3) in this distance range. At the same time, this range should be neither too small (too few points) nor too large (not larger than the size of the smallest zone considered). We found 30-200 km to be the optimum distance range for our purposes.

In Figures 2 and 3 we show examples of evaluations of D_s and D_t for two representative seismic zones, namely Alaska and the Aleutians (9) and Japan (12) (see Fig. 1).

In Figures 2a, 3a we show the $\log(C(r))$ versus $\log(r)$ plots used to evaluate D_s . The interevent distance range used was 32-200Km. Note that we took the shortest distance interval to be slightly over 30 km, which is the epicentre-location error. Notice the linearity of the plots, signifying the scale-invariance of the event spatial distributions.

Figures 2b, 3b show examples of the $\log(m(t))$ versus $\log(t)$ plots used to evaluate D_t with the $M_s \geq 5.5$ catalogue, whereas Figures 2c, 3c represent the $M_s \geq 6.5$ catalogue. We set the maximum interevent distance to be 200Km. The gaps in the plots correspond to time-interval increments without event pairs. Noteworthy is the 'bi-linear' behaviour in Fig. 2b, suggesting a bi-fractal temporal distribution of earthquakes in zone 9.

The summary of the values of D_s and D_t for the sixteen seismic zones considered is presented in Table 1. For evaluating D_s we used the $M_s \geq 5.5$ catalogue. For calculating D_t we used a time-interval range from 2 to 552 days for the 1966-1985 catalogue and from 22 to 1744 days for the 1930-1985 catalogue.

The correlation analysis performed for the D_s data set on the one hand and the two D_t data sets on the other shows weak correlation ($r=0.36$) between D_s and D_t for the $M_s \geq 5.5$ catalogue and significant correlation ($r=0.65$) between D_s and D_t for the $M_s \geq 6.5$ catalogue (see Fig. 4).

DISCUSSION AND CONCLUSIONS

The low values of D_t obtained for both catalogues used (see Table 1) imply considerable earthquake clustering (interaction) in time and seem to preclude quasi-periodic or Poissonian behaviour ($D_t=1$) in the zones studied. Kagan and Jackson (1991), who used five catalogues (four global and one local), found, for shallow seismicity (depth up to 70 km) and a maximum interevent distance of 204.8 km, similar values of D_t . As expected, our $M_s \geq 5.5$ catalogue yields somewhat lower values of D_t , meaning stronger clustering, than the $M_s \geq 6.5$ one. Exceptions are zones 9, 17a and 22, where the larger events show stronger interaction, but such behaviour is not unknown (Kagan and Jackson, 1991). On the other hand, apparently 'anomalous' D_t values may be caused by multi-fractal behaviour of seismicity, as exemplified by zone 9 (Fig. 2b).

Thus, even though our catalogues contain foreshocks and aftershocks, the fact that our $M_s \geq 6.5$ catalogue gives D_s values well below 1 supports Kagan and Jackson's (1991) conclusion that the probability of a large event is highest immediately after a previous large event and decays with time. This pattern is expected to be more pronounced in zones with low D_t than in zones with high D_t . The implication for long-term earthquake prediction is that a region's seismicity is governed by alternating phases of increased-decreased mantle deformation rate and, as a consequence, the longer the time since the last major event, the higher the probability that the region is entering a quieter seismic period (Kagan and Jackson, 1991).

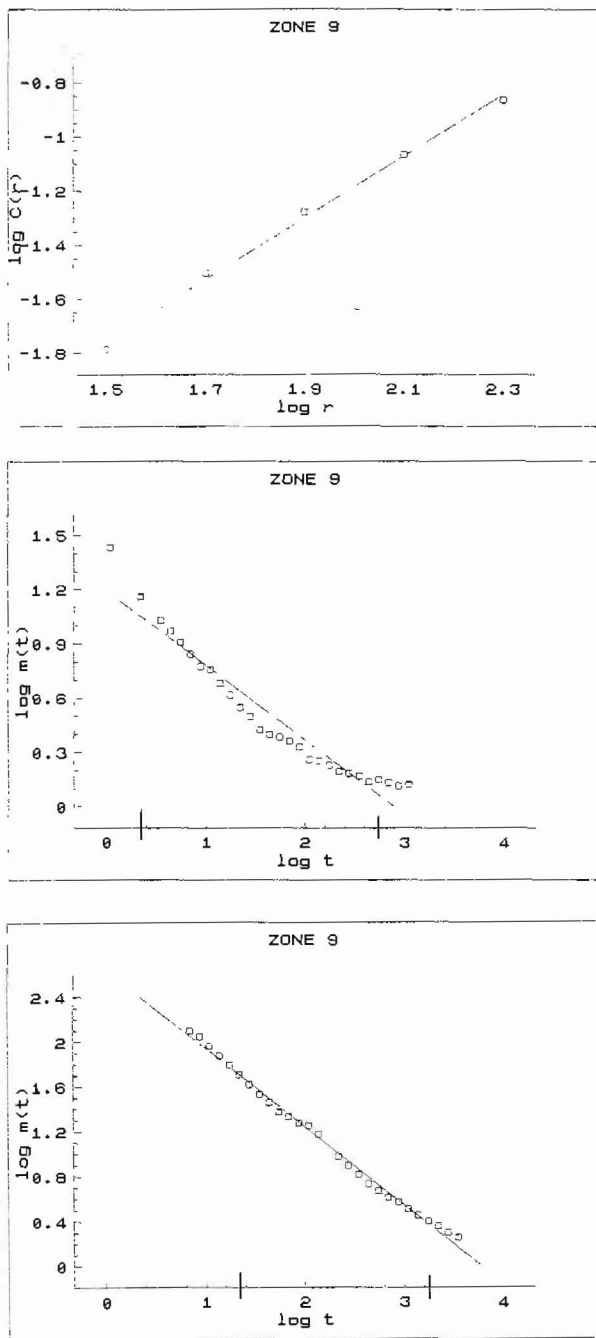


Fig.2. Plots used to evaluate D_s and D_t for the Alaska - Aleutian islands seismic zone (zone 9, see Fig. 1).

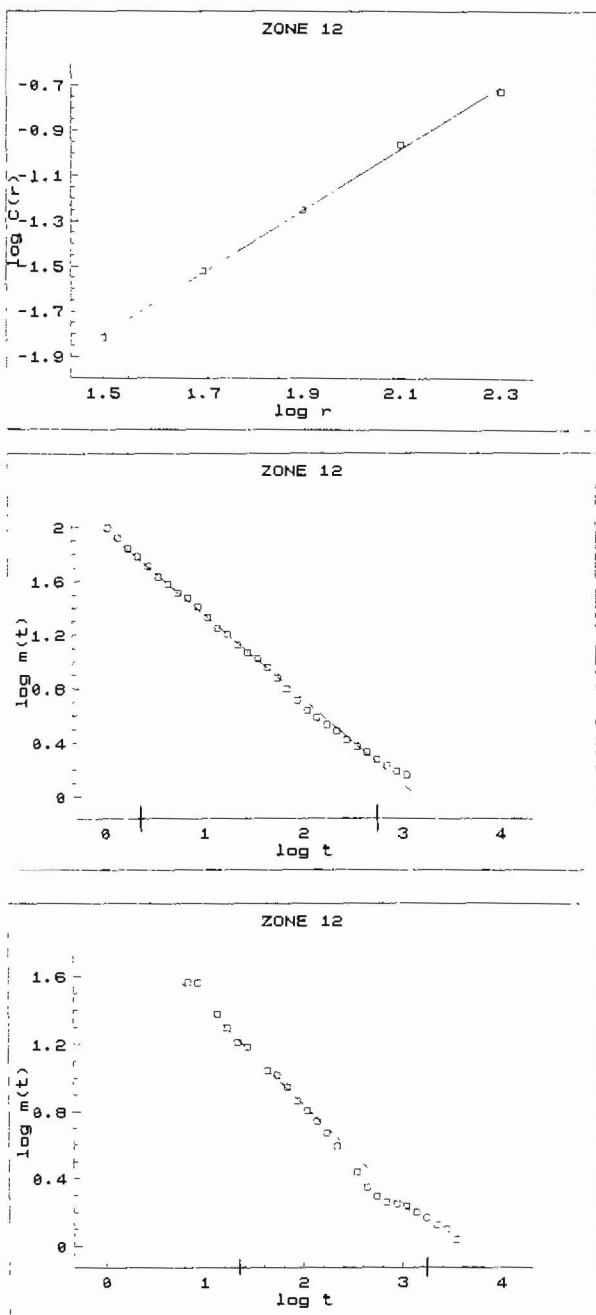


Fig.3. Same as Fig. 2 for the Japan seismic zone (zone 12, see Fig.1).

Table 1. The values of D_s and D_t for the seismic zones considered.

Zone	D_s (std err) $M_s \geq 5.5$ cat.	D_t (std err) $M_s \geq 6.5$ cat.	D_t (std err) $M_s \geq 5.5$ cat.
22	0.75 (0.02)	0.21 (0.01)	0.27 (0.01)
3	0.86 (0.04)	0.23 (0.04)	0.17 (0.01)
1	1.01 (0.09)	0.23 (0.01)	0.24 (0.01)
16	1.09 (0.03)	0.42 (0.04)	0.34 (0.01)
17b	1.13 (0.05)	0.35 (0.04)	0.19 (0.01)
21	1.14 (0.08)	0.37 (0.02)	0.34 (0.01)
9	1.14 (0.04)	0.31 (0.01)	0.57 (0.02)
15	1.14 (0.02)	0.52 (0.03)	0.37 (0.01)
23	1.17 (0.02)	0.49 (0.05)	0.37 (0.04)
18	1.23 (0.06)	0.50 (0.03)	0.51 (0.02)
4	1.25 (0.04)	0.47 (0.04)	0.36 (0.03)
17a	1.30 (0.08)	0.31 (0.02)	0.43 (0.01)
11	1.31 (0.09)	0.55 (0.02)	0.25 (0.01)
17c	1.36 (0.04)	0.44 (0.02)	0.30 (0.01)
12	1.37 (0.03)	0.40 (0.02)	0.36 (0.01)
5	1.59 (0.10)	0.46 (0.03)	0.37 (0.03)

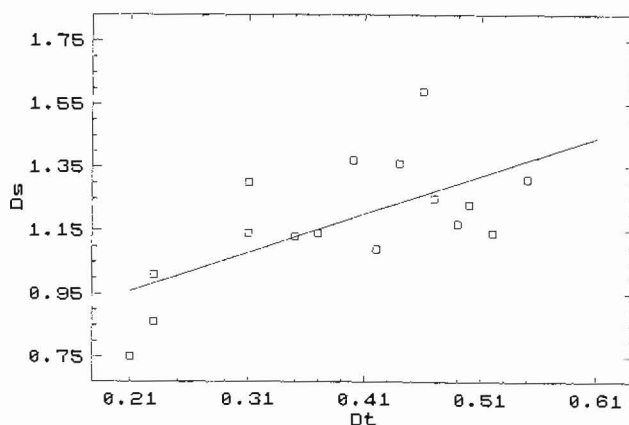


Fig.4. Correlation between D_s (1966-1985 catalogue) and D_t (1930-1985 catalogue) for the 16 shallow-earthquake seismic zones considered (see Fig. 1); correlation coefficient $r=0.65$.

Our D_s values characterize the areal distribution of shocks (we used epicentres) and may reflect regional differences in the stress field and mechanical properties of the rocks.

Tajima and Kanamori (1985) attempted to regionalize world seismicity by quantifying the areal expansion of aftershocks of large earthquakes. They explained the observed differences in

terms of an asperity model in which a fault plane is represented by a distribution of strong spots (asperities) surrounded by weak zones. We use the asperity model to interpret our results and compare them with those of Tajima and Kanamori (1985).

The lowest value of D_s , 0.75, is observed for Asia (zone 22) and reflects the remarkable clustering of seismic activity there. The highest value, 1.59, is found for Mexico (zone 5), indicating a sparse distribution of earthquakes in space. In terms of the asperity model, a low D_s could mean the predominance in a seismic region of sources composed of one or a few relatively large asperities separated by small weak zones, whereas a high D_s could signify the predominance of smaller sparsely distributed asperities. Geometrically, the cases $D_s < 1$, $D_s = 1$ and $D_s > 1$ represent clustered, aligned and sparse asperity distributions, respectively.

Our zonation by the spatial fractal dimension (Table 1) agrees quite well with Tajima and Kanamori's (1985) results, based on a study of aftershock-area expansion. Thus the major shocks in our zones with progressively higher values of D_s also exhibit progressively larger aftershock-area expansions. The only striking exception is zone 5 (Mexico), which has the highest D_s but limited areal expansion. An inspection of our catalogue helps to explain this discrepancy: the two major shocks studied by Tajima and Kanamori (1985) had only a few aftershocks with $M_s \geq 5.5$ and so had little influence on the evaluation of D_s .

A notable result of our study is the significant correlation ($r=0.65$) we found between the values of D_s determined using the $M_s \geq 5.5$ catalogue and the values of D_t computed using the $M_s \geq 6.5$ catalogue. The correlation can be improved if the standard errors of D_s and D_t are considered (in one of the trials we obtained $r=0.83$). This means that the earthquakes with $M_s \geq 6.5$ occurring in zones with a stronger spatial clustering of shocks generally show a stronger temporal interaction (clustering) than the earthquakes in zones with a comparatively weaker spatial clustering of shocks. The implication for earthquake prediction is that a large event is more likely to be followed by another large event if it occurs in a zone with spatially clustered seismicity (low D_s) than in a zone with a sparse distribution of shocks (high D_s).

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