INTERPRETATION OF VERTICAL ELECTRICAL SOUNDING DATA BY COMBINING DIRECT AND ITERATIVE METHODS

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ABSTRACT

Vertical electrical sounding data measured with the Schlumberger array are approximated by a linear combination of simple functions. The fitting operation is carried out by the least-squares method at any desired level by decreasing or increasing the number of fitting functions.The same operation permits to construct the equally sampled data.

The resistivity transform function is determined from the apparent resistivity curve by the same process. The parameters of the layered earth are computed by the direct interpretation method and they are used as input to the iterative software to find out final solution.

ΕΡΜΗΝΕΙΑ ΔΕΔΟΜΕΝΩΝ ΓΕΩΗΛΕΚΤΡΙΚΩΝ ΒΥΘΟΣΚΟΠΗΣΕΩΝ ΜΕ ΣΥΝΔΥΑΣΜΟ ΕΥΘΕΙΑΣ ΜΕΘΟΔΟΥ ΚΑΙ ΜΕΘΟΔΟΥ ΔΙΑΔΟΧΙΚΩΝ ΠΡΟΣΕΓΓΙΣΕΩΝ

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ΠΕΡΙΛΗΨΗ

Τα δεδομένα γεωηλεκτρικών βυθοσκοπήσεων με τη διάταξη Schlumberger προσεγγίζονται με ένα γραμμικό συνδυασμό απλών συναρτήσεων. Η διαδικασία ταύτισης σε οποιοδήποτε επιθυμητό όριο διεξάγεται με την μέθοδο των ελαχίστων τετραγώνων ελαττώνοντας ή αυξάνοντας τον αριθμό των συναρτήσεων. Η ίδια διαδικασία επιτρέπει την ισοκατανομή των δεδομένων. Η συνάρτηση του μετασχηματισμού των ειδικών ηλεκτρικών αντιστάσεων υπολογίζεται από την καμπύλη της φαινόμενης ειδικής αντίστασης με την ίδια διαδικασία. Οι παράμετροι της οριζόντια διαστρωμένης Γης υπολογίζονται με απ' ευθείας ερμηνεία και χρησιμοποιούνται ως είσοδος στο πρόγραμμα των διαδοχικών προσεγγίσεων ώστε να δοθεί η τελική λύση.

INTRODUCTION

In vertical electrical sounding method (VES), the interpretation philosophy is to find a model curve which provides a reasonable fit to the measured curve. It is assumed that the parameters of the fitted model curve represent the real subsurface layers. Nowadays, computers have been used as a standard tool for the fitting operation. The procedure is initialized by giving an initial guess supplied by the interpreter, then a model curve which fits the measured data within a given tolerance is searched. This method of interpretation is known "iterative interpretation", "indirect method" or "inversion". A large number of papers have been published since the pioneering work of Vozoff(1958).

The other interpretation technique, called "direct interpretation", uses the resistivity transform (RT) function to obtain layer parameters without an initial guess. The sample values of the measured apparent resistivity curve are converted into the resistivity transform function. The method has been proposed by Pekeris(1940) by assuming horizontally layered earth model and it has been mainly developed by Koefoed(1970, 1976).

The above mentioned two methods of interpretation have their own advantages and disadvantages. The aim of this paper is to discuss the merits and demerits of the methods and to combine the advantages of each method into one interpretation algorithm.

By improving the quality of data by the help of data processing techniques before starting the interpretation will make the interpretation fast and accurate. In the following, a scheme will be proposed for the processing of VES data. In this way, the whole interpretation procedure will be divided into three stages. Namely, these are i) data improvement, ii) determination of the initial model by the direct interpretation or from the other sources such as drilling, interpretation of neighbour soundings etc., iii) final adjustment of layer parameters by the iterative interpretation method in the apparent resistivity domain.

DATA PROCESSING

The sample values of Schlumberger apparent resistivity (AR)curve are subject to the smoothing, the extrapolating and the re-sampling processes. The smoothing is to estimate new sample values of AR curve which contain less noise than the original curve. The extrapolation is to construct new sample values whose abscissa values are beyond the abscissa range which is used for actual measurements. The re-sampling process permits to estimate equally spaced data along the horizontal logarithmic axis.

Hereafter, an AR curve which contains a few percent of noise will referred as "noisy data". A sample value which is evidently away from the regular shape of the curve will be called as " bad point". Figure 2 shows a noisy Schlumberger curve containing one bad point. If the shape of the curve is not clearly identified, then the data will be referred as " high noisy " .

The bad points can be corrected by the application of linear interpolation. The bad point correction does not reduce the noise, because the new sample value is calculated from the two neighbouring sample values of the noisy data by the use of linear interpolation. However, it prepares the data for the smoothing operation. It is not necessary to apply the bad point correction to the high noisy data.

It will be an advantage to carry out the smoothing, extrapolating and re-sampling operations by the same procedure. The numerical procedure proposed by Santini and

Zambrano(1981) is suitable for these purposes. This involves the approximation of AR data by linear combination of a simple fitting function. Firstly, Schlumberger AR data are converted to a function called y(s):

$$y(s)=(\rho_a(s)-p)/2p$$

where $\rho_a(s)$ stands for the Schlumberger AR and s represents the distance between each current electrode and the centre of a Schlumberger array. p is a constant which is empirically calculated from the first value of the AR curve and represents the resistivity of the first layer. This constant is used for numerical stability of the computer program.

The fitting function for the approximation of Schlumberger AR data is given by Santini and Zambrano(1981) as:

$$f(s;\epsilon_i)=\frac{s^3}{(\epsilon_i^2+s^2)^{3/2}} \qquad (1)$$

where ε are coefficients which define the position of the fitting functions, they are fixed depending on the abscissa range of the electrode spacings and the number of fitting functions.In the case of a perfectly insulating substratum, the Schlumberger AR curve will approach an asymptote with a slope of 45 degrees at large abscissa values. The fitting function which shows the same behaviour is given by Santini & Zambrano(1981) as follows:

$$f(s;\epsilon_i)=(s/\epsilon_i)\,[1-\frac{\epsilon_i}{(\epsilon_i^2+s^2)^{1/2}}] \qquad (2)$$

According to the type of the AR curve, the linear combination of one of the above fitting functions can be used for the decomposition of the y(s) function:

$$y(s_j)-\sum_{i=1}^{m} b_i f(s_j;\epsilon_i)=minimum \qquad j=1,2,3,\ldots,n$$

where n and m are the number of the sample values of AR curve and the number of the fitting functions, respectively and $y(s_j)$ is the j th sample value. The unknown b coefficients can be determined by the least-squares technique using a system of linear equations. The approximated values of the y(s) function can be constructed by the help of known b coefficients at any desired abscissa point:

$$y^*(s_j)=\sum_{i=1}^{m} b_i f(s_j;\epsilon_i) \qquad j=1,2,3,\ldots,n$$

Finally, the sample values of $y^*(s)$ are converted to the AR values. It should be noted that the described procedure is equivalent to the low-pass filtering in the linear filter theory. If the number of the fitting functions is decreased then a smoother curve than the previous one can be obtained. If the interpreter wants to obtain a curve closer to the sample values then the number of fitting functions is increased. The least-squares technique has the advantage that the interpreter can freely decrease or increase the number of the fitting functions by an interactive way in order to find the best smoothed curve among many possible solutions. The other advantage is that the algorithm does not need the equally spaced data and that the output can be estimated for any desired abscissa values.

After the decision of the number of fitting functions by an interactive way, extrapolation and re-sampling are performed by the use of the same construction procedure. Extrapolation becomes necessary when the left or right branches of the AR curve have a few sample values (see Fig. 2). The use of the abscissa values beyond the measurement range permits the extrapolation of the AR curve. In the same way, re-sampling is carried out by the calculation of the approximated AR values at abscissa points which construct equally spaced data along the logarithmic horizontal axis. This process is required because most of the inversion programs operate on equally spaced data. By the help of the re-sampling algorithm,the data may be measured at freely selected electrode spacings which have less sampling rate than that of the inversion program.

DIRECT INTERPRETATION

The well-known expression of the Schlumberger AR for the earth model which consists of homogeneous and isotropic layer is:

$$\rho_a(s)=\rho_1[1+2s^2\int_0^\infty K(\lambda)J_1(\lambda s)\lambda d\lambda]$$

where $J_1(\lambda s)$ the Bessel function of the first kind and first order, λ the integration variable, $K(\lambda)$ the Stefanescu kernel function. ρ_1 is the resistivity of the first layer. The AR data are dependent on the layer parameters by the above integral equation, while the relation between the kernel function and the layer parameters is an algebraic one. The aim of the direct interpretation technique is to solve the layer parameters from the sample values of the resistivity transform function which is obtained from the kernel function by the following relation:

$$T(u)=\rho_1(1+2K(u))$$

where u equals $1/\lambda$. Knowing the fitting function for the Schlumberger array and by applying the inversion formulae of Hankel transforms, the fitting function for the kernel can be derived (Santini and Zambrano 1981):

$$g(u;\varepsilon_i)=\exp(\varepsilon_i/u)\ .$$

The kernel function can be constructed for any desired abscissa values by using the known values of b and ε

coefficients.

$$K(u) = \sum_{i=1}^{m} b_i g(u; \varepsilon_i)$$

The determination of layer parameters from the sample values of RT curve are carried out step by step beginning from the top layer down to the last layer. The procedure starts by dividing the RT curve into branches. The resistivity and the thickness values of one layer are solved for one branch and the resistivity of the last layer is calculated on the last branch. The first, the last and the minimum and maximum points are marked by the program. The other branches which indicates additional layers on an ascending or a descending branch must be identified by the interpreter. In that way, the interpreter makes a decision about the number of layers. The resistivity of the first layer can be calculated from three successive discrete values of RT function by assuming the first branch does not contain any information about deeper layers (Basokur 1984):

$$\rho_1^2 = \frac{T^2(u_{j+1})\,[2T(u_j)\,T(u_{j+2})/T(u_{j+1}) - T(u_j) - T(u_{j+2})]}{T(u_j) - 2T(u_{j+1}) + T(u_{j+2})}$$

when

$$1/u_j - 1/u_{j+1} = 1/u_{j+1} - 1/u_{j+2}$$

and where j represents the key numbers of sample values. u_j and u_{j+2} are the abscissa values which are the same as the electrode spacings used during the field survey or at equally spaced abscissa values with ten points per decade when the measured AR curve has less sample values than this rate. The sample value of $T(u_{j+1})$ is computed at the abscissa point u_{j+1} which satisfies the above condition between three successive abscissa values. By shifting the calculation to the new successive discrete values of the RT function, many resistivity values depending on the number of data points on the branch are obtained. The resistivity of the first layer is determined by the arithmetic mean of the closest values selected from among all calculated values.

Once the resistivity is determined then the thickness of the first layer is calculated by a similar operation using the expression given by Basokur(1984):

$$t_1 = \ln\left[\frac{[(\rho_1 + T(u_j)]\,[\rho_1 - T(u_{j+1})]}{[\rho_1 - T(u_j)]\,[\rho_1 + T(u_{j+1})]}\right] / [1/u_j - 1/u_{j+1}]$$

Many thickness values can be computed from the each successive sample pairs of the RT curve. The arithmetic mean of the selected values gives the final estimation about the thickness of the first layer.

After determining the parameters of the first layer, the reduction equation is used to remove the contribution of the first layer on the RT curve (Koefoed 1970):

$$T_{i+1}(u) = \frac{T_i(u) - \rho_i \tanh(t_i/u)}{1 - T_i(u)\tanh(t_i/u)/\rho_i}$$

The parameters of the second layer are computed on the second branch of the RT curve and the repetition of the procedure on successive branches of the RT curve gives all layer parameters. The resistivity of the last layer is obtained directly from the sample values of the reduced RT function on the last branch, because the reduction equation becomes equal to the resistivity of the last layer at the final step of the direct interpretation.The interpreter decides about the number of layers and at which sample values the branches start and end. The above described procedure gives an equivalent solution without any initial guess. The interpreter has the option of finding the new layer parameters by modifying the number of layers.

Due to error magnification, the reduction equation yields inaccurate results in the abscissa range where abscissa values u are less than the thickness of i th layer at the i th step of reduction. The other direct interpretation methods calculate the resistivity of i+1 th layer at this abscissa range using equations which are identical to the reduction equation as proved by Basokur(1984). The proposed algorithm calculates the resistivity and the thickness of i th layer at the i th step of the reduction. The sample values of the reduced transform curve are computed beginning from the first sample value of one branch up to the last sample point of the RT curve at each reduction step. In that way, the accuracy of the later part of the reduced RT curve remains the same as the measured RT curve. There is another way to check the calculated resistivity values. The sample values of the reduced transform curve must be greater than the resistivity of a layer for an ascending branch and the sample values must be less than the resistivity of a layer for a descending branch. If the resistivity of a layer does not satisfy the above condition or the algorithm produce an unrealistic value for the resistivity or the thickness then the original values of the RT curve can be used instead of the reduced RT curve. That happens because for large values of abscissa ($u>t_i$) where the layer parameters are calculated, $\tanh(t_i/u)$ becomes small and the reduced transform curve becomes approximately equal to the measured transform curve as seen from Fig.1. When the reduced curve is contaminated with high noise, the use of the measured RT curve produces almost correct values in most cases or creates very good approximation to the true values of layer parameters. According to my experience on hundreds field curves, the model AR resistivity curves calculated using the layer parameters solved by direct interpretation method fit to the measured AR curve in a few percent tolerances.

ITERATIVE INTERPRETATION

Among the methods offered for the adjustment of the layer parameters to minimise the differences between measured and model data, Levenberg-Marquardt approach has been pointed out as the most powerful and versatile method by many authors such as Inman(1975), Johansen(1977) and Hoversten et al.(1982). A discussion of the method to solve geophysical problems has been provided by Lines and Treitel (1984).

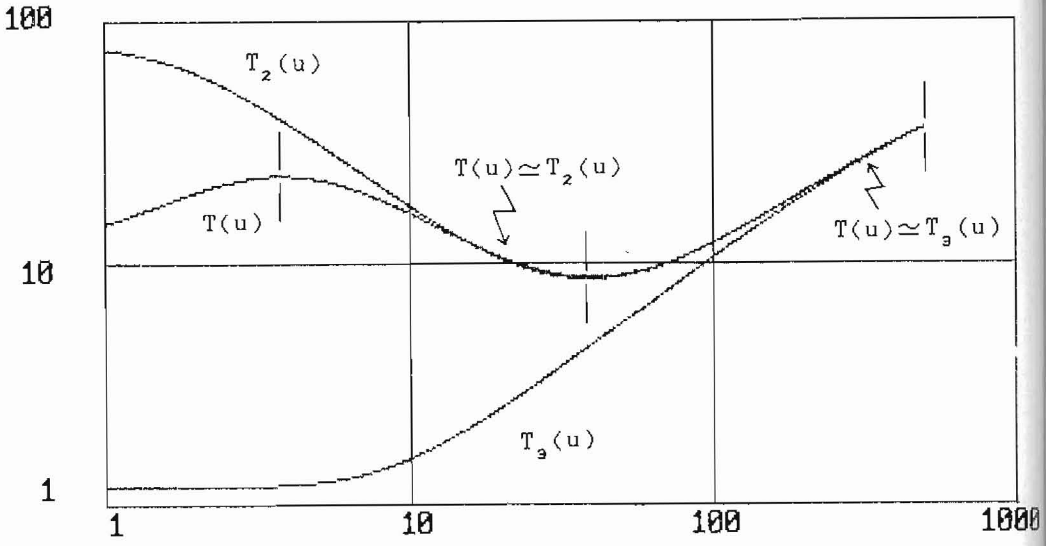


Fig.1. The resistivity transform function T(u) and the reduced transform curves $T_2(u)$ and T_3 (u). The branches are separated by vertical bars. Resistivities are 12, 80, 1.2, and 90 ohm-m. Thicknesses are 1, 2 and 10 meters.

The calculated function is expanded in a Taylor series about an initial guess in the parameter space and it is equated to the measured data. Neglecting the second and higher order terms and taking the differences of the model and measured data, a matrix equation is obtained. A new set of layer parameters which minimise the differences are computed by solving the matrix equation. The iterative adjustment of the parameters is terminated when the differences become less than a predetermined value or any improvement is not obtained by trying new parameters. The matrix equation is solved by the Singular Value Decomposition (SVD) algorithm. Arnason and Hersir(1988) have published a computer program to perform the inversion of the Schlumberger data.

The iterative interpretation method not only solves for the parameters but also provides a statistical information about the parameters.The method shows the dependence between the parameters and it indicates which parameters are determined and with what accuracy. The only disadvantage of method is that it requires an initial guess close to the true solution, because the difference between model and measured curve may have many local minimum for some combinations of model parameters. The algorithm may produce a solution around these local minimums. Starting the inversion with a good initial guess reduce the number of iterations, save the time and makes the algorithm successful. Conversely, a poor initial guess may lead the whole process astray (Arnason and Hersir 1988). If the iteration is started with a good initial guess created by the direct interpretation method, it is possible to reach the solution after a few iterations.

NUMERICAL EXAMPLE

The direct interpretation method and the computer program proposed by Basokur(1984,1990) have been improved according to the idea presented here, that is, by giving a good initial guess even for the high noisy data case. The interpreter is free to make changes the calculated initial guess. For example, a thin layer which is not resolved by the direct method can be added or the other changes can be done according to the geological model and prior knowledge. The fast and well designed computer program of Arnason and Hersir (1988) has been used for the iteration.

A numerical example will be presented to show the applied strategy to interpret the Schlumberger data. The measured AR data are plotted as small plus signs in Fig.2. It is justified that the bad point correction should be applied on the 14 th data point. The corrected value by the linear interpolation is indicated by a small circle. The data have been smoothed by using 8 fitting functions.

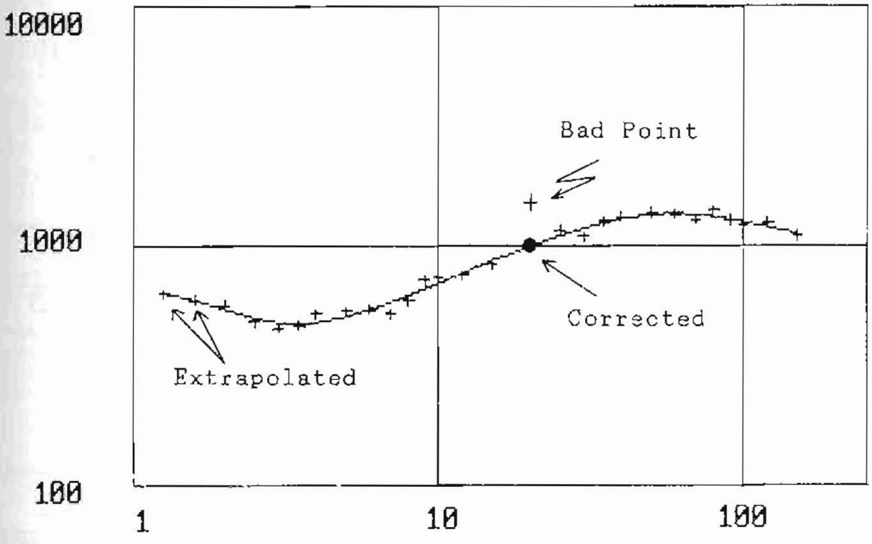
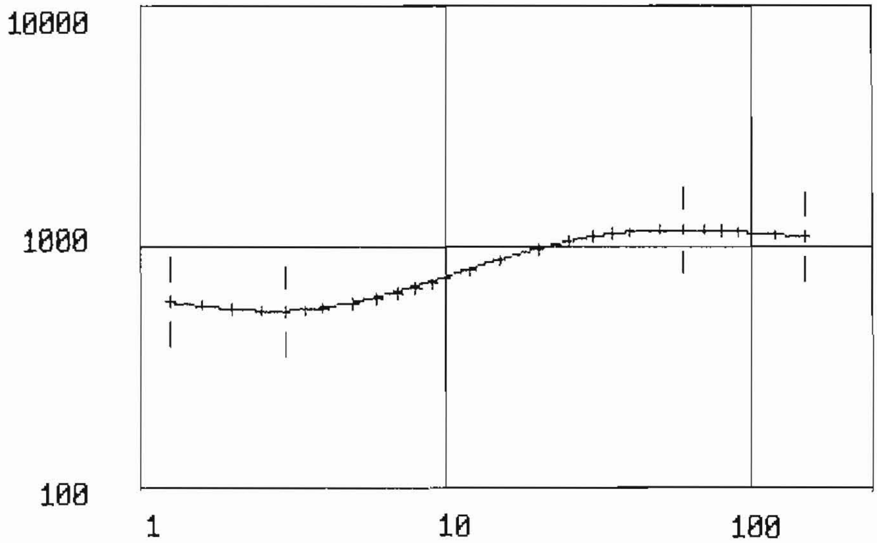


Fig.2. The measured (plus signs) and the smoothed data (continuous curve). The data have been extrapolated two sample values to the left.



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Resis> 653.20   403.86  1685.05   940.61
Thick>   0.95     3.29    34.39
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Fig.3. The measured and model RT curves and layer parameters solved by direct interpretation algorithm without initial guess.

After that, the data have been extrapolated by two sample values to the left since the first branch of the AR curve is too short. The next step is to make decision about the initial guess. This can be done by the use of the direct interpretation. The branches are separated by the program and they are indicated by vertical bars. Figure 3 shows the computed layer parameters. The theoretical RT curve is drawn as unbroken line and the measured RT curve is indicated by the small plus signs.The fit is reasonable in the RT domain.

Accepting the solved layer parameters as an initial guess, the iterative interpretation algorithm creates a new set of layer parameters which produce practically sufficient fit after four iterations as shown in Fig.4. A good fit can be obtained in the RT domain, but the differences between smoothed and model curves are higher in the AR domain, because the AR resistivity curve is more sensitive to the layer parameters than the RT curve.

CONCLUSIONS

The interpretation of VES data is performed by three steps, namely by the data improvement, the decision about an initial guess and the final adjustment of layer parameters. The presented algorithm covers all these sessions. The least-squares technique is used to improve the data quality and for the extrapolation and re-sampling. The interpreter can adjust the level of smoothing by decreasing or increasing the number of fitting functions. An initial guess is supplied by the direct interpretation method. The interpreter is free to modify the initial guess solved by the program.The final adjustment of the layer parameters is made by an iterative method which uses Levenberg-Marquardt optimization technique. The sessions can be repeated and linked several times to try several models by an interactive way.

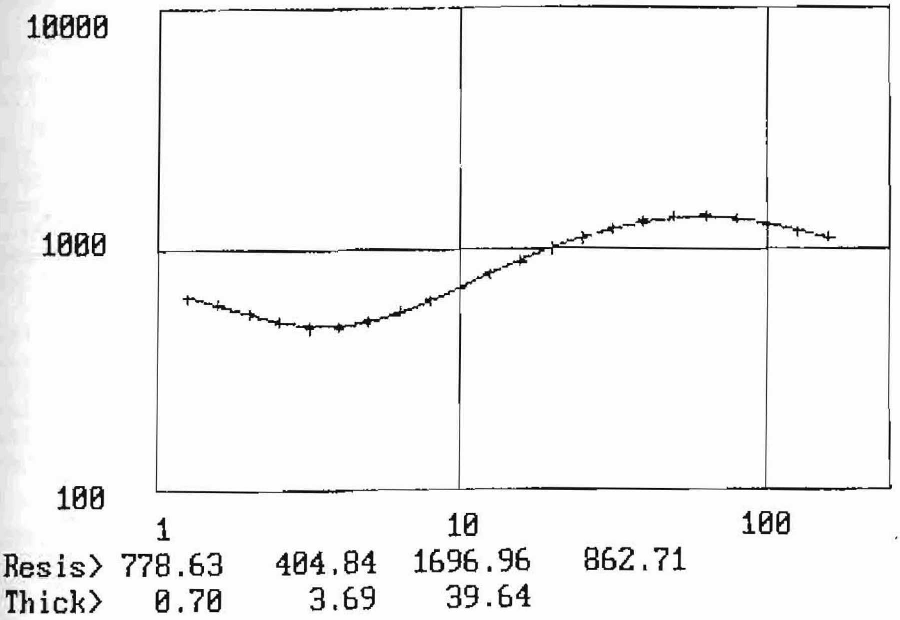

Fig.4. The measured and model AR data. The initial guess created by direct interpretation is improved by the iterative interpretation.

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