A 2-D FORWARD MODEL OF GROUND PROBING RADAR: THE TRANSMISSION LINE MATRIX METHOD

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A B S T R A C T

The use of ground probing radar (GPR) is growing rapidly and especially for shallow depth exploration problems. In order to facilitate data interpretation and explore the applicability of the method to a given environment the GPR's forward problem is addressed within this paper. The Transmission Line Matrix (TLM) method is used to solve the 2-D problem of a ground probing radar's response to a target located in an homogeneous and lossy earth. The basic principles of the TLM method is presented and the modelling procedure is discussed. Modelled time responses of shallow depth targets are calculated and their synthetic radargramms are illustrated. Furthermore, the effects of the earth's conductivity to the GPR response are also examined.

ΑΡΙΘΜΗΤΙΚΗ ΛΥΣΗ ΤΟΥ ΔΙΣΔΙΑΣΤΑΤΟΥ ΕΥΘΕΩΣ ΠΡΟΒΛΗΜΑΤΟΣ ΤΟΥ ΥΠΕΔΑΦΙΟΥ ΡΑΝΤΑΡ: Η ΜΕΘΟΔΟΣ ΤΟΥ ΠΙΝΑΚΑ ΓΡΑΜΜΩΝ ΜΕΤΑΦΟΡΑΣ

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Π Ε Ρ Ι Λ Η Ψ Η

Η χρησιμοποίηση του υπεδάφιου ραντάρ στη γεωφυσική διάσκοπηση στόχων μικρού βάθους αυξάνεται διαρκώς. Στα πλαίσια αυτής της εργασίας η μέθοδος του Πίνακα Γραμμών Μεταφοράς χρησιμοποιήθηκε για τη λύση του ευθέως προβλήματος της απόκρισης του υπεδάφιου ραντάρ σε στόχο μέσα σε ομογενή και απορροφητική γη. Αναπτύσσονται οι βασικές αρχές της μεθόδου καθώς και η διαδικασία της αριθμητικής προσομοίωσης του υπεδάφιου ραντάρ. Παρουσιάζονται συνθετικές αποκρίσεις στόχων σε μικρό βάθος, ενώ παράλληλα εξετάζονται και οι επιδράσεις της αγωγιμότητας της γης στο λαμβανόμενο σήμα.

INTRODUCTION

The use of GPR is growing rapidly especially for shallow depth exploration problems. Its inherent high resolution and non-destructive nature are GPR's main advantages over other geophysical methods. The diversity of the areas in which GPR has been employed ranges from the ice thickness probing at the poles

(Bishop et al., 1980) to the investigation for buried antiquities (Tealby et al., 1992). However, GPR is most commonly employed for the location of buried utility plans for which it has proved to be very effective (Annan et al., 1984). A review of the method, its applications and limitations can be found in an IEE special issue on GPR (Daniels et al., 1988).

However, the GPR data form, in most cases, a complicated image of the subsurface, thereby making the extraction of quantitative or even qualitative information from it difficult. Interpretation of a GPR section is usually based on the "operator's expertise" and could be totally misleading when the targets sought are of complex geometry and composition.

In order to aid interpretation as well as facilitate important decisions about the limitations of GPR in a particular environment, the forward problem of the GPR is addressed within this paper. More explicitly, the solution of the GPR's response to an arbitrary target located in a homogeneous and lossy earth is calculated.

Although, the GPR's antenna illuminates a volume the subsurface image produced is a 2-D one. A proper 3-D analysis is currently being carried out at York. In this paper, the results are presented from the initial 2-D approach to the problem.

The transmission line matrix (TLM) method has been employed for the calculation of the synthetic GPR response. TLM is a time domain method which is being extensively used in the solution of electromagnetic field problems in electronic and electrical engineering. The method is similar to the FD-TD method which have been widely used in geophysics and equivalent schemes of the two methods yield almost identical results (Hoefer, 1992). The former is more a "physical" model in antithesis to the more "numerical" nature of the later. In the following we discuss the basic principles of the TLM method and its application to the GPR forward problem.

THE 2-D GPR PROBLEM

For the simulation of the GPR's response to a target buried in a homogeneous and lossy earth in two dimensions the following assumptions are made:

A. the geometry of the problem is invariant in the target's strike direction, which results in an infinite long target in that direction. Therefore, only the cross-sectional area of the target is considered.

B. the sources are also infinitely long and parallel to the target's strike direction.

The problem's geometry as well as the model used for the simulations are shown in fig. 1.

With the above assumptions the electromagnetic fields are considered to be varying only in a plane containing the target. Lets denote that plane as the x-y plane and consequently assume the invariance of the problem's geometry in the z direction. Furthermore, it follows from the second assumption that the electromagnetic pulse which the transmitting antenna emits is polarized in the z direction. Consequently, H_z is zero and the electromagnetic fields are transverse magnetic (TM).

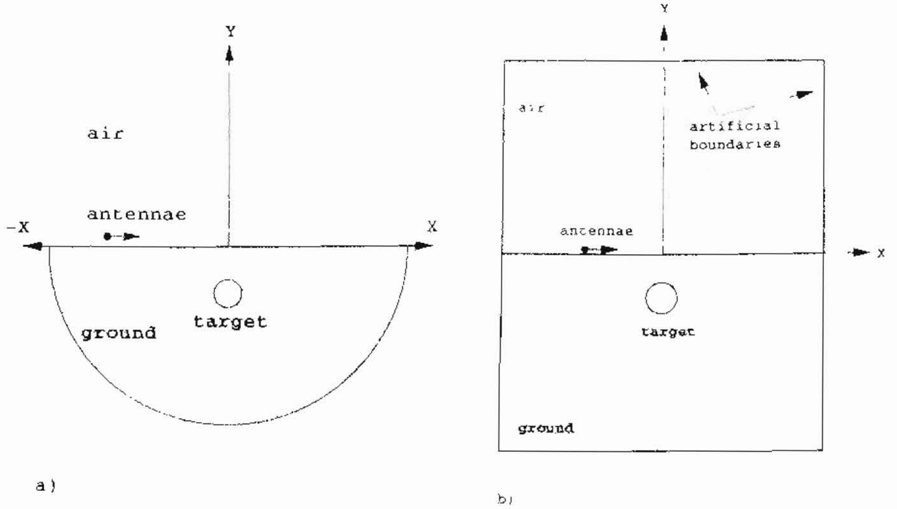

Fig.1. a) The geometry of the 2-D GPR's problem., b) The GPR model.

Maxwell's equations in Cartesian coordinates for the TM-mode of propagation in a source free medium reduce to the following:

$$\frac{\partial Ez}{\partial y} = -\mu \frac{\partial Hx}{\partial t} \tag{1}$$

$$\frac{\partial Hy}{\partial x} - \frac{\partial Hx}{\partial y} = \varepsilon \frac{\partial Ez}{\partial t} + \sigma Ez \tag{3}$$

$$-\frac{\partial Ez}{\partial x} = -\mu \frac{\partial Hy}{\partial t} \tag{2}$$

where μ is the permeability, ε the permittivity and σ the conductivity of the medium.

Differentiating equations (1) and (2) with respect to y and x respectively, adding the resulting equations and combining with equation (3) in order to eliminate the magnetic field components gives:

$$\frac{\partial^2 Ez}{\partial x^2} + \frac{\partial^2 Ez}{\partial y^2} = \mu\varepsilon \frac{\partial^2 Ez}{\partial t^2} + \mu\sigma \frac{\partial Ez}{\partial t} \tag{4}$$

which is a wave equation for 2-D propagation in a homogenous and lossy medium.

THE 2-D TRANSMISSION LINE MATRIX METHOD

General

The starting point for the TLM method is the well known *Huygens principle* (Hoefer, 1991). In order to solve a wave propagation problem with Huygen's model a large number of scatterings, both in space and time, should be modeled. This kind of problem is best approached via digital simulation.

TLM as a modelling technique was introduced by P.B. Johns and R. L. Beurle for the solution of 2-D electromagnetic scattering problems (Johns and Beurle, 1971). They derived the technique from the same principles used by former researchers of analogue methods in which the use of electrical networks for the solution of electromagnetic field problems were employed. The relationship between the TLM method and Huygens model was shown in a later publication (Johns, 1974).

TLM exploits the relationship between Maxwell's field equations and circuit theory. *Space* is modelled as a mesh built up of two-wire transmission lines. Each node of the mesh is a junction of transmission lines and forms an impedance discontinuity in each line (Johns and Beurle, 1971).

At each node a scattering procedure takes place giving rise to pulses propagating in all directions of the mesh. *Time* is discretised and the time-step is the time which a pulse takes to travel from one node to another.

As a time domain approach to electromagnetic modelling problems the solution provided by TLM is the impulse response of the system being modelled. Is a straightforward procedure to obtain frequency or time domain information via a simple Fourier transform or a time convolution of the impulse response.

Exploiting the relation between field and circuit theory in TLM analysis, field quantities are simulated by circuit quantities. Therefore, E and H fields are simulated with voltages (V) and currents (I) and the medium parameters ε (permittivity), μ (permeability), σ (conductivity) with C (capacitance), L (inductance) and G (conductance). When a pulse is launched into a TLM node it will be scattered in all directions. At the next time-step these scattered pulses are incident at the neighbour nodes. With this scattering - connect procedure pulses are propagated in the mesh (Hoefer, 1985).

The scattering mechanism and the calculation of the desired field quantities are performed at the TLM's mesh nodes. A 2-D node is constructed from inductors and capacitors which are connected in such a manner as to model the 2-D wave equation. In order to build a 2-D TLM network a choice between two different node types has to be made:

A.The series node which models TE-modes of propagation,
B. The shunt node which models TM-modes of propagation.

In the following the shunt TLM node we will be described since it is the node which models the wave equation for the GPR's 2-D problem as was derived in the previous section.

Finally, it should be noted that because TLM is a time domain method, frequency domain concepts cannot be used, such as the frequency depended permittivity and the dispersive reflection coefficients. Although there are techniques to include frequency depended parameters in a TLM simulation, these are still in progress and are not used in this paper. Therefore, for the GPR model presented here, it is assumed that the earth's permittivity is frequency independent. However, this assumption, is generally valid, for the GPR's frequency range (Davis and Annan, 1989).

The TLM shunt node

The **TLM** shunt node is constructed from four inductors and one capacitor. It's circuit representation is illustrated in fig. 2. Let a shunt node represent a block of space with dimensions $\Delta l = \Delta x = \Delta y = \Delta z$. For propagation in the x direction applying the Kirchhof's voltage and current laws:

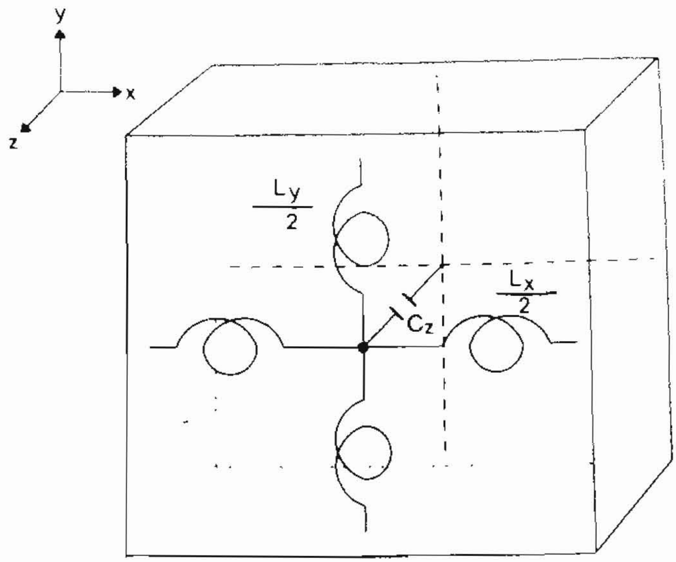

Fig.2. The 2-D TLM shunt node.

$$Lx\frac{\partial Ix}{\partial t} = -\frac{\partial Vz}{\partial x}\Delta x \quad (5)$$

$$Cz\frac{\partial Vz}{\partial t} = -\frac{\partial Ix}{\partial x}\Delta x \quad (6)$$

Differentiating equations (5) and (6) with respect to x and t respectively yields:
In the same manner for propagation in the y direction:

Combining equations (7) and (8) gives:

$$\frac{\partial^2 Vz}{\partial y^2}\frac{(\Delta y)^2}{Ly}=Cz\frac{\partial^2 Vz}{\partial t^2} \tag{7}$$

$$\frac{\partial^2 Vz}{\partial x^2}\frac{(\Delta x)^2}{Lx}+\frac{\partial^2 Vz}{\partial y^2}\frac{(\Delta y)^2}{Ly}=2Cz\frac{\partial^2 Vz}{\partial t^2} \tag{8}$$

In order to calculate the circuit parameters we proceed as follows. From fig. 2 it is evident that the capacitance (Cz) is

$$Cz=\varepsilon\frac{\Delta x\Delta y}{\Delta z} \tag{9}$$

The inductance *Lx* is associated with an x-directed current *I* producing a magnetic field intensity $Hy=I/\Delta y$ (Ampere's Law). The magnetic flux linked with this current is $\Phi=\mu Hy(\Delta x\Delta z)=\mu I\, \Delta x\, \Delta z\, /\Delta y$. Following the same rational for a y-directed current we have:

$$Lx=\mu\frac{\Delta x\Delta z}{\Delta y},\ Ly=\mu\frac{\Delta y\Delta z}{\Delta x} \tag{10}$$

Substituting the relations from equations (10) and (11) into (9) gives:

$$\frac{\partial^2}{\partial x^2}\left(\frac{Vz}{\Delta z}\right)+\frac{\partial^2}{\partial y^2}\left(\frac{Vz}{\Delta z}\right)=(2\varepsilon)\mu\frac{\partial^2}{\partial t^2}\left(\frac{Vz}{\Delta z}\right) \tag{11}$$

The above equation is isomorphic to the wave equation for 2-D TM modes of propagation developed previously for the GPR problem - except of the conductivity term - and establishes the following equivalence between the voltages and currents in the TLM network and the field quantities of equation (4):

$$-\frac{Vz}{\Delta z}\equiv Ez,\ \frac{Ix}{\Delta y}\equiv Hy,\ -\frac{Iy}{\Delta x}\equiv Hx \tag{12}$$

Therefore, a shunt node models a medium of parameters (2ε, μ) which should be taken into account when the capacitance and inductance of the TLM network are being calculated.

The absence of the conductivity term in equation (12) is due to the development of the node in order to model a loss free medium. It would not be difficult to develop a lossy TLM node and consequently a lossy TLM network. However, this is not the most effective way to take into account losses in a TLM model especially in the case where the problem's space is not homogeneous (Hoefer and So, 1991).

The modelling of inhomogeneites

The node described above could model the propagation of a TM-mode electromagnetic wave in a loss free medium. Nevertheless, The need to include regions of different parameters in the same model usually arises in real problems (i.e. the GPR one). Therefore, in order to model inhomogeneites and losses with the TLM method different parts of the model space must model electrical parameters of different media.

However, directly alternating the permittivity and/or permeability of the model's mesh lines will result in a variable velocity of propagation for the pulses in the TLM network. Consequently, that will result in a variable time step or discretisation length within the model. However, in TLM analysis both the time step and the elementary mesh size are constant in order to preserve synchronism and connectivity between the nodes and in particular between the ones at interfaces. As a result, the TLM network is chosen to model a given medium (ε,μ) which is called the *background medium* and usually is free space.

Therefore, modelling of inhomegeneites and losses with TLM is achieved with the introduction of capacitive, inductive and loss stubs at the appropriate nodes to model the extra permittivity, permeability and conductivity which are required. Synchronism is maintained since the round tripe time for the permittivity and permeability stubs is the same with the one of the transmission lines.

Consequently, a pulse scattered in one of these stubs will be incident in the same node at the next time step. On the other hand, a pulse scattered at a loss stub does not return to the node and in circuit terms this stub is described as an infinite matched line connected to the node.

The modelling of boundaries

Boundaries in electromagnetic-field modelling fall into two distinct categories: 1) *Boundaries inherent in the problem description* and 2) *Boundaries posed from the necessity for finite computational space and time.*

In the first case, boundaries could further be classified as non-dispersive and dispersive ones. Non-dispersive boundaries, such as electric and magnetic walls, have a real frequency independent impedance and can be modelled by a single reflection coefficient. Computation of the reflection of an impulse at a magnetic or electric wall is realised by returning it at the next time step with equal or opposite sign to the node of its origin. Therefore, a magnetic wall could be used to exploit a plane of symmetry in the problem. Dispersive boundaries could not be represented as the non-dispersive ones since a dispersive reflection coefficient would result in a distortion of the incident pulse which is not allowed in a TLM simulation.

Boundaries which are posed by the requirement for finite computational space and time are characterised as artificial, since there are not present in the description of the problem. The usual approach to this problem in the TLM method is the termination of the mesh transmission lines with the intrinsic impedance of the TLM network. With this approach, there are no reflections from these artificial boundaries but they perform very well only for normal incidence (Eswarappa et al.,1990).

2-D TLM GPR MODELS

The geometry of the 2-D GPR model is shown in fig. 1. The transmitting and receiving antennae are collocated 2.5 cm over the ground surface. The transmitting antenna emits an electromagnetic pulse into the ground in which the target is located and the transient electric field is measured at discrete steps along the x-direction.

The TLM mesh was constructed from 100 and 63 nodes in the y and x directions respectively and the discretisation length was 2.5 cm. Due to the symmetry of the targets in respect of the y direction only half the problem space was modelled employing one magnetic wall (reflection coefficient 1). The remaining three artificial boundaries introduced by the mesh truncation were absorbing walls (reflection coefficient 0). Furthermore, the ground response was calculated without the presence of the target and subsequently subtracted from the modelled GPR traces. Finally, the impulse response obtained for each trace was convolved with a Gaussian pulse of 1 ns width.

Two GPR models are presented here:

A. The first one is from a simulation of a 15x10 cm void buried at a depth of 25 cm. The relative permittivity and the conductivity of the soil were set at $\varepsilon=8$ and $\sigma=0.01$ mho/m respectively whilst earth is considered to be nonmagnetic. The total simulated time is 17 nanoseconds. With a time step of 0.589 picoseconds, 300 iterations were performed. Forty traces were calculated which form the synthetic radargamm presented in fig. 3.

B. The same modelling procedure was followed for the second one except that the target is now a perfectly conducting metal pipe of radius R=15 cm (staircase approximation) and the relative permittivity of the earth was set at $\varepsilon=4$. The earth's conductivity is the same as for the first model. The synthetic radargramm for this model is presented in fig. 4.

Furthermore, the transient electric response directly over the metal pipe was calculated for different values of conductivity whilst the other parameters of the model were constant (fig.5). The effects of the increase in the earth's conductivity are firstly, the broadening of the response and secondly, a severe reduction in the amplitude as was expected. The broadening of the pulse is ascribed to the higher attenuation of its high frequency components, since attenuation increases linearly with frequency for the GPR frequency range of operation (Vainikainen et al., 1992; Davis and Annan, 1989).

Moreover, since in both models the targets are buried at the same depth, as a result of the decrease in the relative permittivity in the second one a decrease in the apparent depth of the target can be observed.

Finally, real data from a GPR survey over a known void at the York city walls, York, U.K., are presented in fig. 6. The similarities with the synthetic radargramm obtained for a void (fig. 3) are apparent.

The simulated data are presented in two formats: a) as a grey scale image using 64 shades of grey and b) in wiggle traces. Real data are presented in grey scale format only, using the same number of shades as with the simulated ones.

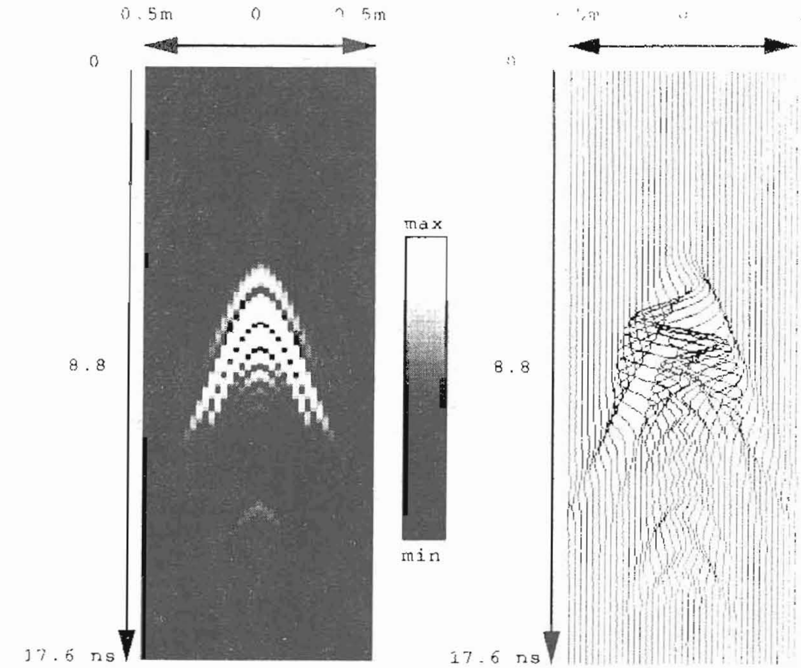

Fig.3. Synthetic radargramm from a void presented as a grey scale image and in wiggle traces.

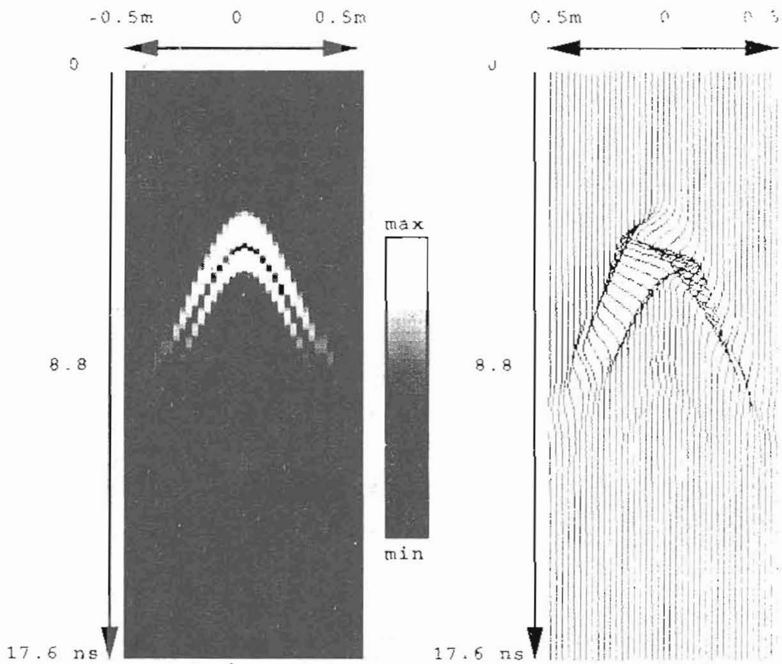

Fig.4. Synthetic radargramm from a metal pipe presented as a grey scale image and in wiggle traces.

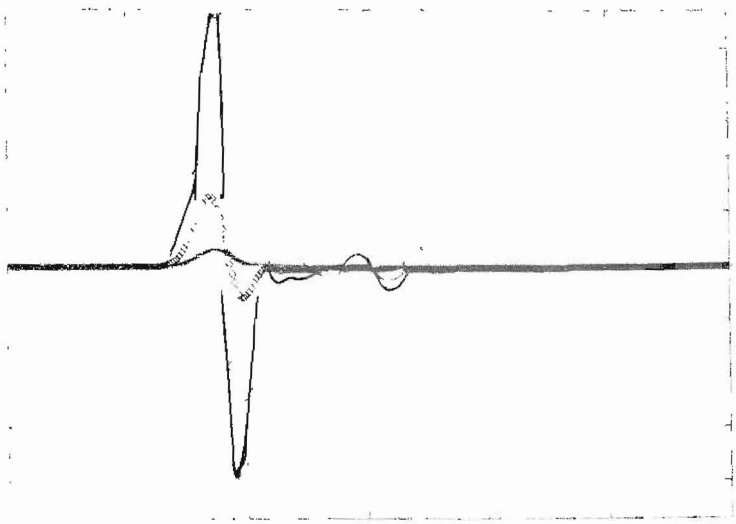

Fig.5. Simulated response over a metal pipe for different values of conductivity.

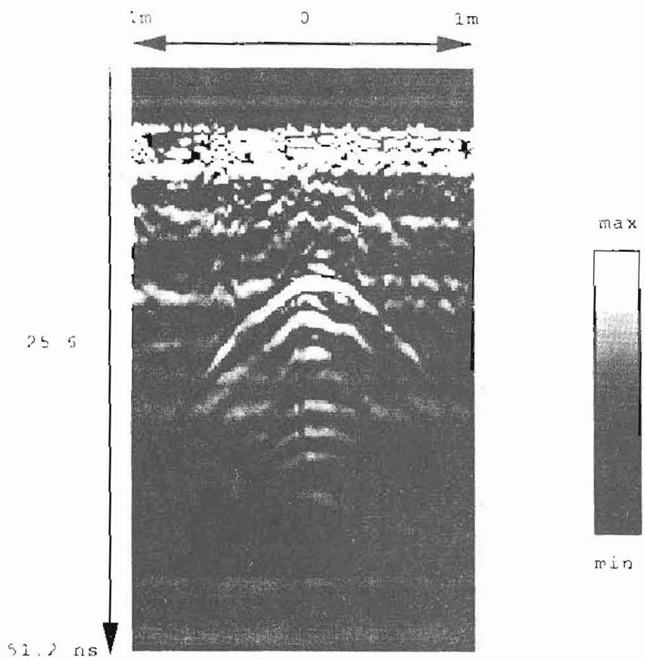

Fig.6. GPR response over a known void recorded at York city walls, using an ERA Technology GPR system.

For the above simulations the Hoefer and So *Electromagnetic wave simulator* (Hoefer and So, 1991) was used and the computational time for one synthetic GPR trace was 113 seconds in a 386 PC with a math co-processor.

CONCLUSIONS

The transmission line matrix method was used for the solution of the 2-D forward problem of the GPR's response to a target buried in an homogeneous and lossy earth. The synthetic radargramms calculated with TLM are in good agreement with the responses observed in real data for the same targets. The significance of the earth's conductivity which sets the penetration limits of GPR in a given environment was examined. Further work on the customisation of the presented method for the GPR case will increase its computational efficiency and - as far as the assumptions made for the construction of the model are valid - it could also serve as a basis for an inversion scheme of GPR data.

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