SOME THEORETICAL EXPERIMENTS FROM THE SOLUTION OF THE INVERSE MAGNETIC PROBLEM WITH FINITE DIPOLES

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A B S T R A C T

On the base of some preliminary experiments, and a thorough review of the respective publications, it is suggested, that for investigation of local magnetic anomalies, the model of finite dipoles is more suitable, as because of the normal distance between the charges with different signs in this case, it usually has better approximating possibilities. Besides, such model is not used yet in the geophysical science and practice. That is why, a suitable computer program was worked out and some numerical experiments were made with it. The obtained results confirm the preliminary assumptions.

ΘΕΩΡΗΤΙΚΑ ΠΕΙΡΑΜΑΤΑ ΑΠΟ ΤΗ ΛΥΣΗ ΤΟΥ ΑΝΤΙΣΤΡΟΦΟΥ ΜΑΓΝΗΤΙΚΟΥ ΠΡΟΒΛΗΜΑΤΟΣ ΜΕ ΠΕΠΕΡΑΣΜΕΝΑ ΔΙΠΟΛΑ

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Π Ε Ρ Ι Λ Η Ψ Η

Με βάση ορισμένα προκαταρκτικά πειραματικα αποτελέσματα και μετά από προσεχτική εξέταση των σχετικών δημοσιεύσεων προτείνεται ότι για τη μελέτη τοπικών μαγνητικών ανωμαλιών, το μοντέλο του πεπερασμένου διπόλου είναι πιό κατάλληλο, αφού λόγω της κανονικής απόστασης μεταξύ των ετερώνυμων φορτίων στην περίπτωση αυτή έχει καλύτερες προσεγγιστικές δυνατότητες. Επιπλέον, ένα τέτοιο μοντέλο δεν έχει χρησιμοποιηθεί ακόμα στη γεωφυσική επιστήμη και πρακτική. Για το λόγο αυτό, ένα κατάλληλο πρόγραμμα Η/Υ εκπονήθηκε και ορισμένα αριθμητικά αποτελέσματα υπολογίστηκαν. Τα αποτελέσματα αυτά επιβεβαιώνουν τις προκαταρκτικές υποθέσεις.

INTRODUCTION

The method for the solution of the inverse problems of the potential fields with a final number of elementary sources, proposed already about twenty years ago (Zidarov, 1965, 1968, 1990), and repeatedly used for solution of local (Georgiev, 1969 ; Zhelev, 1972, 1985, 1990 ; Bochev and Georgiev, 1974 ; Zhelev and Georgiev, 1985 ; Report of GFI, BAS, 1991) as well as of global problems (Zidarov and Bochev, 1965, 1969 ; Bochev, 1976 ; Zidarov and Petrova, 1978, 1979, 1983 ; Zhelev and Georgiev, 1985) of this kind, is one of the most effective in this field.

In the theoretical works (Zidarov, 1965, 1968, 1990) of that author for approximating bodies, mainly point masses for the gravity case and dipoles and circular loops - for the magnetic, are suggested. In practice also only point masses (Georgiev, 1969 ; Zhelev, 1972,1985,1990 ; Bochev and Georgiev, 1974 ; Zhelev and Georgiev, 1985 ; Report of GFI, BAS, 1991) , dipoles (Zidarov and Bochev ,1965, 1969; Bochev, 1976) and circular loops (Zidarov and Petrova, 1978, 1979, 1983) are used so far. However, there is enough grounds to suppose, that in some cases (as for example - in the cases of investigating local and to some extent of regional observations, because of the usual large distances between the poles with different sings, better approximating possibilities can be expected from the final dipoles (FD). In the present work, the possibilities of this model for the solution of the inverse magnetic problem are studied. For this purpose, a suitable computer program is worked out and a number of numerical experiments are made with it. For determination of the corresponding unknown parameters, the method of Marquardt (1963) is applied, widely used for problems of this kind. The obtained results, undoubtedly confirm the preliminary theoretical suggestions.

MATHEMATICAL FORMULATION OF THE PROBLEM

The solution of the problem thus formulated can be reduced mainly to the solution of the following nonlinear system of equations

$$f\ (\ x\ ) = y\ ,$$

where x $(x_1, x_2, \ldots x_n)$ is the vector of the unknown parameters - coordinates of the gravity center - ξ_k, η_k, ζ_k, the corresponding masses - m_k , directional cosines - α_k, β_k, γ_k and lengts l_k , $k = 1$, ... , $n/7$ of a system of $n/7$ finite dipoles , which must be determined on the basis of the vector of the observations y^* $(y_1, y_2, \ldots, y_N)$, and f^* $(f_1, f_2, \ldots, f_N)$ is a vector of N nonlinear functions (the symbol * means transposition). Specially in this case, the functions f_i = 1, ..., N are defined from the corresponding analytical expression for the magnetic field of a system of $n/7$ elementary sources of this kind in N points of observation with coordinates - X_i, Y_i, Z , $i = 1$, ... , N , located over the Earth's surface.

For representation of the background of the field, when necessary, a linear regression equation can be used, whose coefficients can be determined in the process of optimization, together with the rest of the unknowns (Zhelev, 1990). Instead, more elementary sources can be included in the model for this purpose (Zhelev, 1990) . Their parameters can be specified in the same way. Usually, in order to represent the background, they must lie significantly deeper from the rest. Of course, the best thing to do here is to try to remove the background before the interpretation, but this is not always possible with the needed precision. Regardless of this, here is suggested, that the background is successfully removed from the observations, before starting the father analyses of the concrete anomalies.

Obviously in this way solutions, which represent not the real source of the observations but the corresponding maternal body (Zidarov, 1968, 1990 ; Zidarov and Zhelev, 1970 ; Zhelev, 1977) (the most concentrated element) of the family of bodies with identical exterior potential fields, approximately defined with the given observations, can be obtained . Of course, from the so determined concentrated sources (in the form of finite dipoles), on the basis of the method for obtaining equifamily bodies (Zidarov, 1968, 1990 ; Zidarov and Zhelev, 1970 ; Zhelev, 1977) and some additional information (Zidarov and Zhelev,1970) (as for example - other geophysical or geological) , concrete bodies from the family can be determined, close (in some sense) to the real. But we will not treat this problem in detail here, as it is out of the frames of the present work. Besides, in a number of cases, the so defined concentrated source bodies give sufficiently good idea about the configuration of the real disturbers.

COMPUTER PROGRAM AND PRELIMINARY RESULTS

On the basis of the method described above a program in FORTRAN for "PRAVETS-16" (equivalent to "IBM-PC") computer was worked out and some numerical experiments (clarifying its possibilities) were made with it. The optimization was carried out after the Marquardt method (1963). A part of the results obtained are presented in the tables - 1 and 2. Besides the parameters of the source of the observations, the initial and final (t.e. - the solution) approximations of the unknowns, the following parameters are listed there for convenience :

- the corresponding functional

$$F(x)=\sum_{i=1}^{N}[y_i-f_i(x)]^2,$$

- the respective mean square deviation (Zhelev, 1990)

$$\sigma=[\frac{F(x)}{(N-n)}]^{1/2},$$

- the gradient of the functional

$$G(x)=(\sum_{k=1}^{N}[\frac{\partial F(x)}{\partial x_k}]^2)^{1/2}$$

- the coefficient of non-representativeness (Zhelev, 1972)

$$K_v=100(\frac{F}{E})^{1/2}\ \%,\qquad E=\sum_{i=1}^{N}y_i^2$$

at the point of the minimum, etc.

The values of the functional, the corresponding mean square deviation and the coefficient of non-representativeness show (see the tables), that satisfactory solutions were found, and the models fit the observations comparatively well in almost all

Table 1. Solution of the inverse magnetic problem with a system of FD- two FD represent the field of two elementary sources of the same kind with 0 , 5 and 10 % errors in the observations.

Parameters	F	G σ	k	ξ_k α_k	η_k β_k	ζ_k γ_k	m_k l_k
Initial approximations :	$.96.10^{3}$	$.35.10^{3}$	1	8.5	9.	5.	260.
		-		.52	.83	.21	9.64
			2	9.5	8.5	5.	190.
				.51	.51	.69	5.83
Solution with 0 % errors	$.59.10^{-14}$	$.39.10^{-5}$	1	8.5	9.	5.	240.
in the observations :		0.		.70	.60	.40	10.05
			2	10.5	7.5	4.	200.
Number of iterations = 25				.51	.51	.69	5.83
Source :			1	8.5	9.	5.	240.
				.70	.60	.40	10.05
Nx = Ny = 7 ; XO = YO = ZO = 0 ; Dx = Dy = 3 ;			2	10.5	7.5	4.	200.
Ymax = 54.9				.51	.51	.69	5.83

See the continuation on the next page:

continuation of Table 1.

P a r a m e t e r s :	F	C / σ	k	ε_k / α_k	η_k / β_k	ζ_k / γ_k	m_k / l_k
Solution with 5 % error	$.23.10^{3}$	$.67.10^{-1}$	1	8.03	7 52	4.41	411.87
in the observations :		2.58		.74	.63	.21	7.70
			2	6.77	4.01	3.48	264.65
Number of iterations = 78				-.57	-.71	.42	6.25
Solution with 10% error	$.76.10^{3}$	$.29.10^{-5}$	1	3.58	8.37	2.62	143.81
in the observations :		4.65		.73	.68	.57	5.07
			2	9.77	6.06	4.60	1084.30
Number of iterations = 29				.73	-.02	.68	2.73

Table 2. Solution of the inverse magnetic problem with a system of FD - two FD represent the field of tow elementary sources of the same kind with 0 , 5 and 10 % errors in the observations.

Parameters :	F	G / σ	k	ξ_k / α_k	η_k / β_k	ζ_k / γ_k	m_k / l_k
Initial approximations :	$.11.68^{4}$	$.42.10^{3}$	1	8.	10.	5.5	200.
		-		.67	.67	.33	9.0
			2	10.5	9.	4.5	160.
				.71	.57	.42	7.07
Solution with 0 % errors	$.85.10^{-13}$	$.59.10^{-5}$	1	9.	9.	4.5	220.
in the observations :		0.		.67	.67	.33	9.
			2	10.5	9.	3.5	180.
Number of iterations : 23				.41	.82	.41	7.35
Source :			1	9.	9.	4.5	220.
				.67	.67	.33	9.0
Nx = Ny = 7 ; XO = YO = ZO = 0 ; Dx = Dy = 3 .			2	10.5	9.	3.5	180.
Ymax = 50.				.41	.82	.41	7.35

see the continuation on the next page :

continuation of Table 2.

Parameters.	μ	ε / σ	k	ξ_k / α_k	η_k / β_k	ζ_k / γ_k	m_k / l_k
Solution with 5% errors in the observations :	$.15.10^{3}$	$.88.10^{-5}$	1	5.89	9.53	1.81	77.86
		4.92		.67	.72	.19	12.96
			2	10.62	7.64	4.87	373.87
Number of iterations = 53 .				.55	.44	.63	8.16
Solution with 10% errors in the observations :	$.85.10^{3}$	$.67.10^{-1}$	1	7.32	12.30	4.70	251.75
		4.92		.55	.81	.20	10.24
			2	10.72	9.13	2.97	197.39
Number of iterations = 78 .				.59	.74	.32	9.28

of the cases. The corresponding gradient points out, that in all of them the optimum of the functional has been achieved.

As it is not difficult to understand, two different examples (with verious sources and initial approximations) are considered here for significance (presented on the tables 1 and 2 respectively). The only difference between them is that the second one (placed in the table 2) is with a little more rough initial approxi-mations. The corresponding results confirm in general the preliminary theoretical suggestions. Satisfying solutions are obtained both at exact observations and at random errors in them -with zero mean and different dispersions s-, representing 5, 10 and 15 % of the corresponding maximum values - Y_{max} of the observations. However, on the tables only these with the exact observations and with the first two dispersions are shown. As it can be seen from the results, at precise observations, the corresponding exact solutions are obtained without any difficulties. It must be said however, that exact solution of these examples with the models used so far for this purpose is obviously impossible, and no doubt such problems can be often met in the geophysical science and practice. At 5 % errors in the observations also comparatively precise solutions are obtained, which describe the details of the sources' configurations sufficiently well. Despite that at larger errors in the observations some indications of unstability are observed, and some details of the solution are missing, its main features are saved comparatively well.

CONCLUSIONS

It can be said in conclusion, that the obtained results confirm undoubtedly the preliminary theoretical suggestions. Really, the new model has better approximating possibilities, and obviously the corresponding method (based on it) is more effective of the ones used so far for investigation of local and regional anomalies. Of course , this inference is based not only on the experiments placed on the tables, but on a number of other investigations, which because of the space limitations here, are not included in this preliminary paper on this subject.

Despite that the obtained results are comparatively good, it must be said however, that this problem is so complicated, that the achieved solution must not be regarded as a final one. That is why, the investigations in this field must continue in the future, mainly with experiments for improvement of the computer program worked out and with applications in different spheres of the geophysical science and practice.

Probably it is worth mentioned at the end, that the computer program developed, with small transformations can be used and for analysis of global observations.

Besides, observations of other celestial bodies, can be analyzed analogously.

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