

**DETERMINATION OF CRUSTAL THICKNESS BY INVERSION OF  
TRAVEL TIMES: AN APPLICATION IN THE AEGEAN AREA**

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**A B S T R A C T**

Inversion tomography is usually dealing with the determination of the variations of velocity within the studied medium. On the other hand, it is well known that Pn residuals are mainly associated with changes of the crustal thickness. In the present paper, the Pn residuals are attributed to these changes and are incorporated in an inversion scheme for the determination of the Moho depth in the Aegean area. The main assumption made is that the granitic and basaltic layers follow the undulations of the Moho and that the Pg, Pb and Pn velocities are generally stable in the examined area. The results are compatible with the Moho image from previous work in the area.

**ΚΑΘΟΡΙΣΜΟΣ ΤΟΥ ΠΑΧΟΥΣ ΤΟΥ ΦΛΟΙΟΥ ΜΕ ΑΝΤΙΣΤΡΟΦΗ ΤΩΝ ΧΡΟΝΩΝ  
ΔΙΑΔΡΟΜΗΣ: ΕΦΑΡΜΟΓΗ ΣΤΗΝ ΠΕΡΙΟΧΗ ΤΟΥ ΑΙΓΑΙΟΥ**

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**Π Ε Ρ Ι Λ Η Ψ Η**

Η τομογραφία αντιστροφής αφορά συνήθως τον καθορισμό των μεταβολών της ταχύτητας μέσα στο μέσο που μελετάται. Είναι γνωστό όμως, ότι τα Pn χρονικά υπόλοιπα σχετίζονται κυρίως με μεταβολές του πάχους του φλοιού. Στην παρούσα εργασία, τα Pn χρονικά υπόλοιπα συσχετίζονται με αυτές τις μεταβολές και περιλαμβάνονται σε ένα σύστημα αντιστροφής για την μελέτη του βάθους της Moho στην περιοχή του Αιγαίου. Η βασική υπόθεση είναι ότι το γρανιτικό και βασαλτικό στρώμα ακολουθούν τις διαταραχές της Moho και ότι οι Pg, Pb και Pn ταχύτητες είναι γενικά σταθερές στην υπό εξέταση περιοχή. Τα αποτελέσματα είναι συμβατά με την εικόνα της Moho από προηγούμενες μελέτες στην περιοχή.

**INTRODUCTION**

Inversion of travel time residuals for determination of three dimensional velocity structure is often applied for mapping of the earth's structure. Since the first applications of these technique (Aki and Lee, 1976; Aki et al., 1977) a lot of improvements have been made aiming mainly to the efficiency, accuracy, speed and memory requirements of the inversion algorithm (e.g. Paige and Saunders, 1982) or to the selection

of a more appropriate ray tracing technique (e.g. Thurber and Elsworth, 1980; Moser et al., 1992). In some cases, the result of these studies was the construction of spectacular 3-D images of the subsurface structure (e.g. Spakman et al., 1992). However, the target of all these studies is the determination of the velocity variations whether it is real or if it is the result of the undulations of the boundaries of layers with more or less constant velocities. In the latter case an alternative approach can be followed where one can estimate these undulations using a priori information about the layer velocities. A typical example is the case of Pn residuals which are strongly influenced by the undulations of the Moho boundary. Therefore, these residuals can be used for the determination of the crustal thickness. In some cases, this is performed by estimating the average Pn residuals in all the available seismological stations and then by directly calculating the crustal thickness from this average residual (Panagiotopoulos, 1984).

In the present paper, the Pn travel time residuals are incorporated in an inversion scheme for the determination of the crustal thickness assuming that the velocities are more or less stable in the crust and that the layers forming the crust generally follow the undulations of the Moho boundary.

#### MODELLING OF THE Pn TRAVEL TIMES

In the following analysis, we assume that the crust is consisting of  $n$  layers and has a variable thickness, as shown in figure 1. In the same figure we have a Pn wave ray which is refracted on the Moho at distances  $d_1$  and  $d_2$  from the epicentre,  $i$ , and the recording station,  $j$ , respectively. The remaining part,  $D$ , of the epicentral distance is divided into  $M$  segments of a preselected length  $d'$ . At each point,  $p$ , we define the crustal thickness,  $Z_p$ , and the sub-Moho velocity,  $V_p$ . Then, we assume that:

a) Within each layer the velocity,  $U_k$ , is considered to be constant.

b) The  $n$  layers that form the crust generally follow the undulations of the Moho. This assumption means that if  $L_{kp}$  is the thickness of the  $K$  layer at point  $P$  then the following relation holds:

$$L_{kp} = B_k \cdot Z_p, \quad \sum_{i=1}^{n-1} B_i = 1 \quad (1)$$

where  $B_k$  is constant for each of the layers that form the crust.

We can now calculate the travel time  $T_{ij}$  for the Pn waves using the following expression:

$$T_{ij} = T_s + T_0 + T_1 \quad (2)$$

where  $T_s$  is the travel time under the Moho and  $T_0$  and  $T_1$  are the travel times of downgoing and upgoing part of the ray.

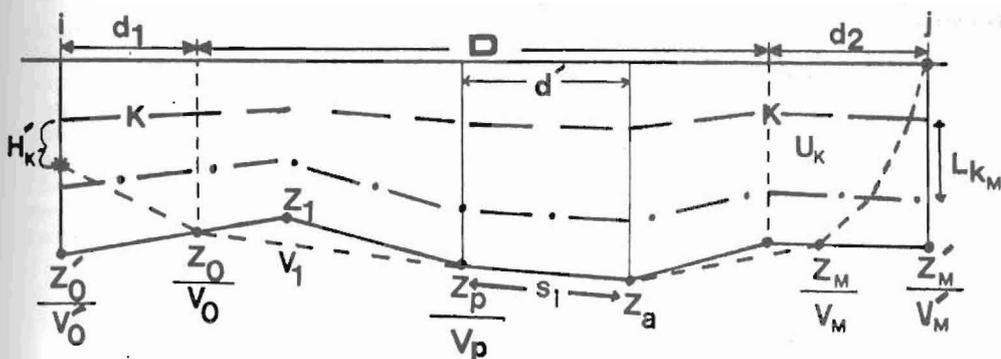


Fig.1. Crustal model and Pn wave propagation path used throughout this study.

The travel time under the Moho,  $T_s$ , is calculated as a simple sum of the travel times of the segment,  $S_i$ , between the points where the Moho depths are defined:

$$T_s = \sum_{i=0}^M \frac{S_i}{V_i} \quad S_i = S_i(Z_p, Z_a, d'), \quad V_i = \frac{(V_p + V_a)}{2} \quad (3)$$

where  $V_i$  is the interval velocity between the starting,  $Z_p$ , and ending,  $Z_a$ , points of the ray segment,  $S_i$ . The length of this segment is obviously calculated by  $Z_p$ ,  $Z_a$  and  $d'$ .

The upgoing and downgoing travel times can be calculated using the well known formulas for the flat earth, if we assume that we have a quite dense grid of Moho points so that locally the Moho discontinuity is almost horizontal. Hence, the upgoing travel time is given by the relation (see also figure 1):

$$T_1 = \sum_{\tau=1}^{n-1} \frac{B_\tau V_M (Z_M + Z'_M)}{2U_\tau \sqrt{V_M^2 - a_\tau^2 \cdot U_\tau^2}} \quad (4)$$

A similar relation holds for the downgoing part of the ray.

In equation (4) we have incorporated, in an approximate way, the earth's spherical shape by introducing a correcting factor,  $a_p$ , calculated for each layer  $p$ , at each point  $Z_q$ , by using the relation:

$$a_p = 1 - b'_p \frac{Z_p}{R_0} \quad \text{where} \quad b'_p = \sum_{q=p}^{n-1} B_q - 0.5 \cdot B_p \quad (5)$$

where  $R_0$  is the local mean earth radius. This correction is necessary since for rays up to 600Km it can contribute up to 1 sec in the travel time.

In the following we will consider only the case where the upper mantle velocity is also considered constant, i.e.  $V = \text{const}$ .

We consider on the earth's surface a grid where we define the depth of the Moho discontinuity,  $X_i$ , based on our starting model.

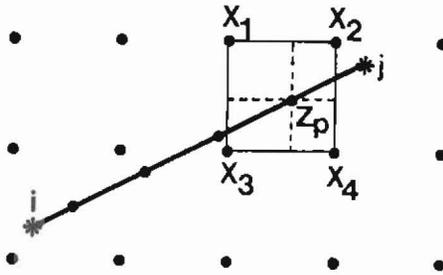


Fig.2. Surface grid of Moho depths,  $X_i$ , and ray path along which the Moho depths,  $Z_p$ , are defined.

Therefore, the Moho depths at each point of our ray (figure 2) are calculated by:

$$Z_p = \sum_{q=1}^4 C_{pq} X_q \quad (6)$$

The  $P_n$  travel time residual can be attributed to the correction of the depth of each point  $Z_p$  of our ray and through equation (6) to corrections of the depths of the Moho grid:

$$RES_{ij} = T_{ij-obs} - T_{ij-cal} = \sum_{p=0}^M \frac{\partial T_{ij}}{\partial Z_p} \cdot dZ_p = \sum_{q=1}^4 \left( \sum_{p=0}^M \frac{\partial T_{ij}}{\partial Z_p} C_{pq} \right) dX_q \quad (7)$$

or

$$Y = W \cdot X \quad (8)$$

We reach a linear system where the  $P_n$  residuals (vector  $Y$ ) are linearly connected through matrix  $W$  to the undulations of the Moho discontinuity (vector  $X$ ). This linear system can be sequentially inverted for the estimation of the crustal thickness in an area.

The previously described method has certain limitations due to the fact that the equations used (3 and 4) are approximate. However, the most serious disadvantages lie on the validity of the hypothesis that are made. If we have significant velocity variations within each layer this will result in mapping fictitious undulations of the Moho. These limitations should be seriously taken into account when applying this technique in some areas not fulfilling the assumptions previously made.

## APPLICATION IN THE AEGEAN AREA

We applied the previously described procedure in the Aegean area. The data used, came from the Annual Bulletins of the Geophysical Laboratory of the University of Thessaloniki for the period 1981-1987. We also used data from three local experiments conducted in Mygdonia basin (1985), Peloponnesus (1986) and Southern Aegean (1988) by L.G.I.T of the University of Grenoble and the Geophysical Laboratories of the University of Thessaloniki and Athens. In order to avoid problems with non Pn waves but also due to the limitations of the method for short rays we used ray paths for events in the crust with lengths from

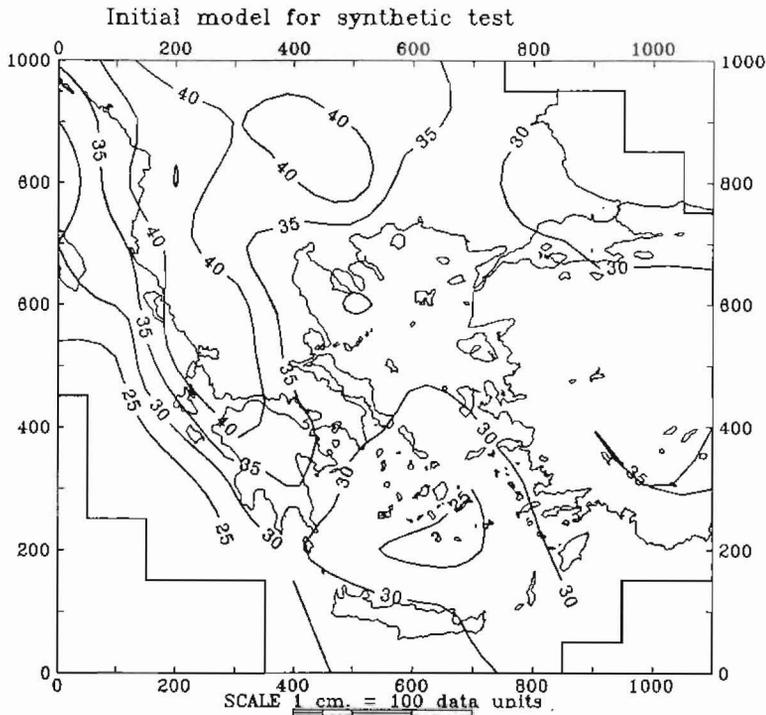


Fig.3. Initial crustal thickness model used in the present study, compiled on the basis of previously published work.

200 up to 600 Km. In total we had approximately 60000 rays. The crust was roughly divided into a granitic and a basaltic layer, since no recording station lies on a sedimentary layer. The velocities used were  $U_g=6.1$  Km/sec,  $U_b=6.7$  Km/sec and  $V=7.9$

Km/sec whereas the granitic layer was estimated to be about  $B_g=0.62$  of the total crustal thickness, based on previous work (e.g. Papazachos et al., 1966; Makris, 1973, 1977, 1978; Calcagnile, et al., 1982; Panagiotopoulos, 1984; Delibasis et al, 1988; Hashida et al., 1988; Spakman, 1988; Drakatos, 1989; Ligdas et al., 1990; Voulgaris, 1991;). The initial model was also constructed on the basis of this literature and can be seen in figure 3. The velocities used are quite well constrained except for the  $U_h$  velocity which shows a variation with depth.

Initially, the resolving ability of our modelling was tested by a synthetic example. The model of figure 3 and the coordinates of the earthquake foci of our data set were used for the calculation of the travel times. Random errors with standard deviations of 15 Km for the horizontal and 5 Km for the vertical were added to the epicentral coordinates. Also, random errors of 0.75 sec were added to the calculated travel times. The synthetic data were inverted and the resulting model is shown in figure 4. We see that we have a quite good recovery of our initial Moho image.

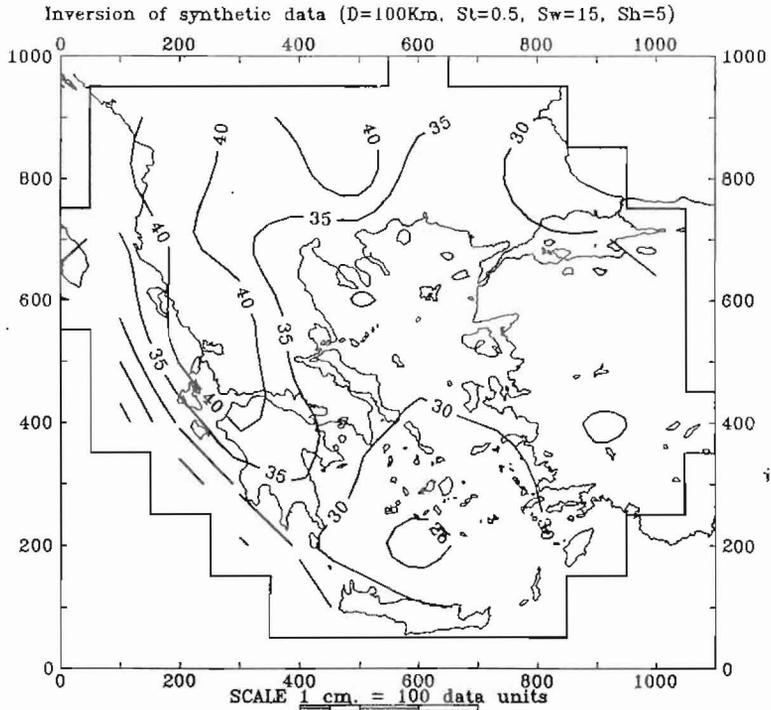


Fig.4. Result of the inversion of synthetic Pn travel times data set calculated using the model of figure 3.

In figure 5 we see the result of the inversion of our real data. The result was almost the same whether we used a initial flat Moho model or the model of figure 3 and this supports the credibility of this result. Although, the final image does not resemble very much with the one of figure 3, its qualitative and

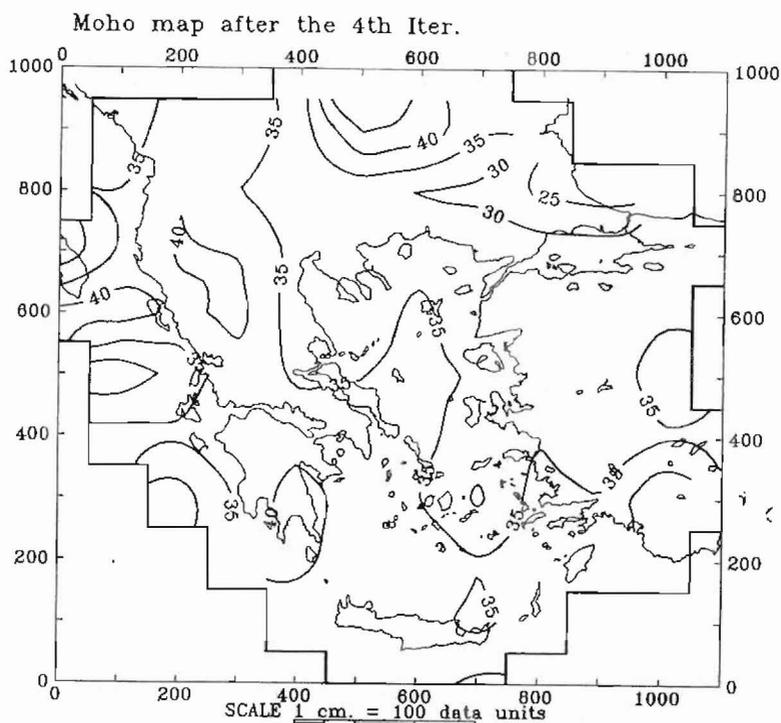


Fig.5. Crustal thickness in the Aegean area from the inversion of the real Pn travel time data.

partly its quantitative characteristics are the same. We observe a thick crust under the Hellenides mountain chain reaching up to more than 40 Km. In the southern Aegean the crust is thinning up to less than 30 Km. These two results are in very good agreement with the results of Makris (1973, 1977, 1978), although the crust in the southern Aegean is somewhat thinner. We also observe a thin crust under the NE Aegean towards the Black Sea (less than 25 Km). Panagiotopoulos (1984) had already suggested such a thinning but also noticed that this was in contradiction with the positive Bouguer anomalies of Makris (1973, 1977, 1978). However, this conclusion seems to be valid, except if a sudden change of the Pn velocity occurs in this area. The crust is also thickening towards the inner part of the Balkan peninsula, in agreement with previous studies (e.g. Shanov and Kostadinov, 1992).

## CONCLUSIONS

A method is proposed for the determination of the crustal thickness by the use of Pn residuals in an inversion scheme. The main hypothesis made is that the velocities do not vary significantly within each of the crustal layers and that these layers generally follow the undulations of the Moho discontinuity. The method seems to work well, even in a complicated area such as the Aegean where we have rapid changes of the crustal thickness. The obtained results are in good agreement with previous work in the area, showing a thick crust under the Hellenic mountain chain and thin crust in the southern Aegean and the Marmara-southwest Black Sea areas. However, the method has a medium accuracy due to the approximations and assumptions made. When applied, one should consider these assumptions, bearing in mind that significant velocity variations within and especially below the crust result in fictitious Moho undulations. The method can be expanded in order to include also the velocity variations beneath the crust. A possible complete expansion of this method could include all kind of P arrivals. In this case a more appropriate ray tracing technique should be used in order to calculate the variations not only of the Moho but of all the discontinuities in the crust.

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