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Networks as a method for portfolio selection

Τα δίκτυα ως μέθοδος επιλογής χαρτοφυλακίου

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ΔΙΑΤΜΗΜΑΤΙΚΟ ΠΡΟΓΡΑΜΜΑ ΜΕΤΑΠΤΥΧΙΑΚΩΝ ΣΠΟΥΔΩΝ στα

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Τιτλος Εργασιας

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Networks as a method for portfolio selection

Προκόπιος Καραδήμος

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ABSTRACT

This thesis is an extension to the existing literature on applications of network theory for portfolio selection. Until now, networks for portfolio selection are identified via Pearson correlation -a linear correlation measure- of stock returns while the stocks from which the portfolios are constructed are chosen based on common centrality measures. In the current thesis not only Pearson but also non-linear measures are applied which lead to both symmetric and asymmetric adjacency matrices. In total, 6 different types of (overlapping) networks are identified: Net I: Pearson correlation with replacement of negative values by zero, ii) Net II: Absolute values of Pearson correlation, iii) Net III: Normalized Mutual Information, iv) Net IV: Directed Normalized Mutual Information, v): Net V: Information Interdependence, vi): Net VI: Information Dependence (Asymmetric). For each node of those overlapping networks, the following are computed: strength, closeness centrality, betweenness centrality, eigenvector centrality, eccentricity; portfolios are constructed from the stocks with the highest and the lowest score on those measures. Apart from the application of non-linear measures and the identification of directed networks, a separate study for the 2008 financial crisis era is performed in order to also come up with the best performing networks during periods of extreme volatility. Portfolios are evaluated based on returns, total risk, systemic risk, adjusted to total risk return and adjusted to systemic risk return.

Regarding the most crucial conclusions to be made, networks identified through Pearson Correlation achieve higher returns. However, non-linear measures are superior when it comes to building portfolios of less risk (both total and systematic). Concerning adjusted to risk return, top performance is shared between linear measures and the best performing non-linear ones. However, during the crisis the superiority of non-linear measures is evident, with the importance of directed networks during high volatility eras becoming lucid as well.

KEY WORDS

Networks, Portfolio Selection

ΠΕΡΙΛΗΨΗ

Η παρούσα διπλωματική εργασία αποτελεί επέκταση της έρευνας που έχει γίνει για την αξιοποίηση της θεωρίας δικτύων στη δημιουργία χαρτοφυλακίων. Μέχρι τώρα με βάση τις αποδόσεις των μετοχών αναγνωρίζονται δίκτυα μέσα από τον συντελεστή συσχέτισης Pearson -ο οποίος αποτελεί ένα γραμμικό μέτρο συσχέτισης- και επιλέγονται μετοχές με βάση γνωστά μέτρα κεντρικότητας. Στην παρούσα εργασία αναγνωρίζονται δίκτυα όχι μόνο με τον Pearson αλλά και με μη γραμμικά μέτρα που οδηγούν σε συμμετρικούς αλλά και μη συμμετρικούς πίνακες γειτνίασης. Συνολικά αναγνωρίζονται 6 τύποι (επικαλυπτόμενων) δικτύων: Net I: Pearson correlation με μηδενισμό αρνητικών βαρών, ii) Net II: Απόλυτες τιμές Pearson correlation, iii) Net III: Normalized Mutual Information, iv) Net IV: Directed Normalized Mutual Information, v): Net V: Information Interdependence, vi): Net VI: Information Dependence (Asymmetric). Για καθένα από τα επικαλυπτόμενα δίκτυα κάθε τύπου υπολογίζεται το strength, η closeness centrality, η betweenness centrality η eigenvector centrality και η eccentricity κάθε κόμβου και σχηματίζονται χαρτοφυλάκια από τις μετοχές που έχουν τις χαμηλότερες και τις υψηλότερες τιμές σε κάθε ενα από αυτά τα μέτρα. Εκτός από την εφαρμογή μη γραμμικών μέτρων και την αναγνώριση κατευθυνόμενων δικτύων, γίνεται και ξεχωριστή μελέτη για την περίοδο της χρηματοοικονομικής κρίσης του 2008 έτσι ώστε να διαπιστωθεί τα χαρτοφυλάκια ποιών δικτύων αποδίδουν καλύτερα σε περιόδους πολύ υψηλής μεταβλητότητας. Τα χαρτοφυλάκια αξιολογούνται με βάση την απόδόση τους, τον κίνδυνο (συνολικό και συστημικό) αλλά και την απόδοση προς τον κίνδυνο τους.

Όσον αφορά τα βασικότερα συμπεράσματα, τα δίκτυα που αναγνωρίζονται μέσω Pearson σημειώνουν καλύτερες επιδόσεις όσον αφορά το κέρδος, ενώ τα μη γραμμικά μέτρα υπερτερούν στη δημιουργία χαρτοφυλακίων χαμηλότερου κινδύνου (συνολικού και συστηματικού). Αναφορικά με την προσαρμοσμένη στον κίνδυνο απόδοση, δίκτυα μέσω Pearson εναλλάσσονται στην κορυφή με κάποια από τα μη γραμμικά δίκτυα. Μέσα στην κρίση όμως η υπεροχή των μη γραμμικών μέτρων είναι σαφής, με την σημασία των κατευθυνόμενων δικτύων να αναδεικνύεται εξίσου κατά την περίοδο υψηλής μεταβλητότητας.

ΛΕΞΕΙΣ ΚΛΕΙΔΙΑ

Δίκτυα, Επιλογή Χαρτοφυλακίου

Contents

ΣΥΝΟΨΗ

Η διαχείριση χαρτοφυλακίου και η επιλογή των κατάλληλων περιουσιακών στοιχείων για τη διάρθωση του είναι ένα από τα μείζονα ζητήματα των χρηματοοικονομικών και απασχολεί τόσο οργανισμούς και εταιρίες όσο και απλούς πολίτες. Ένας από τους θεμελιώδεις στόχους της διαχείρισης χαρτοφυλακίου είναι η διαφοροποίηση του, η επιλογή δηλαδή περιουσιακών στοιχείων οι αποδόσεις των οποίων συσχετίζονται αρνητικά έτσι ώστε να μειώνεται ο συνολικός κίνδυνος του χαρτοφυλακίου.

Η επιλογή περιουσιακών στοιχειών γίνοταν στο παρελθόν κυρίως εμπειρικά και βασιζόμενη πάνω σε ιστορικά στοιχεία για τις αποδόσεις των μετοχών και των υπολοίπων προϊόντων. Αυτό άλλαξε όταν ο Markowitz το 1952 δημιούργησε τη σύγχρονη θεωρία χαρτοφυλακίου, η οποία εκτός από τις ιστορικές τιμές κάθε περιουσιακού στοιχείου λαμβάνει υπόψη τον κίνδυνο του αλλά και τις συσχετίσεις των στοιχείων αυτών και αποσκοπεί στην επιλογή του άριστου χαρτοφυλακίου, αυτού δηλαδή με την υψηλότερη αναμενόμενη απόδοση δεδομένου του κινδύνου ή αυτού με το χαμηλότερο δυνατό εκτιμώμενο κίνδυνο δεδομένης της αναμενόμενης απόδοσης. Ο Markowitz μάλιστα βραβεύθηκε με νόμπελ οικονομικών το 1990.

Βασιζόμενοι πάνω στη λογική της θεωρίας του Markowitz, κάποιοι συγγραφείς έχουν ήδη καταφύγει στη θεωρία δικτύων ώστε να καταφέρουν να επιλέξουν με μεγαλύτερη επιτυχία περιουσιακά στοιχεία για τη δημιουργία ενός βέλτιστου χαρτοφυλακίου. Οι Pozzi, Di Matteo, & Aste (2013) προσπαθουν να δημιουργήουν χαρτοφυλακία μετοχών «αντι-συσχετιζόμενων» κατά το δυνατόν περισσότερο. Για να το επιτύχουν αυτό, αναγνωρίζουν επικαλυπτόμενα δίκτυα χρονοσειρών μετοχών μέσω του συντελεστή συσχέτισης Pearson και στη συνέχεια υπολογίζουν για κάθε κόμβο των δικτύων αυτών την degree centrality, την closeness centrality, την eigenvector centrality, την betweenness centrality καθώς και την eccentricity. Το βασικό τους συμπέρασμα είναι ότι χαρτοφυλάκια που επενδύουν σε μετοχές-κόμβους της περιφέρειας αποδίδουν καλύτερα όσον αφορά την προσαρμοσμένη στον κίνδυνο απόδοση. Οι Peralta, Zareei (2016) προβαίνουν στην επονομαζόμενη ρ-dependent strategy θεωρώντας ότι με την στρατηγική αυτή προσεγγίζουν ορθότερα την θεωρία του Markowitz. Σύμφωνα με τη στρατηγική τους, δεν επιλέγουν πάντα περιφερειακούς κόμβους-μετοχές αλλά εξετάζουν τη συσχέτιση της eigenvector centrality με τον δείκτη Sharpe των μετοχών και ανάλογα με το αν αυτή η τιμή ξεπερνάει η όχι ένα συγκεκριμένο κατώφλι επενδύουν στις 20 περισσότερο ή λιγότερο central μετοχές αντίστοιχα. Επίσης, ενώ οι Pozzi et al. (2013) δημιουργούν κάποια συνθετικά μέτρα κεντρικότητας βάσει των μέτρων που υπολογίζουν, οι Peralta, Zareei (2016) βασίζονται αποκλειστικά στο eigenvector centrality.

Σκοπός της παρούσας διπλωματικής εργασίας είναι να επεκτείνει την έρευνα που έχει γίνει όσον αφορά την αξιοποίηση των δικτύων στη δημιουργία χαρτοφυλακίου. Βασικό θέμα που εξετάζεται είναι η καταλληλότητα του συντελεστή συσχέτισης Pearson ως μέτρου για την αναγνώριση δικτύων. Ο Pearson, ο οποίος χρησιμοποιείται τόσο στη θεωρία χαρτοφυλακίου όσο και στα paper των Pozzi et al. (2013) & Peralta, Zareei (2016) είναι ένα γραμμικό μέτρο συσχέτισης. Στη βιβλιογραφία όμως υπάρχουν σημαντικές ενδείξεις ότι οι σχέσεις μεταξύ των χρηματοοικονομικών περιουσιακών στοιχείων είναι μη γραμμικές. Έτσι λοιπόν αναγνωρίζονται 6 διαφορετικά είδη επικαλυπτόμενων δικτύων: i) Net I: Pearson correlation με μηδενισμό αρνητικών βαρών, ii) Net II: Απόλυτες τιμές Pearson correlation, iii) Net III: Normalized Mutual Information, iv) Net IV: Directed Normalized Mutual Information, v): Net V: Information Interdependence, vi): Net VI: Information Dependence (Asymmetric). Απο τα παραπάνω δίκτυα, τα πρώτα δύο είναι γραμμικά ενώ τα υπόλοιπα μη γραμμικά, με τα δίκτυα iv και vi να είναι μη συμμετρικά. Την αξία εξέτασης μη συμμετρικών δικτύων υπογραμμίζουν μάλιστα και οι Peralta, Zareei (2016).

Αφού αναγνωριστούν αυτοί οι 6 διαφορετικοί τύποι επικαλυπτόμενων δικτύων, για κάθε ένα απο τα επικαλυπτόμενα δίκτυα υπολογίζονται τα μέτρα που υπολόγισαν και οι Pozzi et al. (2013). Κάθε φορά δημιουργούνται χαρτοφυλάκια ίσης στάθμισης με τις 20 μετοχές με υψηλότερους και χαμηλότερους δείκτες και υπολογίζεται η επίδοση τους όσον αφορά τα παρακάτω κριτήρια: απόδοση (return), συνολικός κίνδυνος (variance), συστημικός-μη διαφοροποιήσιμος κίνδυνος (beta), απόδοση προσαρμοσμένη στο συνολικό κίνδυνο (μία παραλλαγή του γνωστού δείκτη Sharpe), απόδοση προσαρμοσμένη στο συστημικό κίνδυνο (μία παραλλαγή του γνωστού δείκτη Treynor). Οι επιδόσεις αυτές υπολογίζονται για περίοδο διακράτησης από 51 μέχρι 250 ημέρες. Στη συνέχεια υπολογίζεται η μέση επίδοση καθενός από τα έξι δίκτυα όσον αφορά το κάθε από τα 5 κριτήρια (μέσος όρος των επιδόσεων των επικαλυπτόμενων δικτύων ανά ημέρα διακράτησης).

Μέσα απο την ανάλυση, τα μέτρα κεντρικότητας strength, closeness, eigenvector φαίνονται να είναι αυτά που οδηγούν σε χαρτοφυλάκια με καλύτερη απόδοση –προσαρμοσμένη στον κίνδυνο και μη-, χαμηλότερο κίνδυνο και επιβεβαιώνουν και τη βιβλιογραφία περί επένδυσης σε μετοχές της περιφέρειας του δικτύου. Η σύγκριση μεταξύ των διαφορετικών δίκτυων δείχνει ότι τα δίκτυα που αναγνωρίζονται μέσω Pearson σημειώνουν καλύτερες επιδόσεις όσον αφορά το κέρδος, ενώ τα μη γραμμικά μέτρα υπετερούν αναφορικά με τη δημιουργία χαρτοφυλακίων χαμηλότερου κινδύνου (συνολικού και συστηματικού). Όσον αφορά την προσαρμοσμένη στον κίνδυνο απόδοση, δίκτυα μέσω Pearson εναλλάσσονται στην κορυφή με κάποια από τα μη γραμμικά δίκτυα.

Για την εξαγωγή περαιτέρω συμπερασμάτων, ακολουθείται η ίδια διαδικασία για την περίοδο της χρηματοοικονομικής κρίσης του 2008, η οποία οριοθετείται στα πλαίσια αυτής της εργασίας μεταξύ 01/08/2007 και 31/03/2009. Την περίοδο αυτή η υπεροχή των μη γραμμικών δικτύων είναι εμφανής, ιδιαίτερα στην κατασκευή χαρτοφυλακίων χαμηλότερου κινδύνου. Την περίοδο της κρίσης καταδεικνύεται επίσης η σημαντικότητα και των μη συμμετρικών δικτύων αφού είναι περισσότερες οι περιπτώσεις στις οποίες σημειώνουν την καλύτερη επίδοση.

Αναφορικά με το ποιο δίκτυο αποδίδει καλύτερα, κρίνεται ότι το vi (Information Dependence) αποδίδει καλύτερα σε σύγκριση με το έτερο μη συμμετρικό iv (Directed Normalized Mutual Information). Το δίκτυο iii (Normalized Mutual Information) βρίσκεται με συνέπεια στις πρώτες θέσεις, το net v (Information Interdependence) διακρίνεται όμως για τις επιδόσεις του κατά τη διάρκεια της κρίσης.

Για την επεξεργασία των δεδομένων χρησιμοποιήθηκαν τα λογισμικά R και Matlab. Από την R χρησιμοποιήθηκαν οι βιβλιοθήκες: igraph, sna, entropy.

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PROLOGUE

This thesis examines the issue of selection of the most suitable assets in order for a well-diversified portfolio to be constructed. As stated in Farlex Financial Dictionary, portfolio diversification entails the "investing in different asset classes and in securities of many issuers in an attempt to reduce overall investment risk and to avoid damaging a portfolio's performance by the poor performance of a single security, industry or country." Risk is usually measured by the volatility (variance) of the returns. Portfolio selection is crucial for practical purposes for both institutions and individuals, and plays a key role to the sustaining of lifetime consumption and bequest. On the one hand, institutions, mutual funds, pension funds and hedge funds are only examples of those that face this decision when managing large portfolios. On the other hand, even individuals are confronted with this issue when making their financial plans and thinking about the consequences of their choices (Detemple, 2012). Regarding diversification in particular, -as mentioned in Pozzi et. al (2013)- managing risk is of the utmost importance in periods of financial turmoil (Meucci, 2013 ; Hull J.C, 2012). A look at the recent Global Financial Stability Reports by the International Monetary Fund is indicative of the fact that we are living in an era characterized by such potential instability and risk. However, irrespective of the economic status, solely focusing on the risk can have positive results in the returns as well. According to theory, there is no point in a mere minimization of risk -not considering the expected returns- by the investor (Sharpe, 1964). However, there is extensive empirical evidence of low risk anomalies, according to which low risk assets outperform high risk ones (Unger, 2015; Blitz and van Vliet, 2007; Clarke et. al, 2006). This is another reason which renders the minimization of risk important for portfolio optimization.

Subject thesis is an extension on the existing literature of exploiting network science in order to build well-diversified portfolios, consisting of stocks as anti-correlated as possible. After the prologue, the main part includes: i) a literature review on modern portfolio theory and applications of network science on the matter, ii) the data processed, iii) the applied methodology, iv) the results. On the epilogue, the main conclusions as well as proposals for further research are mentioned. The appendices include images and graphs for further study.

MAIN PART

Literature Review

Modern Portfolio Theory

Before 1952, portfolio selection was a result of ad hoc methods lacking any mathematical background. Emphasis was mainly given to the individual returns of each investment. Markowitz (1952, 1959) was the one that came up with a revolutionary mathematical way for portfolio construction.

Instead of focusing solely on returns, he stressed the importance of risk in a portfolio as well. The optimum portfolio would be the one with the maximum expected return given the level of risk or - given the expected return -the one with the minimum risk among all the possible portfolios. In order for this problem to be solved, it is also of the utmost importance to study how each asset co-moves along with the other ones.

Markowitz was the first one that demonstrated how diversification can help to reduce the total portfolio risk without losing on return. He initiated a new logic in investing by proposing a focus on overall risk-reward characteristics rather than solely on individual ones. The assumptions of this model have to be comprehended in order for it to be properly utilized:

- Investors examine each asset assuming that it is represented by a probability distribution of expected returns to be generated during a holding period. It is also assumed that this distribution is normal.
- Investors maximize their one-period expected utility and their utility curves are characterized by decreasing marginal utility of wealth. In other words, investors' utility increases as their wealth does so as well, but each unit of wealth being added results in progressively lower utility increase.
- Investors calculate portfolio risk based on the volatility of its expected returns
- Investment decisions are made depending on expected return and risk, the investor's utility curves are therefore functions of expected return and variance (or standard deviation)

- Single period investment horizon. In the beginning, the investor opts for the suitable assets (or asset classes) and allocates his wealth accordingly by assigning weights to each asset. During the holding period each of them generates a random rate of return. In the end, the investor's return is a weighted average of the returns from each asset.
- Investors opt for higher returns to lower risk and lower risk for the same return
- Risk-averse investors. They will accept more risk only if this will increase expected return.
- Markets are perfectly efficient (no taxes or transaction costs)

Return & risk

Return

Let N be the assets of the portfolio with returns R_k, k=1,2…N. In addition, let:

- R_p: portfolio return
- w_k: weight of asset k
- σ_k: standard deviation of asset k
- $E(R_k)$ the mean or expected return of asset k

Then portfolio return is calculated as follows:

$$R_p = \sum_{k=1}^{N} w_k \, E_{(r_k)} \quad (1)$$

Risk

According to Markowitz model, the risk of a portfolio is a function of the risk of each asset (the variance of each asset's returns) and the covariance between the returns of all the assets of which the portfolio consists. Portfolio risk is calculated as follows:

$$\mathrm{Var}_{(r_p)} = \sigma_P^2 = \sum_{k=1}^{N} \sum_{l=1}^{N} w_k \, w_l \sigma_{kl} \quad (2)$$

, where σ_{kl} is the covariance of the returns of stock k and stock l. Covariance is a measure of the degree to which two variables tend to move in tandem. However, its interpretation can sometimes turn out to be ambiguous. For instance, a high covariance between two risky assets could indicate either a strong positive correlation or a weak positive correlation between asset returns, depending on whether subject time series are characterized by low or high volatility respectively. In order therefore for such ambiguity to be eschewed, the covariance can be replaced by the product of the correlation coefficient and the standard deviation of each of the assets returns. Taking that into consideration, the following formula for portfolio risk is also applicable:

$$\mathrm{Var}_{(r_p)} = \sigma_P^2 = \sum_{k=1}^{N}\sum_{l=1}^{N} w_k\, w_l \rho_{kl}\sigma_k\sigma_l \qquad (3)$$

Portfolio risk depends on the variance of each asset's returns, the covariance between the assets and the portfolio weights assign to each asset. Covariance is of higher importance than the individual risk of each asset, and the more assets are the portfolio includes, the higher is the importance of covariance in contrast to individual risk.

Systematic risk

Risk in stock markets is divided in systematic and unsystematic risk. The unsystematic risk is the one inherent in each company or sector someone invests. Unsystematic risk is also called diversifiable as this is the one which can be decreased through diversification. Systematic is the risk that characterizes the whole market. Systematic risk cannot be diversified; therefore it is also called undiversifiable risk. Systematic risk can be measured through beta (Sharpe, 1963). Beta is calculated as follows:

$$\beta_k = \frac{\sigma_{km}}{\sigma_m^2} \qquad (4)$$

σ_{km}: Covariance of stock's k returns with market returns (a market index is chosen as representative of the market)

σ_m^2: Variance of market returns

Beta is indicative of whether and in what extent a stock moves in tandem with the market. Table 1 shows how beta is interpreted:

Table 1: Beta coefficient values and interpretation

β<0	The stock moves opposite to the market
β=0	The stock moves in a manner uncorrelated to the market
0<β<1	Stock-market moves are correlated but the stock moves less than the market
β=1	Stock-market moves are totally correlated
β>1	Stock-market moves are correlated but the stock moves more than the market

The larger the beta of a stock, the more systematic risk this stock is believed to have.

Efficient portfolio

An efficient Markowitz portfolio can contain assets of any number. For starters, the asset allocation between two assets is examined. In such a portfolio, an investors invests w_1 and w_2 in the two assets, with $w_2=1-w_1$. The logic is similar for portfolios of more assets. In the case of a two-asset portfolio, 3 extreme cases can be examined at first:

a. Correlation Coefficient of 1 between the two assets

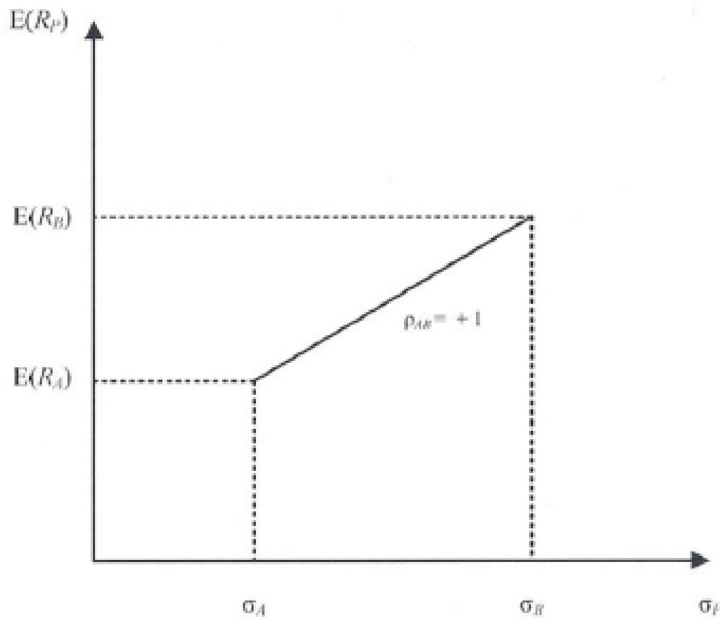

Image 1: Expected return and risk for correlation coefficient of 1 between two assets

In this first case, portfolio risk and return are merely linear combinations of the risk and return of each asset.

b. Correlation Coefficient of -1 between the two assets

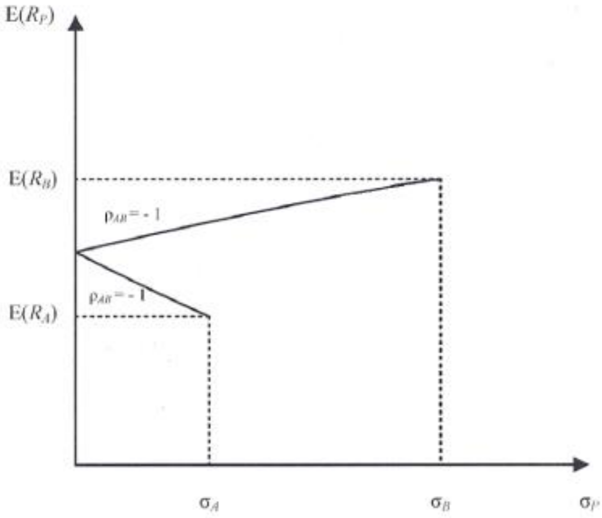

Image 2: Expected return and risk for correlation coefficient of -1 between two assets

In this second case, portfolio risk and return remain linear combinations of the risk and return of each asset. It should also be noted that portfolio risk of assets with a correlation coefficient of -1 is always lower that the respective of perfectly positively correlated assets. When two assets are perfectly negatively correlated, a portfolio of zero risk can be constructed.

c. Correlation Coefficient of 0 between the two assets

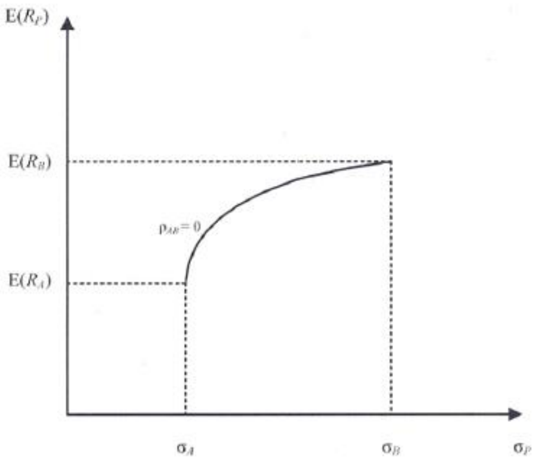

Image 3: Expected return and risk for correlation coefficient of 0 between two assets

In this third case, the portfolio risk is the square root of the weighted mean of the variance of the two risky assets, while the portfolio's expected return logic remains the same.

Comparing the three scenarios, it can be inferred that ρ=1 is the only case in which no benefit from diversification exists. Irrespective of the wealth allocation between the two assets, both the portfolio mean return and risk are simple weighted averages. No portfolio can be regarded as inefficient, investors choose among the possible portfolios only with risk as a criterion. On the other hand, when asset returns correlation is less than 1, there is a diversification effect. Investors can therefore reduce the individual asset risk they are subjected to through a diversified portfolio. Such a constructed portfolio will allow the same expected return with less amount of risk.

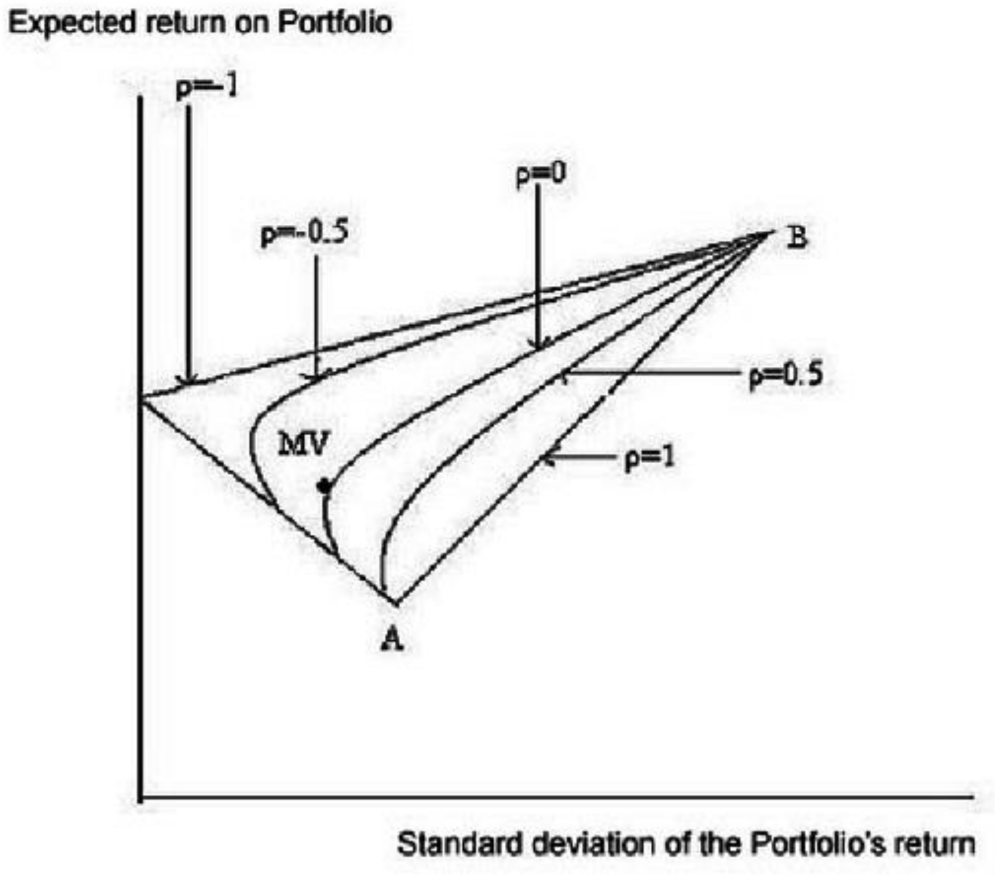

Image 4: Combinations of risk and return for different values of correlation coefficient (Ross et al, 2002).

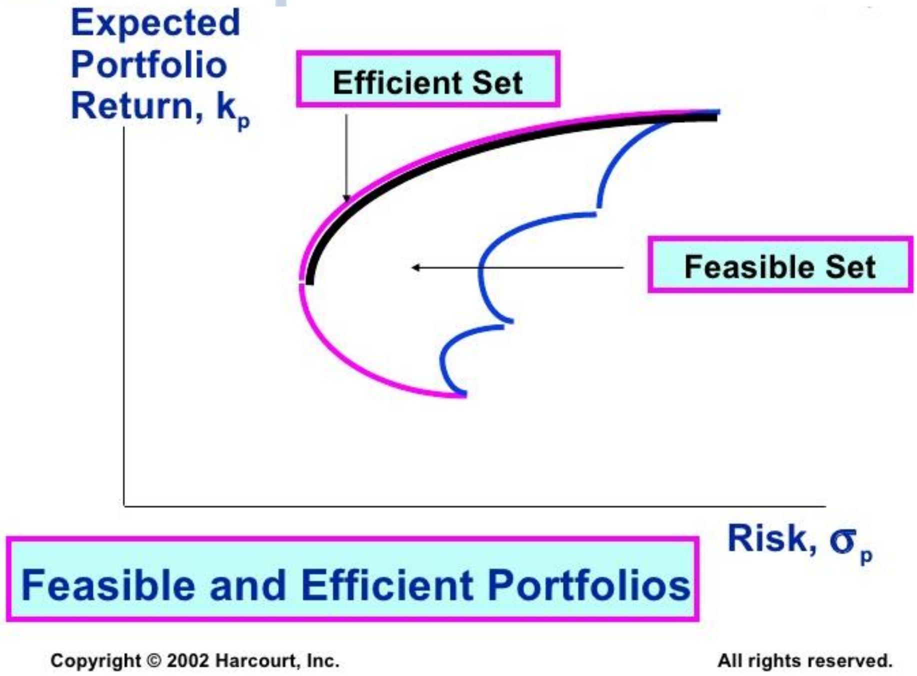

Image 5: Efficient Frontier and Feasible Set of Portfolios

Adding more assets to the analysis, the goal of the Markowitz model is the construction of an efficient portfolio. The latter refers to a portfolio with the best possible expected return given the level of risk or the least possible risk given the amount of expected return. The set of all the efficient portfolios is called the efficient frontier. The efficient frontier is located "on the northwest" of the feasible set of portfolios. The feasible set refers to all the portfolios that can be constructed with the available assets by the investor. In other words, the efficient frontier consists of all the dominant portfolios in terms of risk and return in comparison to all the possible ones.

Image 6 is indicative of the efficient frontier. It begins from the Minimum Variance Portfolio (point V) and ends at the maximum return portfolio (point A). Therefore the curve VA is the efficient frontier and contains all the efficient portfolios. Subject image changes if short selling is allowed. Short selling is common market practice nowadays. Short selling involves the borrowing and selling of assets that are not owned by the investor. It is incited by a belief that the subject asset's price will decrease. In that case, the investor can buy the security at the lower price after some time, return the shares to the owner and earn the difference of the borrowing and the afterwards buying price. Apparently if the security price rises after the short position, the investor incurs a loss.

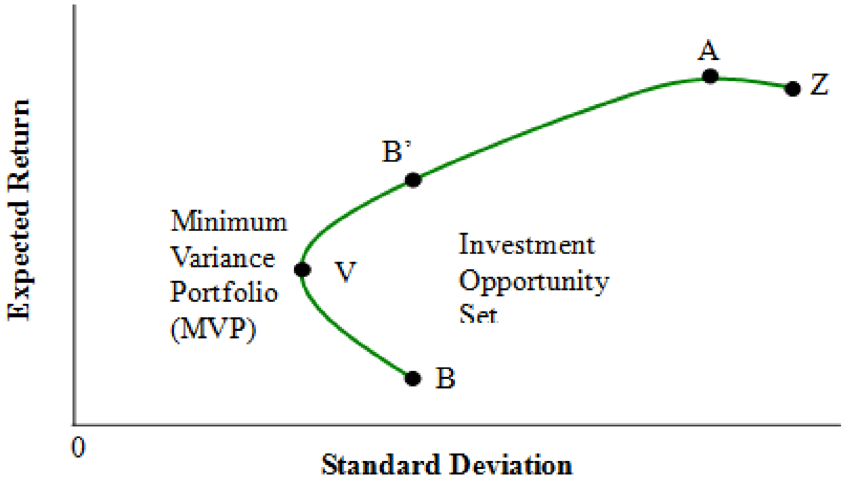

Image 6: MVP and Efficient Frontier with no short selling (Lee, Lee, Lee, 2010)

Short selling increases the efficient frontier bounds to plus/minus infinity, as depicted below. An investor can continuously short sell B and reinvest in A. That would result in upper infinity as the maximum expected return. On the other hand, if someone short sells A and reinvests in B, this can result in an infinitely negative expected returns, which explain the minus infinity as the new bound.

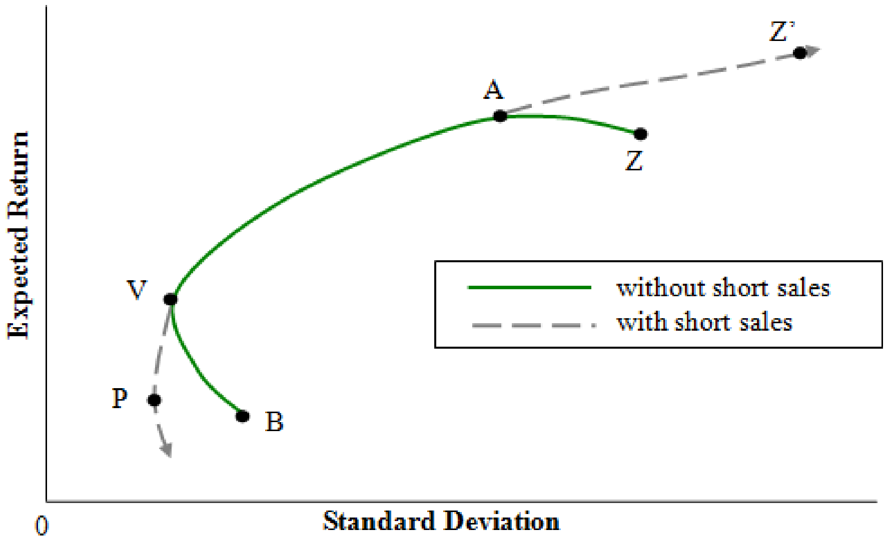

Image 7: MVP and Efficient Frontier with short selling (Lee, Lee, Lee, 2010)

Constructing efficient portfolios

In order to find the minimum variance portfolio weights, one can minimize the Lagrange function F for portfolio variance (Lee, Lee, Lee, 2010).

$$Min\ \sigma_p^2 = \sum_{k=1}^{N}\sum_{l=1}^{N} w_k w_l \rho_{kl}\, \sigma_k \sigma_l$$

Subject to $w_1+w_2+w_3+\ldots+w_{N-1}+w_N=1$

$$F = \sum_{k=1}^{N}\sum_{l=1}^{N} w_k w_l \sigma_{kl} + \lambda_1\left(1 - \sum_{k=1}^{N} w_l\right) \quad (5)$$

, where λ_1 is the Lagrange multiplier, ρ_{kl} the correlation coefficient between the asset returns, w_k, w_l the asset weights and σ_k and σ_l the assets standard deviation.

By adding a condition about the expected return, one can find other points of the efficient frontier curve (Lee, Lee, Lee, 2010).

$$Min\ \sigma_p^2 = \sum_{k=1}^{N}\sum_{l=1}^{N} w_k w_l \rho_{kl}\, \sigma_k \sigma_l$$

, Subject to

$\sum_{k=1}^{N} w_k\ \ E(R_k) = R^*$, where R*=expected return

$$\sum_{k=1}^{n} W_k\ = 1$$

The Lagrangian objective function is rewritten as follows:

$$F = \sum_{k=1}^{N}\sum_{l=1}^{N} w_k w_l \sigma_{kl} + \lambda_1\left[R^* - \sum_{k=1}^{N} w_k\, E(r_k)\right] + \lambda_2\left(1 - \sum_{k=1}^{N} w_k\right) \quad (6)$$

In case short selling is allowed, then the second constraint is replaced by $\sum_{k=1}^{N}|w_k| = 1$

and the Lagrangian function is the following (Lee, Lee, Lee, 2010):

$$F = \sum_{k=1}^{N}\sum_{l=1}^{N} w_k w_l \sigma_{kl} + \lambda_1\left[R^* - \sum_{k=1}^{N} w_k\, E(r_k)\right] + \lambda_2\left(1 - \sum_{k=1}^{N}|w_k|\right) \quad (7)$$

This altered constraint still demands the sum of the wealth to be 1 but allows for negative positions, in line with the essence of short selling. A similar alteration is needed for Minimum Variance Portfolio if short selling is allowed.

The final choice of the optimal portfolio among the efficient ones depends on the each investor's type and in particular on the risk tolerance of each investor. This is depicted in the investor utility curves. An investor is indifferent between portfolios on the same curve. The optimum portfolio for each investor is located on the intersection of the utility curve with the efficient frontier.

For instance, in the Image 8 portfolios A and B are the optimum for two different investors. Both portfolios are efficient as they are located on the efficient frontier. However investor B is less risk averse than A and therefore opts for a portfolio with higher risk (and higher expected return).

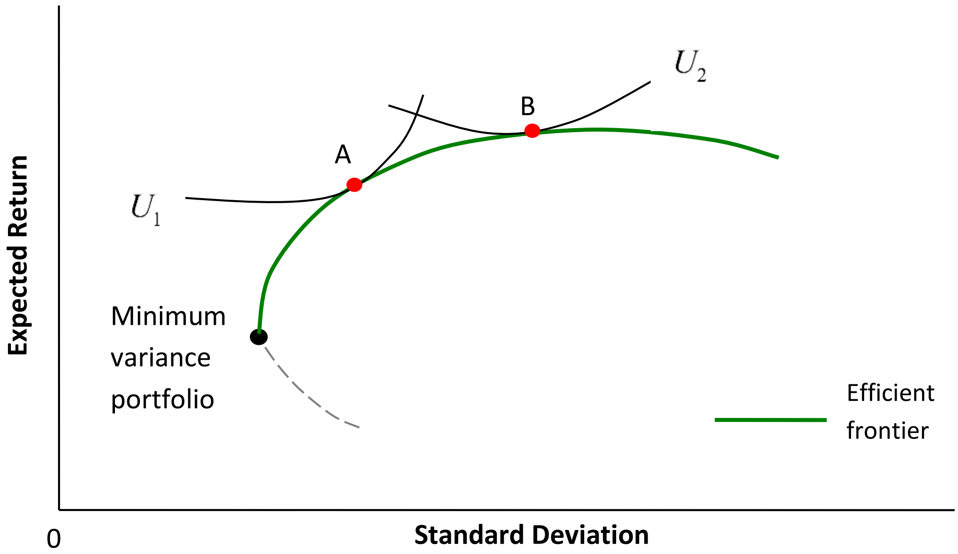

Image 8: Selection of the optimal portfolio (Lee, Lee, Lee, 2010)

Assessing Portfolio Performance

Investors often judge a portfolio's performance only by the return achieved. However, they should always keep in mind the risk incurred in order for this return to be attained. In literature, there are several ratios and formulas which are utilized in order for the performance of the portfolio to be evaluated. Two of the most basic ones, which take both the risk and the return into consideration, can be found below:

Sharpe Ratio

The Sharpe Ratio is a method for calculating the adjusted to risk return, developed by Nobel laureate William F. Sharpe (Sharpe, 1966). The ratio is calculated as follows:

$$S_k = \frac{r_k - r_f}{\sigma_k}, \quad (8)$$

,where r_k : the expected return of asset k

r_f: the risk free rate (e.g. Treasury bill interest rate)

σ_k: asset k standard deviation

Treynor Ratio

The Treynor Ratio (Treynor, Black, 1973) is similar to Sharpe Ratio in the sense that it calculates risk adjusted return. The difference is that instead of total risk (standard deviation), Treynor Ratio has systematic risk (beta) as its denominator.

$$T_i = \frac{r_k - r_f}{\beta_k} \quad (9)$$

Networks

Networks and Graphs

As stated by Newman (2010), “a network -also called a graph in the mathematical literature- is a collection of vertices joined by edges”. Vertices are also called nodes while edges are often reported as links. Barabasi (2016) defines a network as “a catalog of a system’s components often called nodes or vertices and the direct interactions between them, called links or edge”.

In most cases, there is at most one link between two nodes. If there is more than one edge between two vertices, then those edges that connect the same pair of vertices constitute a multiedge (Newman, 2010), while a network containing multiedges is called a multigraph (Newman, 2010). In addition, vertices are not usually connected to themselves; however, if they do, such a link connecting a node to itself is known as a self-edge or self-loop (Newman, 2010).

There are different ways in order for a network to be mathematically represented. The adjacency matrix belongs to the rudiments of those ways. The adjacency matrix A of a simple graph A is one with elements A_{kl} such that:

A_{kl} = 1, if there is a link between node *k* and node *l*

A_{kl}= 0, otherwise

An example of adjacency matrices can be found on image 9.

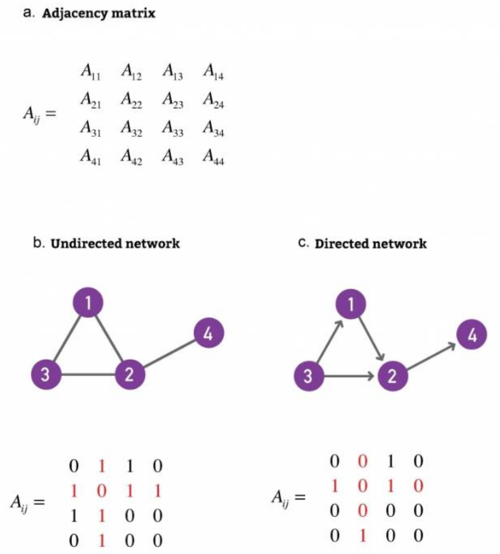

Image 9: Networks and Adjacency Matrices (Barabasi, 2016)

It can be noticed that networks containing no self-loops have adjacency matrices whose diagonal elements equal to zero.

It should also be mentioned that if the matrix is symmetric (such the matrix in b. Undirected network), then if there is an edge between k and l, there will be an edge between l and k as well. If this is not the case, (such as in matrix C. Directed Network), then the matrix is called asymmetric. In asymmetric matrices, the fact that there is a link from node k to l does not mean that there is also a node from l pointing to k.

Apart from networks whose adjacency matrix elements equal to either 1 or 0, there are also the so called weighted networks, in which each link from node k to node l has a unique weight w_{kl} (Barabasi, 2016). Weights are usually positive but there is no reason why they cannot be negative (Newman, 2010). An example of weighted adjacency matrix can be found below in image 10.

$$\mathbf{A} = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 0.5 \\ 1 & 0.5 & 0 \end{pmatrix}$$

Image 10: Example of a weighted adjacency matrix

A distinction should also be made between undirected and directed networks. A directed network or directed graph or a digraph is a network that contains edges pointing from one node to another. The adjacency matrix of a digraph is not symmetric (Newman, 2010).

Distance is also a fundamental notion when studying graph theory and describes numerically how remote objects are. Distance between objects belonging to a set X is a number given to any pair K,Λ ∈ X. The pair (X, d) is called metric space. Distance in space X is defined in terms of an equivalence (≈) between the objects of X, satisfying the below conditions:

d: YxY ->ℝ: (K, Λ) → d (K,Λ)

- Positivity: d(K,Λ) ≧ 0
- Identical beings are Indiscernible: K≈Λ => d(K,Λ)=0, d(K,Λ)>0 => K≉Λ
- Identity of Indiscernibles: d(K,Λ)=0 => K≈Λ
- Triangle Inequality: d(K,Λ)≤ d(K,Ψ)+d(Ψ, Λ)
- Symmetry: d(K,Λ)=d (Λ,K)

In statistics, networks and geometry two generalizations are applicable in order for real-world challenges to be met: the divergence meeting the first three aforementioned conditions and the asymmetric distance which satisfies the first four (Antoniou).

Graph Filtering

In literature, several methods have been proposed so that complex data sets are filtered out and a subgraph of representative links is extracted, being regarded as the key information. This need for filtering a densely connected graph has been found to be of great importance in the case of correlation networks, in which, if any filtering procedure is not present, links among all elements exist (Tumminello, Aste, Matteo, & Mantegna, 2005).

In order for the below to be better comprehended, it should be noted that a tree is a connected, undirected network containing no closed loops while a graph is planar if it is possible to draw it on a plane without any of its links crossing (Newman, 2010).

One of the most fundamental methods for graph filtering is the Minimum Spanning Tree (Mantegna, 1999). This filtering method results in a spanning subgraph of a connected, weighted, undirected graph. This subgraph connects all the nodes together; however no cycles are reported (the MST is a tree) and the selected edges have the minimum possible total edge weight.

Another filtering procedure is the Planar Maximally Filtered Graph (Tumminello et al., 2005). It is similar to the Minimum Spanning Tree, with the main difference being that the resulting spanning subgraph must be a planar graph.

The MST and the PMFG can be summarized in the below steps (Tumminello et al., 2005). First, a similarity measure between the different nodes is set. For example, in case of correlation networks, Pearson correlation coefficient can be such a measure. Then, a list G is created by sorting the similarities in a decreasing order. After the sorting, in order for an MST to be constructed, starting from the very first element of G, the respective edge is added if and only if the graph remains a forest (acyclic graph consisting only of trees) or a tree. In order for a PMFG to be constructed, the process is similar. However, after creating the list G and starting from its very first element, the respective edge is added if and only if the produced graph remains planar.

Centrality/Peripherality Measures

Degree Centrality- Strength

The concept of centrality deals with the issue of finding the most important or the most central nodes in a network. Degree centrality is the simplest measure of all. Degree centrality refers to the degree of a vertex; in other words, degree is the number of edges connected to a vertex. In case of directed networks, in-degree and out-degree are applicable and refer to the number of edges stemming from or pointing to a vertex respectively. In case of weighted networks, the strength is computed. Strength is the sum of the weights of the edges with which a node is connected. Strength of a vertex k in a graph with adjacency matrix α_{kl} and N vertices is defined as follows (Barrat, Barthélemy, Pastor-Satorras, Vespignani, 2004)

$$s_k = \sum_{l=1}^{N} a_{kl} w_{kl} \qquad (10)$$

Eigenvector Centrality

Eigenvector centrality (Bonacich, 1987) is actually an extension of degree centrality. Degree (or strength) does not discriminate between the nodes connected to a vertex; in other words, it assigns the same importance to all connections. This is not the case with eigenvector centrality. Many times, a nodes' importance exists in the fact that it is connected to nodes which are themselves important, which is exactly the concept behind eigenvector centrality. The latter assigns each vertex a score which is proportional to the sum of the scores of the vertices with which a vertex is connected. For a graph G:= (V,E) with $W=(w_{k,l})$ its weighted adjacency matrix, the relative centrality score of a vertex k can be defined as:

$$x_k = \frac{1}{\lambda} \sum_{l \in G} w_{k,l} x_l \qquad (11)$$

,with λ being a constant. This can be also written as the eigenvector equation

$$Wx=\lambda x \qquad (12)$$

There may be several eigenvalues; the λ, however, is the largest eigenvalue and the eigenvector centrality is the eigenvector corresponding to the largest eigenvalue. Eigenvector centrality is computed for both directed and undirected networks. In case of directed networks, the right eigenvector refers to the in-eigenvector centrality, while the left eigenvector to the out-eigenvector (Newman, 2010). The out-eigenvector can be computed by computing the right eigenvector of the transposed adjacency matrix. It should also be noted that mathematically, only nodes of a strongly connected component of at least two nodes or the out-component of such a component can have an eigenvector centrality different from zero. It should thus be inferred that acylic networks –which have no such strongly connected components- will have all their vertices have zero eigenvector centrality. Therefore eigenvector centrality is considered to be of no use for acyclic networks (Newman, 2010).

Closeness Centrality

Closeness centrality demonstrates the average distance of a node from the others nodes in the network. If d_{kl} is the geodesic path from vertex k to vertex l, then closeness centrality of a vertex k is calculated as follows (Freeman, 1978)

$$c_k = \frac{1}{\sum_{k \neq l} d_{kl}} \qquad (13)$$

In other words, one can say that closeness is the reciprocal of farness. In case of directed networks, in-closeness and out-closeness are defined accordingly.

Two issues are often reported with regards to closeness. The first is that the values tend to have a small range from the smallest to largest, thus rendering the discrimination between central and less central vertices relatively difficult (Newman, 2010). The second one is that if two vertices are in different components or if there is no path between two vertices (in case of directed networks), the distance between those two vertices is infinite and the C_k is zero. A common practice in order for this issue to be eschewed is the computation of closeness only inside the different components or the computation of the harmonic mean distance (the mean of the inverse distances). Another practice, which is implemented in this thesis, is to list the total number of vertices instead as the path length in case there is no (directed) path between two vertices.

Betweenness Centrality

Betweenness centrality (Freeman, 1977) measures how much a node is located in paths between other nodes. The betweenness centrality of a node k is calculated as follows:

$$b_k = \sum_{m,n: m \neq k \neq n} \frac{\delta_{mn}(k)}{\delta_{mn}} \qquad (14)$$

δ_{mn}: total number of shortest paths from vertex m to vertex n

$\delta_{mn}(k)$: total number of those shortest paths which pass through k

Eccentricity

The eccentricity of a node is the highest geodesic distance of this node and any other node. In other words, eccentricity captures how far a vertex is from the most distant vertex in the graph. In case of directed networks, in-eccentricity and out-eccentricity are applicable.

Networks and portfolio selection in literature

Network theory has also been exploited in the portfolio selection process. Pozzi, Di Matteo and Aste (2013) have initiated the use of the network centrality measures in order to build well diversified portfolios. Working on equity data from the American Stock Exchange market, they first identify moving weighted correlation networks. Further to the construction of the dependency matrices, the authors proceed with the identification of filtered networks. In order to filter the matrices, they use two very well-known tools: the Minimum Spanning Tree (Mantegna, 1999) and the Planar Maximally Filtered Graph (Tumminello et. al, 2005). Having identified the moving filtered networks, they compute the following centrality and peripherality indices: degree, betweenness, eccentricity, closeness and eigenvector. Their proposed strategy involves the selection of nodes of low centrality in favor of highly centralized ones. They also come up with two synthetic centrality indices which are found to be performing better than the regular centralities. The main finding is that investors should opt for stocks that belong to the network periphery, with the network centralities being the criterion of selection. This strategy is found to be resulting in portfolios of lower risk and better returns in comparison to portfolios constructed with other traditional methods.

Peralta and Zareei (2016) also try to exploit networks in the portfolio selection process. Their method has many similarities with the one of Pozzi et al (2013), as they also propose the use of nodes centrality for asset selection. The originality of their work is found in the following two factors: i) instead of the synthetic centrality indices they use eigenvector centrality, ii) they come up with an investing method that is said to be more in accordance with the logic of the Markowitz model. They argue that Pozzi et al. (2013) do not take the individual performance of assets into consideration, which can have a negative impact on the portfolio performance. For this reason they propose the so- called ρ-dependent strategy. According to this strategy, the correlation between the nodes' centrality and their Sharpe Ratio is computed. If the correlation between the centrality and Sharpe ratio values is found to be below a certain threshold, they naively invest in the 20 stocks with the lower centrality (which is similar to Pozzi et al, 2013). If, however, ρ exceeds a certain limit, then they naively invest in the 20 stocks with the higher centrality.

Information Theory

Information theory studies -among other issues- the quantification of information. A basic notion of information theory is information entropy (Shannon, 1948). The latter refers to the mean amount of information which a probabilistic stochastic source of data produces. If we let $x_1^\nu, x_2^\nu, \ldots, x_{n_\nu}^\nu$ be the $\tilde{n}_\nu \leq T$ distinct observed values of each Variable X_ν , ν=1,2,…,N, Shannon's (1948) entropy is defined as follows:

$$\tilde{\mathcal{S}}_\nu = -\sum_{i=1}^{n_\nu} \tilde{\rho}(x_i^\nu) log_2 \tilde{\rho}(x_i^\nu) \qquad (15)$$

Information entropy is often regarded as a measure of uncertainty. Joint Shannon entropy which measures such an uncertainty characterizing a set of variables is defined for two variables X_κ , X_λ as:

$$\tilde{\mathcal{S}}_{\kappa\lambda} = -\sum_{i=1}^{\tilde{n}_\kappa}\sum_{j=1}^{\tilde{n}_\lambda} \tilde{\rho}\left(x_i^\kappa, x_j^\lambda\right) log_2 \tilde{\rho}\left(x_i^\kappa, x_j^\lambda\right) \quad (16)$$

, where $\max\left\{\tilde{\mathcal{S}}_\kappa, \tilde{\mathcal{S}}_\lambda\right\} \leq \tilde{\mathcal{S}}_{\kappa\lambda} \leq \tilde{\mathcal{S}}_\kappa + \tilde{\mathcal{S}}_\lambda$. Joint entropy is not a distance as it does not satisfy all the below conditions (Antoniou):

- Positivity: $\mathcal{S}$[K,Λ]≧0
- $\mathcal{S}$[K,K]=$\mathcal{S}$[K]≠0 **Not satisfied**
- $\mathcal{S}$[K,Λ] = 0 ⟺ K,Λ are deterministic **Not satisfied**
- Triangle Inequality: $\mathcal{S}$ [K ,Z] ≤ $\mathcal{S}$ [K ,Λ] + $\mathcal{S}$ [Λ ,Z]

- Symmetry: $\mathcal{S}$ [K ,Λ] = $\mathcal{S}$ [Λ , K]

Mutual information $\mathcal{S}$[K, Λ] of two variables X_κ , X_λ is the defined as the sum of their information entropies $\tilde{\mathcal{S}}_\kappa$ and $\tilde{\mathcal{S}}_\lambda$ minus their joint entropy $\tilde{\mathcal{S}}_{\kappa\lambda}$ or else:

$$\mathcal{S}[K, \Lambda]= \tilde{\mathcal{S}}_\kappa+\tilde{\mathcal{S}}_\lambda-\tilde{\mathcal{S}}_{\kappa\lambda} \quad (17)$$

Mutual Information is not a distance since it only satisfies the first and the fifth property (Antoniou):

- Positivity: $\mathcal{S}$[K ;Λ]≧0
- $\mathcal{S}$ [K; K] = $\mathcal{S}$ [K] **Not satisfied**
- $\mathcal{S}$ [K; Λ] = 0 ⟺ K,Λ Independent **Not satisfied**
- Triangle Inequality: $\mathcal{S}$ [K ; Z] ≤ $\mathcal{S}$ [K ; Λ] + $\mathcal{S}$ [Λ ; Z] **Not satisfied**
- Symmetry: $\mathcal{S}$ [K ; Λ] = $\mathcal{S}$ [Λ; K]

Conditional Entropy, illustrating the uncertainty about a variable K after observing a variable Λ is defined as follows:

$$\mathcal{S}[\Lambda|K]=\mathcal{S}[K ,\Lambda]- \mathcal{S}[K] \quad (18)$$

Conditional entropy satisfies the first properties and is thus regarded as an asymmetric distance (Antoniou):

- Positivity: $\mathcal{S}$[K,Λ]≧0
- $\mathcal{S}$ [K|K] = 0
- $\mathcal{S}$ [K|K] = 0 ⟺K=φ(B) ⟹ K~Λ
- Triangle Inequality: $\mathcal{S}$ [K |Z] ≤ $\mathcal{S}$ [K |Λ] + $\mathcal{S}$ [Λ |Z]

The sum

$$\mathcal{S}[K|\Lambda] + \mathcal{S}[\Lambda|K] = d(K ,\Lambda) \qquad (19)$$

of the conditional entropies of two random variables defines a distance in random variables algebra and is known as Rokhlin information distance [Rokhlin, 1961;

Rokhlin, 1967; Martin, England, 1981; Katok, 2007]. This sum satisfies the following properties (Antoniou):

- Positivity: d(Κ ,Λ)≧0
- Identical beings are Indiscernible: Κ≈Λ ⟹d(Κ ,Λ)=0
- Identity of Indiscernibles: d(Κ ,Λ)= 0 ⟹ Κ≈Λ
- Triangle Inequality: d(Κ ,Λ)≦ d(Κ ,Z)+ d(Z ,Λ)
- Symmetry: d(Κ ,Λ)=d(Λ ,Κ)

Information distance can take the following values:

$$0\leq\mathcal{S}[\mathrm{K}|\Lambda]+\mathcal{S}[\Lambda|\mathrm{K}]\leq\mathcal{S}[\mathrm{K},\Lambda]\leq\mathcal{S}[\mathrm{K}]+\mathcal{S}[\Lambda] .$$

In order for a distance with values from 0 to 1 to be available, the following normalized distance is applicable:

$$d^R(K,\Lambda)=\frac{S[K|\Lambda]+S[\Lambda|K]}{S[K,\Lambda]}=2-\frac{S[K]+S[\Lambda]}{S[K,\Lambda]}=1-\frac{S[K;\Lambda]}{S[K,\Lambda]} \qquad (20)$$

Mutual information and Pearson Correlation Coefficient achieve maximum values in case of deterministically dependent variables. This can be estimated through affinity, similarity and proximity, which are defined by the distance in a manner that objects with large affinity will have a small distance (Deza, Deza, 2013)

Affinity between Κ,Λ in a set X is a number w: (Κ,Λ)⟼ w (Κ->Λ)=$w_{\mathrm{K}\Lambda}$, satisfying the following properties (Antoniou):

- A1. Values: −1≤w(Κ→Λ)≤1
- A2. $w_{\mathrm{K}\Lambda}$=1 Λ is positively determined by Κ
- A3. $w_{\mathrm{K}\Lambda}$>0 Λ positively depends on Κ
- A4. $w_{\mathrm{K}\Lambda}$=0 Λ is not influence by Κ
- A5. $w_{\mathrm{K}\Lambda}$<0 Λ negatively depends on Κ
- A6. $w_{\mathrm{K}\Lambda}$=−1 Λ is negatively determined by Κ

Similarity is the symmetric affinity A7. $w_{\mathrm{K}\Lambda}$=$w_{\mathrm{B}\Lambda}$, while proximity is the positive symmetric affinity A8. $w_{\mathrm{K}\Lambda}$≥0 .

In order for someone to switch between proximity to distance and vice versa, subtracting from 1 is applicable as follows:

- Proximity from distance: w=1-d
- Distance from proximity: d=1-w

Apart from mutual information, normalized mutual information (proximity), defined as follows:

$$\tilde{I}_{\kappa\lambda} = \frac{\tilde{S}_{\kappa} + \tilde{S}_{\lambda} - \tilde{S}_{\kappa\lambda}}{\min\{\tilde{S}_{\kappa}, \tilde{S}_{\lambda}\}} \qquad (21)$$

,where $0 \le \tilde{I}_{\kappa\lambda} \le 1$.

Based on information distance, the following proximity can be defined, named as information dependence

$$\widetilde{K}_{\kappa\lambda} = \frac{\tilde{S}_{\kappa} + \tilde{S}_{\lambda} - \tilde{S}_{\kappa\lambda}}{\tilde{S}_{\kappa\lambda}} \qquad (22)$$

,where $0 \le \widetilde{K}_{\kappa\lambda} \le 1$.

The above formula is extracted as follows:

$$\widetilde{K}_{\kappa\lambda} = 1 - d^{R}(K,\Lambda) = \frac{S[K|\Lambda] + S[\Lambda|K]}{S[K,\Lambda]} = \frac{S[K] + S[\Lambda] - S[K,\Lambda]}{S[K,\Lambda]} = \frac{S[K;\Lambda]}{S[K,\Lambda]} \qquad (23)$$

Finally, the following positive affinity is also applicable, originating from conditional entropy and named as Information Dependence:

$$\widetilde{\Pi}_{\kappa\lambda} = \frac{\tilde{S}_{\kappa} + \tilde{S}_{\lambda} - \tilde{S}_{\kappa\lambda}}{\tilde{S}_{\kappa}} \qquad (24)$$

, where $0 \le \widetilde{\Pi}_{\kappa\lambda} \le 1$. The above formula is extracted as follows:

$$\widetilde{\Pi}_{\kappa\lambda} = 1 - \mathcal{I}[\Lambda|K] = 1 - \frac{S[\Lambda|K]}{S[\Lambda]} = \frac{S[K] + S[\Lambda] - S[K,\Lambda]}{S[\Lambda]} = \frac{S[K;\Lambda]}{S[\Lambda]} \qquad (25)$$

Critique on existing literature

As already mentioned, Markowitz’s work has been groundbreaking and in the field of portfolio optimization. The impact of his work was so remarkable that he received a Nobel Prize in Economics in 1990.

However, there are some serious issues to point out when it comes to the application of the theory. It has been recognized that a Markowitz optimal portfolio is highly sensitive to even a small alteration in expected asset returns (Black and Litterman, 1992). Jorion (1985) found that changing a few observations in the sample is able to significantly alter the asset allocation. Best and Grauer (1991) go on to note that a mere slight increase in an asset mean can lead to exclusion of half of the assets from the portfolio. Nevertheless, such a dramatic change will only slightly affect the portfolio expected return and volatility.

This issue of over-sensitivity to returns becomes more important if we consider the following point. Someone has only a finite number of past observations, based on which the expected returns are computed. However, since many years there is evidence that a historical average is not an accurate estimator of future returns (Merton, 1980; Jorion, 1985). Similar estimation errors have been reported regarding the covariance matrix (Jobson and Korkie, 1980).

However, Chopra and Ziemba (1993) also noted that for an investor with a mediocre affinity for risk, mean-variance optimization is eleven times more sensitive to estimation error in returns in contrast to estimation error in risk (variance), while the model is two times more prone to estimation error in risk (variance) in comparison to estimation error in covariance.

Not only are the inputs prone to estimation error, but it has also been reported that the Markowitz model as an optimization procedure has an error-maximizing property (Michaud, 1989). In other words, it has the tendency to increase the influence of estimation errors as it assigns high weights to assets with high expected returns, small variance or negative covariance.

In addition, according to Sharpe (1964), investors should hold the market portfolio as this is theoretically the one achieving the highest Sharpe ratio. One may be thus tempted to note that according to theory there is no point in ignoring returns and focusing solely on the minimization of risk (Scherer, 2010). Nevertheless, there is significant empirical evidence of low risk stocks performing better than high risk ones and of the minimum variance portfolio outperforming the market one. (Haugen and Baker, 1991; Blitz and van Vliet, 2007; Clarke et. al, 2006). This is another argument in favor of excluding expected returns from portfolio selection.

An important point to be made about network theory in portfolio selection is that the usability of financial filtered networks could turn out questionable. Pozzi et. al (2013) use both the Minimum Spanning Tree and the Planar Maximally Filtered Graph while Peralta and Zareei (2016) choose only the Minimum Spanning Tree in order to filter the correlation matrix. The reason of this choice is the reduction of data complexity as a fully connected network is regarded as difficult to analyze. However, the analyst has to wonder whether such filtering is in line with the research objective and whether this is a proper way to handle correlations from a finance point of view. Tse et. al (2010) report that both MST & PMFG are characterized by serious information loss. The reason for this is that high correlations edges can be neglected while low correlation ones are kept in order to satisfy the topological criteria set. They mention that this reduces the usefulness of the aforementioned filtered networks, especially regarding their ability to identify the correlations among assets. This ability is of the utmost importance in portfolio construction, whereas restrictions such as the filtered network being a tree or planar do not seem to add anything to the analysis as far as finance is concerned. Instead, if high correlations are removed so that such criteria are satisfied, this can turn out to be detrimental to the selected portfolio performance.

However, the most important issue to point out is the choice of the correlation measure. Both Markowitz and the existing literature in networks & portfolio selection try to identify the correlations between the asset returns with the Pearson correlation coefficient. The latter refers to a linear measure of dependence between the two variables. However, as stated by Fiedor (2014) there is strong evidence in literature that financial markets are characterized by non-linearity. Fiedor (2014) comes up with a series of examples from literature having provided us with evidence of non-linearity in financial markets. Such examples include rates of return in commodities (Frank, Stengos, 1989), currency rate changes (Hsieh, 1989; Brock, Hsieh, LeBaron,1991; Meese,Rose,1991; Brooks, 1996; Qi, Wu, 2003), financial indexes (Scheinkman, Lebaron, 1989), the FTSE-100 index (Abhyankar, Copeland, Wong, 1995). Stock returns [Qi, 1999; McMillan, 2001; Sornette, Andersen, 2002; Oh, Kim, 2002] and market index returns [Franses, Van Dijk, 1996; Chen, 1996; Abhyankar, Copeland, Wong, 1997; Ammermann, Patterson 2003], which are more close to the scope of this thesis, have also been found to be characterized from non-linearity. Furthermore, apart from symmetric relations between stocks, directed ones could also be examined. Lead-lag relationships have already been studied in financial literature [Kullmann, Kertesz, Kaski, 2008; Curme et al., 2014; Sandoval, 2013; Billio et. al, 2012] As also stated by Peralta & Zareei (2016), who have already adopted network theory in portfolio selection, the construction of directed networks of stocks is regarded as extremely appealing. Therefore, the identification of directed networks as well as symmetric networks through non-linear measures does seem to have very valid grounds.

Data

The dataset used consists of daily closing prices of 185 highly capitalized stocks of the S&P-500 index, which demonstrate non-negative total equity from October to December 2012. Stocks chosen are mentioned in the appendix. Data source is Datastream by Thomson Reuters. Testing time period is between 01/10/2002 and 31/12/2012. This is a dataset resembling the one used by Peralta & Zareei (2016).

Methodology

For all stocks of the dataset, daily logarithmic returns are computed as follows:

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right) \qquad (26)$$

- r_t : the daily logarithmic return
- p_t: price of a stock at day t
- p_{t-1}: price of a stock t-1

The returns are split in overlapping windows of 125 observations, with the oldest 25 observations being replaced by 25 new ones as we move from window to window. For each of the windows, the networks mentioned in Table 2 are identified.

Table 2: List of Networks identified and studied

Net I: Pearson correlation with negative values replaced by zero	$\tilde{r}^{+}_{\kappa\lambda} = [\![\tilde{r}_{\kappa\lambda} > 0]\!] \cdot \tilde{r}_{\kappa\lambda}$	$0 \leq \tilde{r}^{+}_{\kappa\lambda} \leq 1$
Net II: Absolute values of Pearson correlation	$\lvert\tilde{r}\rvert_{\kappa\lambda}$	$0 \leq \lvert\tilde{r}\rvert_{\kappa\lambda} \leq 1$
Net III: Normalized Mutual Information	$\tilde{I}_{\kappa\lambda} = \dfrac{\tilde{S}_{\kappa} + \tilde{S}_{\lambda} - \tilde{S}_{\kappa\lambda}}{\min\{\tilde{S}_{\kappa}, \tilde{S}_{\lambda}\}}$	$0 \leq \tilde{I}_{\kappa\lambda} \leq 1$
Net IV Directed Normalized Mutual Information	$\tilde{I}_{\kappa\to\lambda} = \tilde{I}_{\kappa\lambda}[\![\tilde{S}_{\kappa} \geq \tilde{S}_{\lambda}]\!]$	$0 \leq \tilde{I}_{\kappa\to\lambda} \leq 1$

Net V: Information Interdependence	$\widetilde{K}_{\kappa\lambda} = \dfrac{\tilde{S}_\kappa + \tilde{S}_\lambda - \tilde{S}_{\kappa\lambda}}{\tilde{S}_{\kappa\lambda}}$	$0 \le \widetilde{\boldsymbol{K}}_{\kappa\lambda} \le 1$
Net VI: Information Dependence (Asymmetric)	$\widetilde{\Pi}_{\kappa\lambda} = \dfrac{\tilde{S}_\kappa + \tilde{S}_\lambda - \tilde{S}_{\kappa\lambda}}{\tilde{S}_\kappa}$	$0 \le \widetilde{\Pi}_{\kappa\lambda} \le 1$

As shown in the third column of Table 2, the networks are weighted. Net I, NetII, NetIII and Net V are symmetric, while Net IV and Net VI are asymmetric. The networks are not exposed to any filtering due to the reasons described in the literature review section. The formulas used for the aforementioned networks are depicted on Table 3.

Table 3: Formulas for the different networks

$\tilde{r}_{\kappa\lambda} = \begin{Bmatrix} 0, & if \left(\sum_{\tau=1}^{T} \lvert\chi_\tau^\kappa - \widetilde{m}_\kappa\rvert\right) \cdot \left(\sum_{\tau=1}^{T} \lvert\chi_\tau^\lambda - \widetilde{m}_\lambda\rvert\right) = 0 \\ \dfrac{\sum_{\tau=1}^{T}(\chi_\tau^\kappa - \widetilde{m}_\kappa)(\chi_\tau^\lambda - \widetilde{m}_\lambda)}{[\sum_{\tau=1}^{T}(\chi_\tau^\kappa - \widetilde{m}_\kappa)^2]^{1/2}[\sum_{\tau=1}^{T}(\chi_\tau^\lambda - \widetilde{m}_\lambda)^2]^{1/2}}, & otherwise \end{Bmatrix}$	**(27)- Pearson Correlation**
$\widetilde{m}_\nu = \dfrac{1}{T}\sum_{\mu=1}^{T} \chi_\tau^\nu$	**(28)-the empirical mean of the Variable X_ν**
$[\![Statement]\!] = \begin{Bmatrix} 1, if\ Statement\ is\ true \\ 0, if\ Statement\ is\ not\ true \end{Bmatrix}$	**The Iverson bracket of the** $Statement$ **[Knuth D.1992]**
$x_1^\nu, x_2^\nu, \ldots, x_{n_\nu}^\nu$	**the $\tilde{n}_\nu \le T$ distinct observed values of each Variable X_ν , ν=1,2,…,N**
$\tilde{S}_\nu = -\sum_{i=1}^{n_\nu} \tilde{\rho}(x_i^\nu) log_2 \tilde{\rho}(x_i^\nu)$	**The empirical Entropy of the Variable X_ν** $0 \le \tilde{S}_\nu \le log_2(\tilde{n}_\nu)$
$\tilde{\rho}(x_i^\nu) = \dfrac{\sum_{\tau=1}^{T}[\![\chi_\tau^\nu = x_i^\nu]\!]}{T}$, **i=1,2,…, $\tilde{n}_\nu$**	**(30)- The empirical probability of the variable X_ν**

$\tilde{S}_{\kappa\lambda} = -\sum_{i=1}^{\tilde{n}_\kappa}\sum_{j=1}^{\tilde{n}_\lambda} \tilde{\rho}(x_i^\kappa, x_j^\lambda)\, log_2\tilde{\rho}(x_i^\kappa, x_j^\lambda)$	**The Empirical Joint Entropy of the Variables** X_κ , X_λ $\max\{\tilde{S}_\kappa , \tilde{S}_\lambda\} \leq \tilde{S}_{\kappa\lambda} \leq \tilde{S}_\kappa + \tilde{S}_\lambda$
$\tilde{\rho}(x_i^\kappa, x_j^\lambda) = \frac{\sum_{\tau=1}^{T} [\![\chi_\tau^\kappa = x_i^\kappa]\!] [\![\chi_\tau^\lambda = x_j^\lambda]\!]}{T}$	**(31)- The empirical joint probability of the Variables** X_κ , X_λ

For the purpose of identification of Networks III, IV, V, VI, the observations are discretized into 8 distinct states. This is a number chosen in previous literature, too (Navet, Chen, 2008). Besides, the binning in 8 states was chosen based on the famous Sturges' rule (Sturges, 1926), which indicates the optimal number of bins (denoted here as k):

$$k=1 +\log_2(N) \quad (32)$$

Furthermore, as shown by David Scott (Scott, 2009) , Sturges' rule concides with some more modern rules such as the Terrell-Scott inequality for the optimal number of bins for sample sizes $n \approx 100$.

For each of the windows and the respective networks identified, the following are computed -where applicable- for each of the stocks- vertices:

- Strength, in-Strength, out-Strength
- in-Eigenvector, out-Eigenvector
- Betweenness
- Closeness, in- Closeness, out- Closeness
- Eccentricity, in- Eccentricity, out- Eccentricity

For the degree and eigenvector centrality, the link between vertex κ and vertex λ is $\tilde{r}^+_{\kappa\lambda}$, $|\tilde{r}|_{\kappa\lambda}$, $\tilde{I}_{\kappa\lambda}$, $\tilde{I}_{\kappa\to\lambda}$, $\tilde{K}_{\kappa\lambda}$, $\tilde{\Pi}_{\kappa\lambda}$ depending on the network tested, while for betweenness, closeness and eccentricity the link between vertex κ and vertex λ equals to 1 minus the aforementioned weights ($\tilde{r}^+_{\kappa\lambda}$, $|\tilde{r}|_{\kappa\lambda}$, $\tilde{I}_{\kappa\lambda}$, $\tilde{I}_{\kappa\to\lambda}$, $\tilde{K}_{\kappa\lambda}$, $\tilde{\Pi}_{\kappa\lambda}$).

After the above centrality/peripherality measures are computed, the stocks-vertices with the 20 highest and the 20 lowest values are chosen per window/network.

Thus, two portfolios are constructed per window/network, one from the most central and one from the least central stocks-vertices.

In order for the performance of those portfolios to be tested (short selling is prohibited), the performance criteria mentioned in Table 4 are computed for each of them for a holding period of 51, 52, 53…248, 249, 250 days:

Table 4: Portfolio performane criteria

Return (33)	$R_p = \sum_{k=1}^{N} w_k \, E_{(r_k)}$
Variance (34)	$\mathrm{Var}_{(r_p)} = \sigma_P^2 = \sum_{k=1}^{N}\sum_{l=1}^{N} w_k \, w_l \sigma_{kl}$
Beta (35)	$\beta_p = \sum_{k=1}^{N} w_k \beta_k$
Risk adjusted return (36)	$S_P' = \frac{R_p}{\sigma_p}$
Systemic risk adjusted return (37)	$T_k' = \frac{R_p}{\beta_p}$

S_P' and T_k' are similar to Sharpe and Treynor ratios respectively. The only difference is that the risk free rate is not included. Risk free rate was excluded as it would not add anything to the analysis, it terms of which network is the best-performing one.

Then, for each network & centrality/peripherality measure, I have computed the mean of the results for each performance criterion per holding day. The images in the results section as well as in the appendix namely depict how the portfolios stemming from each network type and network centrality have performed on average per holding day in terms of each of the 5 performance criteria set for the whole testing period. The same methodology has also been applied only for the financial crisis period (August 2007-March 2009) in separate, so as to check which networks and centrality measures perform better at times of extreme volatility.

Results

One of the first points to be made is that, in accordance with the findings of Pozzi et. al. (2013) portfolios made of stocks/vertices with lower strength, closeness and eigenvector centrality perform better in terms of return, risk and adjusted to risk return in contrast to stocks/vertices of high centrality values. The same is the case with stocks/vertices of higher eccentricity in comparison to the ones of lower eccentricity.

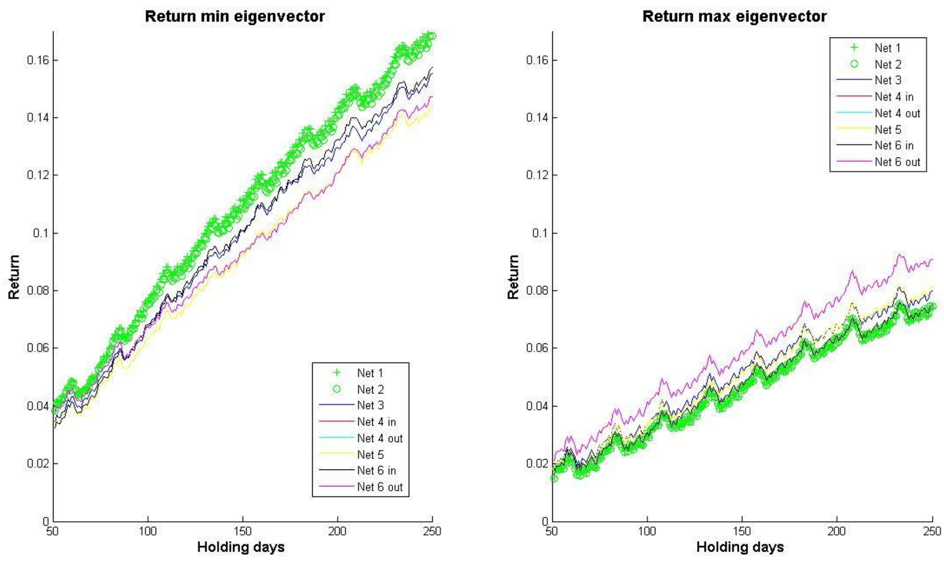

Image 11: First example- Less central vertices outperform more central ones (higher return)

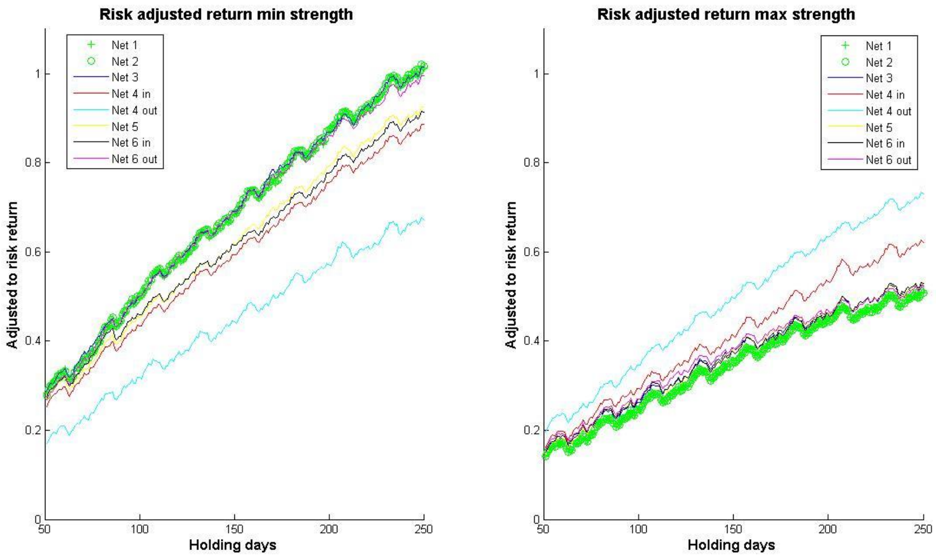

Image 12: Second example- Less central vertices outperform more central ones (higher return adjusted to risk)

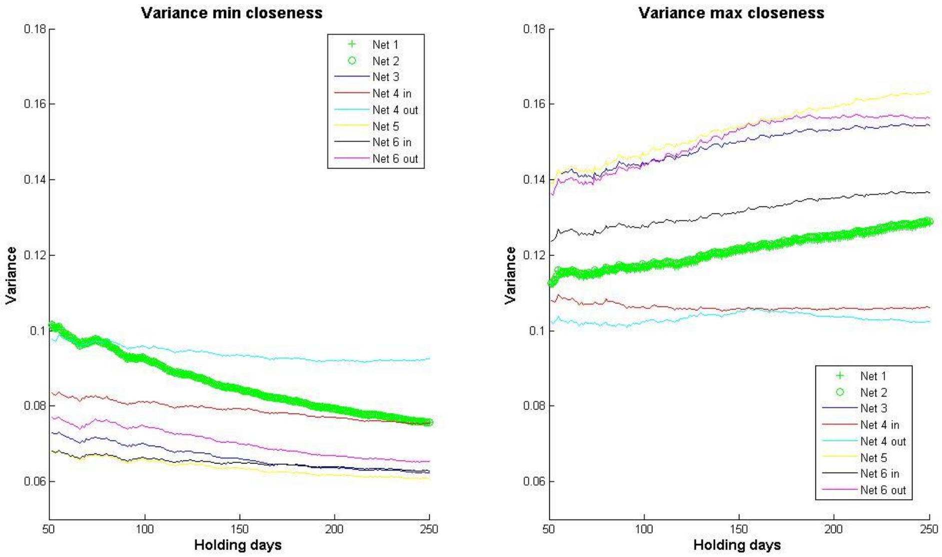

Image 13: Third example-Less central vertices outperform more central ones (lower risk)

Regarding strength, the nets recognized by Pearson Correlation have in most cases the best results in Return. In only 9.5% of the cases (all of them for holding periods of less than 150 days) the results of Net 6 in are better. However, the case is exactly the opposite when risk criteria and not return are set as targets. Net 6 out produces the best results in 100% of the testing holding periods regarding beta, while Net 5 the best results for variance in 99% of the holdings periods. Net 3's results are also close to optimal regarding beta and variance while Net 6 in also demonstrates satisfactory results regarding variance. The results regarding risk-adjusted return are mixed between linear and non-linear measures. As shown in the relevant graph, too, the results of Nets 1 &2 almost coincide with those of Net 3 and Net 6 out, while the percentage table reveals that the split between linear and non-linear nets is close to 50% - 50%. The same networks are the optimal for systemic risk adjusted return as well. However, the networks recognized by non-linear measures produce the best results in most of the cases as shown by both the respective image and the percentage table (Net 6 out being the best in 67.5% of the cases)

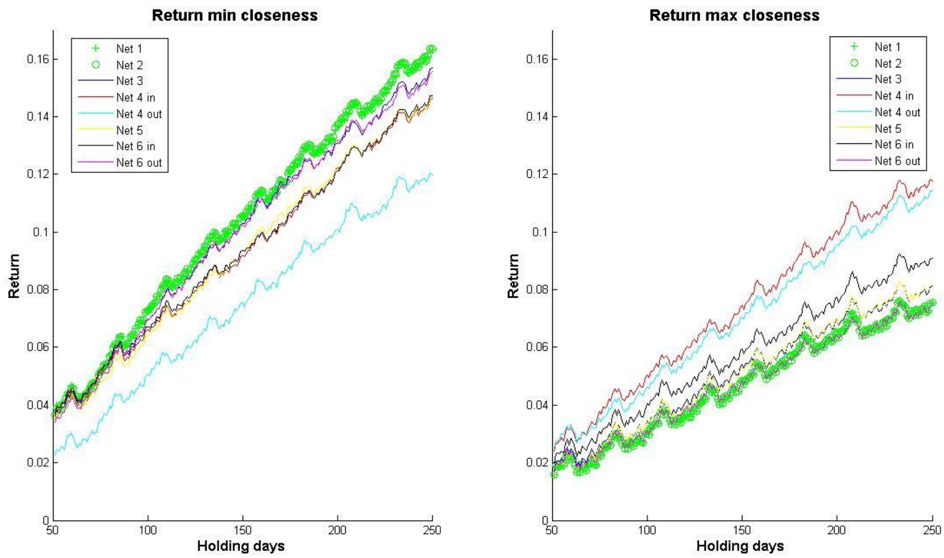

Image 14: Closeness- Pearson correlation networks outperform the rest in terms of return

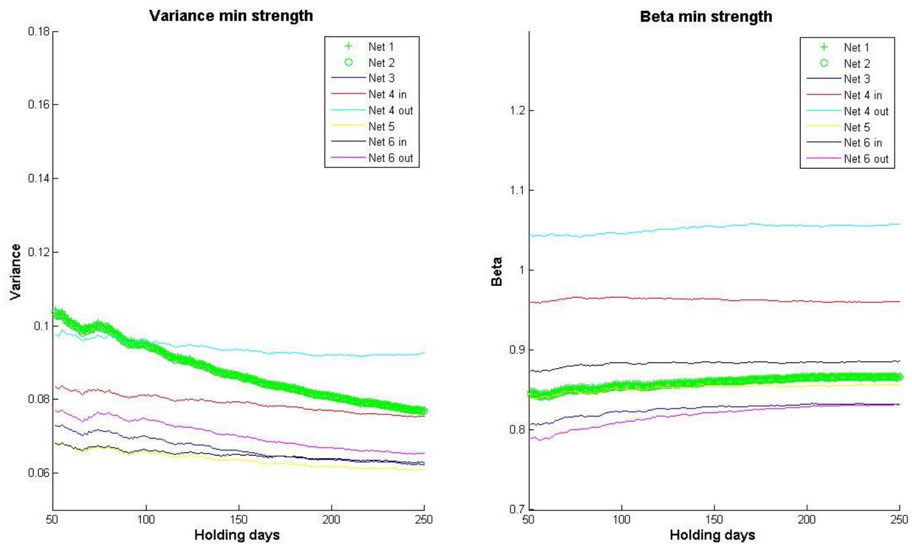

Image 15: Strength- Networks identified by non-linear measures outperform the rest in terms of risk

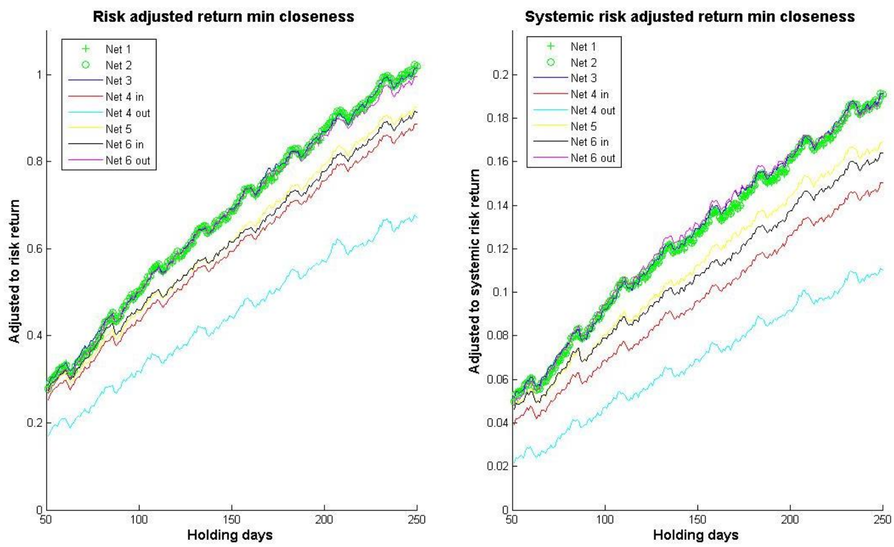

Image 16: Closeness- The best non-linear measures perform equally well or sometimes better than the linear measures in terms of adjusted to risk return

The results of closeness centrality demonstrate significant similarity with those of the strength. The Pearson Correlation Networks 1&2 produce again the best results in Return, while the bigger the holding period, the bigger the gap between those aforementioned networks and the second best performing Net 3 and Net 6 out. This is identical to the image of the strength return results. While the linear measures stemming networks are the best performing ones about return, the non-linear stemming ones are -exactly as for strength- the best performing ones about risk measures. Net 3 produces satisfactory -though not the best- results concerning both beta and variance; Net 6 (in) is among the best options for variance while the best results stem again from Net 5 and Net 6 (out) for variance and beta respectively. The results for adjusted to (systemic) risk return are no objection to the general empirical result of similarity between strength and closeness. Nets 1,2 as well as Net3 and Net 6 (out) in are among the best performing regarding risk adjusted return, with the best performance per holding day being almost equally split between linear and non-linear measures. Just like strength, the same networks are the best performing for systemic risk adjusted return as well, with the non-linear Net 6 (out) being the best performing one for the majority of the holding days.

Although not identical to the results of strength and closeness, the results of eigenvector centrality also have significant similarities with those of the aforementioned centrality measures. Regarding return, Nets 1 & 2 are the best performing ones. Unlike strength and closeness, Net 1 is the best performing one. However, the differences between two Pearson stemming networks are negligible. Therefore, the main conclusion to be drawn regarding return is the same as in the previous cases; namely that the networks recognized by Pearson correlation appear to be leading to better results. As far as risk criteria are concerned, non-linear networks constitute once more the best options. Net 3 and Net 6 (in) are the best regarding beta, while Net 5 and Net 6 out (and then Net 3) the best concerning variance. In comparison to strength/closeness, two points about risk measures and eigenvector centrality could me made: i) Net 6 (in) appears as one of the best choices for the first time, ii) there are different networks to be opted for depending on the investing horizon. For a short investing horizon, the directed non-linear networks are better in 100% of the holding days (Net 6 in for beta, Net 6 out for variance). On the other hand, the undirected non-linear networks are the best performing in most of the cases in which the holding period is longer (Net 3-93%-Beta, Net5-63%-

Variance). Although the difference of the results between Nets 3,5 & 6 is not significantly large, this different image per investment horizon (and especially the fact that directed networks perform better the longer this investing horizon) cannot be neglected.

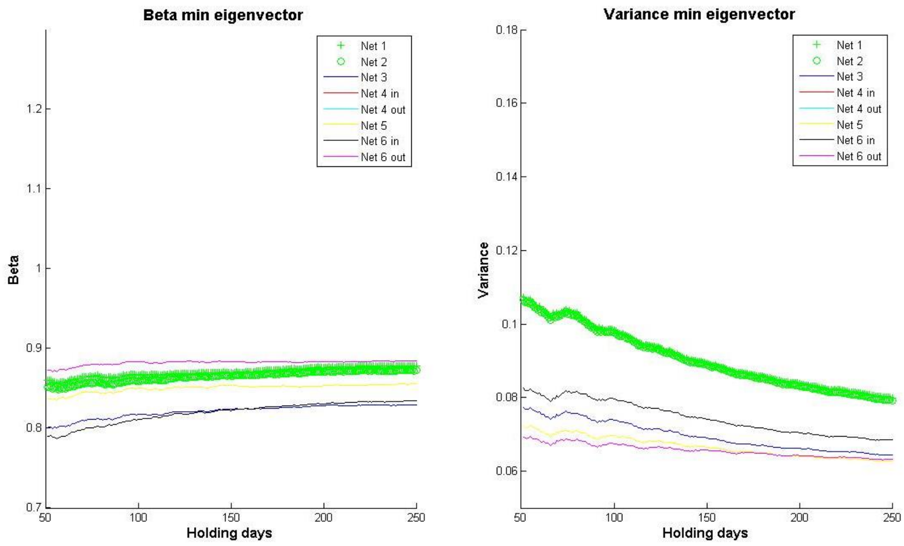

Image 17: Eigenvector- Asymmetric measures prevail for a shorter horizon in terms of risk minimization

Regarding eccentricity, the best results [biggest return, (systemic) risk adjusted return and smallest beta, variance] stem from the portfolios constructed from stocks (nodes) with the highest eccentricity. This may appear as a difference compared to the centrality measures mentioned above, for which the nodes with the minimum centrality scores were chosen in order for portfolios to be constructed. However, this should not provoke confusion. As already mentioned in literature, “the lower the eccentricity of a particular node, the closer it is to every other node, that is, the more central it is” (Kaya, 2013). However, unlike the centrality measures, the best results do not necessarily come from Pearson correlation recognized networks. In fact, 70% of the best results stem from non-linear measures and mainly Net 6 (in) and Net 5. Eccentricity’s results resemble, however, those of centrality measures concerning risk criteria as the best results for beta and variance come from networks recognized from non-linear measures. Net 6 (out) is the best for beta, which was also the case for strength and closeness centrality. Net 6 (in) is the best option for variance. Finally, non-linear networks and especially Net 5 are the best options for risk-adjusted return. Net 3 is among the best options for adjusted to (systemic) risk return while Net 6 (out) only for systemic risk adjusted return.

The above analysis about eccentricity is only worth it from such a point of view that the linear measures are compared to the non-linear ones. In fact, the portfolio performance of eccentricity is significantly inferior to the one of portfolios constructed with strength, closeness and eigenvector centrality. Based on the results of this thesis, eccentricity is therefore suggested to be an unsuitable criterion in order for optimal portfolios to be constructed.

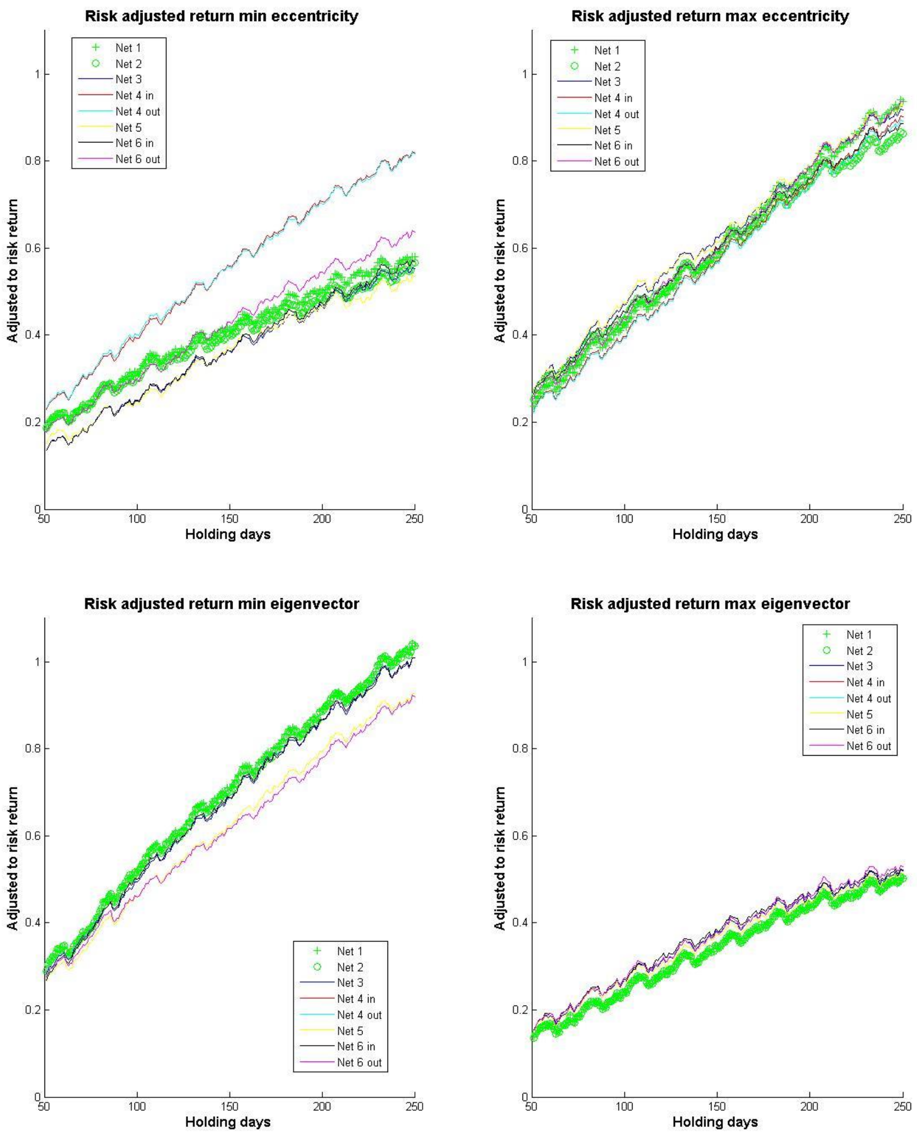

Image 18: Eccentricity- High eccentricity vertices outperform low eccentricity ones but are outperformed by strength, closeness, eigenvector.

Betweenness centrality is also found to be unsuitable for the purposes of this essay. The performance of portfolios constructed from nodes with minimum betweenness are in most cases equal or inferior to the one of high-centrality portfolios, while neither maximum or minimum betweenness centrality nodes can lead to results that minimum strength, closeness eigenvector can, judging from the results of this thesis. The reason behind this may be that many nodes have betweenness centrality equal to zero; therefore Betweenness itself may not be a sufficient criterion in order to select the optimal stocks (20 in our case).

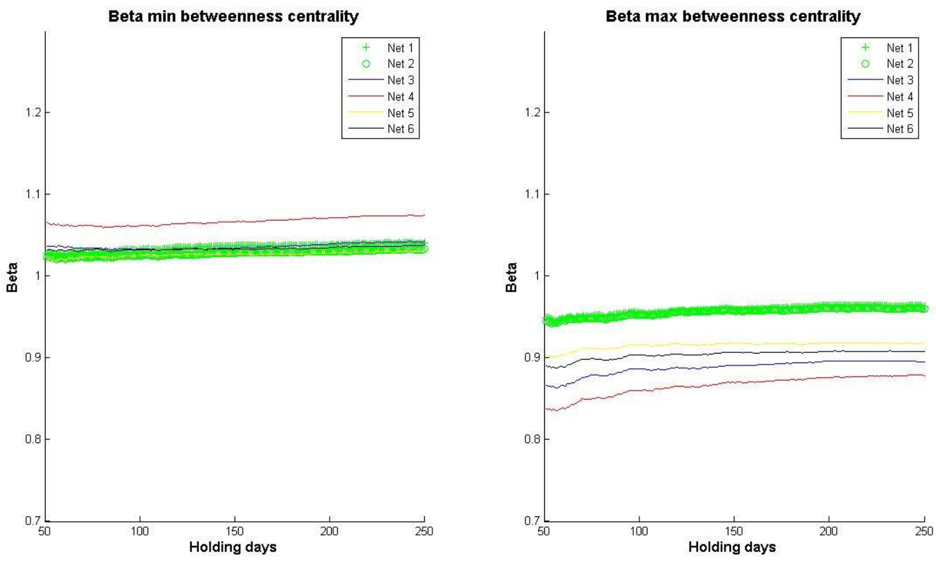

Image 19: Beta betweenness

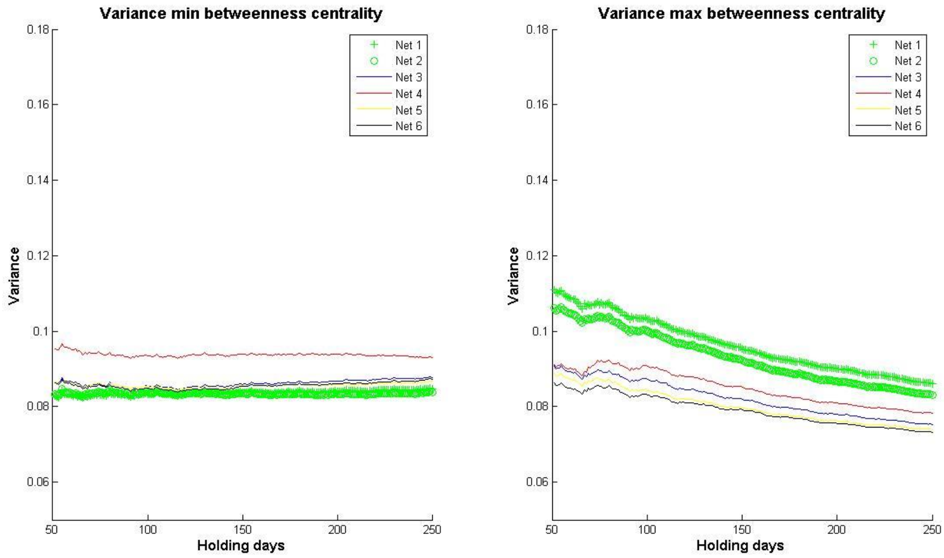

Image 20: Variance Betweenness

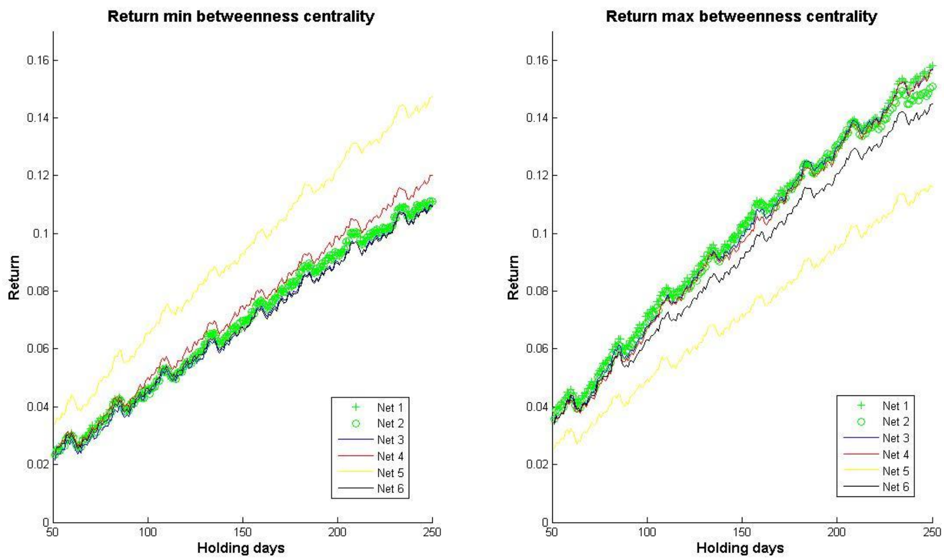

Image 21: Betweenness Return

In a nutshell, the results of this thesis indicate that eccentricity and betweenness would better be not regarded as someone's first option when trying to implement network theory for portfolio selection. On the other hand, opting for stocks/vertices with the minimum strength, closeness or eigenvector centrality appears as the optimal decision. The main conclusion until now is that such networks recognized by Pearson Correlation more or less to the best results regarding return, whereas networks recognized by non-linear measures lead to far better results as far as risk in concerned.

In order to further check the latter results, the same methodology (with an overlap of 115 observations) was applied specifically for the period of the financial crisis of 2008. In particular, the financial crisis testing period was chosen to be between 01.08.2007 and 31.03.2009. The logic of setting the start point on August 2007 is that BNP Paribas froze three of their funds at that month, admitting their inability to price the CDOs (collateralized debt obligations), more or less accepting their high exposure to subprime loans. On 02/04/2009, a global stimulus package of $5tn was agreed, which is why the end of the turmoil was set by the writer at the end of March 2009.

Regarding strength, the best results for return do not only stem from Net1 & Net 2 anymore, as for about 60% of the tested holding days the best results stem from some of the non-linear networks. Nevertheless, the most impressive results are observed in the case of risk and risk adjusted return. When examining the whole testing period, it was already shown that portfolios constructed by networks recognized from non-linear measures perform better in terms of risk, meaning that portfolios of less risk are constructed. This finding is confirmed during the crisis period, in which, however, the difference between the risk of the two categories of portfolios becomes much bigger. Images 22 & 23 about variance and beta respectively are indicative.

During both the whole period and the crisis period, the least containing risk portfolios are those that have been constructed from non-linear networks. However, during the crisis there is a much greater gap between linear and non-linear ones. In addition, during the crisis all non-linear networks result in portfolios of less risk compared to the linear ones, which was not the case for the whole period. Non-linear measures are also found to be superior also in terms of (systemic) risk adjusted return, as their portfolios achieve higher adjusted to risk return for 100% of the holding days.

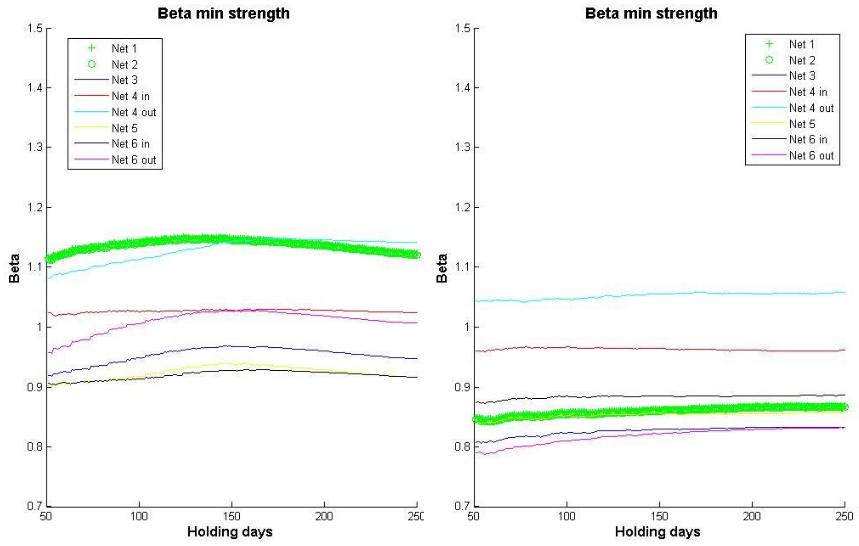

Image 22: Strength Beta during the whole period (right) and crisis (left).Superiority of non-linear measures in contrast to linear ones is more evident during the crisis

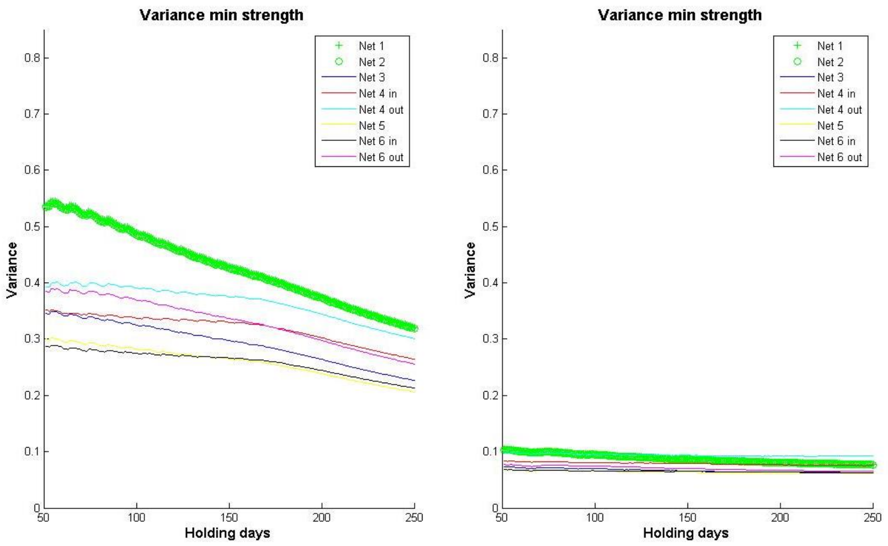

Image: 23: Strength Variance during the whole period (right) and crisis (left).Superiority of non-linear measures in contrast to linear ones is more evident during the crisis

Just as for the whole testing period, the results of closeness centrality are almost identical to the ones of strength for the crisis period as well. In other words, non-linear nets (Net 4 out, Net 5) achieve the best performance in return for most of the holding days tested, with the based on Pearson correlation Net 2 also achieving the best performance for an important amount of holding days. Net 6 (in) and Net 5 are the optimal choices as far as beta and variance is concerned. Concerning risk adjusted return, Net 4 out and Net 3 have the greatest performance. However, it should be noted that in contrast to risk, the differences between the networks and the constructed portfolios are not notably big for risk adjusted return.

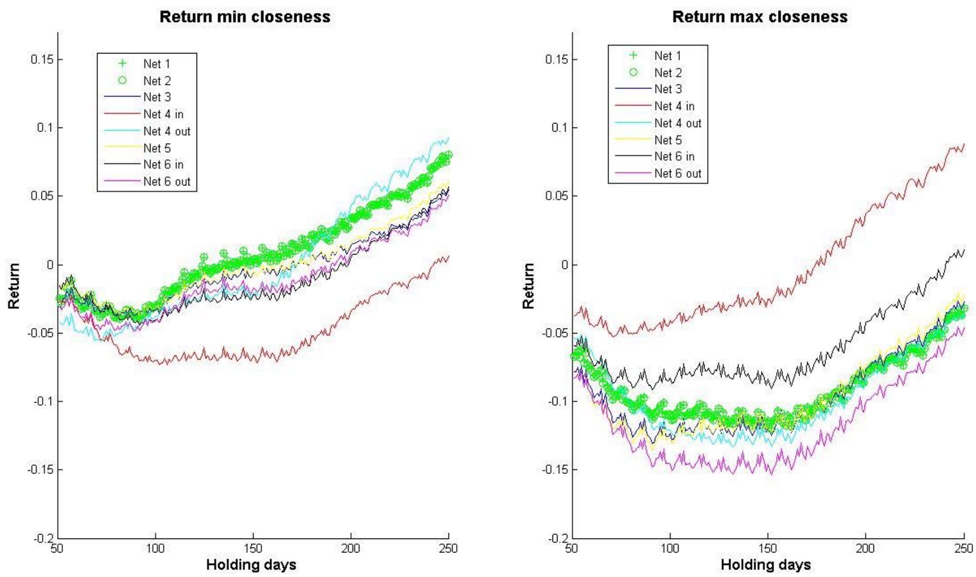

Image 24: Return (closeness) during the crisis- Less difference between central and less central vertices; Non-linear measures are also taking over in terms of performance

As far as eigenvector centrality is concerned, Net 1 & Net 2 still manage to achieve the best performance in return, with this superiority being greater the larger the investing horizon. The results about risk show some similarity to the ones of closeness and strength, in terms of choosing between linear and nonlinear measures. However, Net 6 out (and not Net 6 in which was the best for strength & closeness) is the optimal choice for eigenvector centrality, with Net 5 being the second best performing network. Regarding (systemic) risk adjusted return, the results of the different networks do not notably differ with each other. Nevertheless, Net 6 out and Net 3 are the best performing networks.

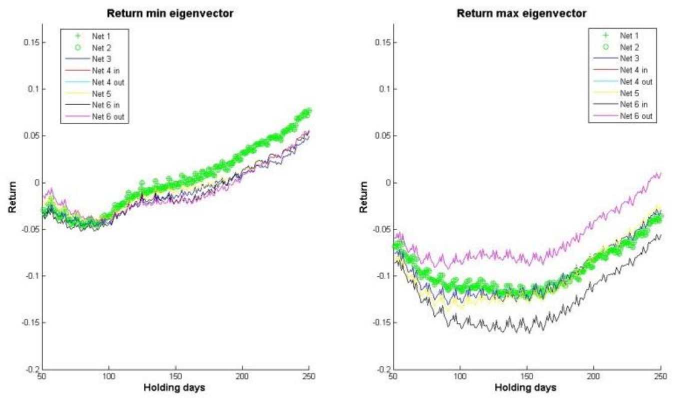

Image 25: Eigenvector Crisis; Linear measures remain the best regarding return

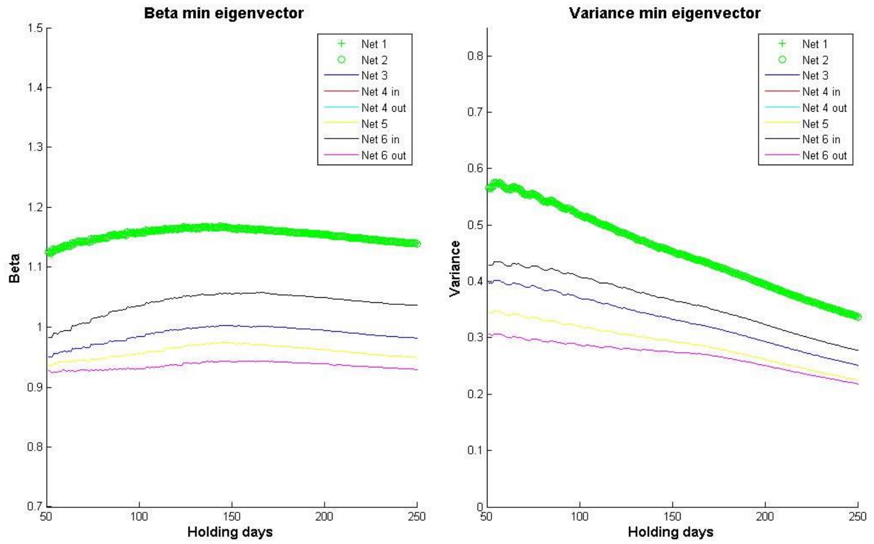

Image 26: Eigenvector Crisis-Net 6 is the best performing in terms of risk minimization

Until now, the main point of the discussion is the comparison between networks recognized by linear and nonlinear measures. However, a comparison should also be made between symmetric and asymmetric networks. One point to be made is that (excluding betweenness), Net 4 is not the best performer for any one of the tested investing horizons for the whole testing period. However, Net 4 (out) seems to be quite a good performer during the crisis period. In particular, it's the optimal choice for 32% of the holding days in return (strength), 58% in risk adjusted return and 32.50% in systemic risk adjusted return. In addition, those percentages are significantly bigger for a larger investing horizon (64%, 98%, 65% for Return, risk adjusted and systemic risk adjusted return respectively). The corresponding percentages are (almost) identical for closeness due to the aforementioned similarity of the results of the two centrality measures.

The above comments about Net 4 indicate that directed networks seem to be of greater value during the crisis period. This turns out to be true if Table 5 is consulted.

Table 5: Frequency table of best performance per criterion; Division in i)networks from linear measures, symmetric networks from non-linear measures, asymmetric networks from non-linear measures, ii)whole testing period, crisis period

WHOLE PERIOD					CRISIS			
Strength	**Linear**	**NL Symmetric**	**NL Asymmetric**		**Strength**	**Linear**	**NL Symmetric**	**NL Asymmetric**
Return	90,00%	0,50%	9,50%		**Return**	42,50%	21,50%	36,00%
Beta	0,00%	0,00%	100,00%		**Beta**	0,00%	10,50%	89,50%
Variance	0,00%	99,00%	1,00%		**Variance**	0,00%	57,00%	43,00%
Risk adj. return	51,00%	43,50%	5,50%		**Risk adj. return**	0,00%	36,50%	63,50%
Systemic risk adj.return	3,50%	29,00%	67,50%		**Systemic risk adj.return**	0,00%	59,00%	41,00%
Eigenvector	**Linear**	**NL Symmetric**	**NL Asymmetric**		**Eigenvector**	**Linear**	**NL Symmetric**	**NL Asymmetric**
Return	100,00%	0,00%	0,00%		**Return**	77,00%	5,00%	18,00%
Beta	0,00%	46,50%	53,50%		**Beta**	0,00%	0,00%	100,00%
Variance	0,00%	31,50%	68,50%		**Variance**	0,00%	0,00%	100,00%
Risk adj. return	100,00%	0,00%	0,00%		**Risk adj. return**	26,00%	31,50%	42,50%
Systemic risk adj.return	85,50%	0,00%	14,50%		**Systemic risk adj.return**	27,00%	33,50%	39,50%
Closeness	**Linear**	**NL Symmetric**	**NL Asymmetric**		**Closeness**	**Linear**	**NL Symmetric**	**NL Asymmetric**
Return	93,00%	0,00%	7,00%		**Return**	45,50%	19,00%	35,50%
Beta	0,00%	0,00%	100,00%		**Beta**	0,00%	10,50%	89,50%
Variance	0,00%	99,00%	1,00%		**Variance**	0,00%	57,00%	43,00%
Risk adj. return	54,00%	41,50%	4,50%		**Risk adj. return**	1,50%	35,00%	63,50%
Systemic risk adj.return	5,00%	27,50%	67,50%		**Systemic risk adj.return**	0,00%	59,00%	41,00%

The above summarizes what has been described in the analysis of each centrality measure's results. In the whole period, linear networks seem to be superior as far as return is concerned while nonlinear in terms of risk with asymmetric ones performing better in terms of beta and symmetric ones in terms of variance. Regarding adjusted to risk return, linear measures perform better for eigenvector centrality, while the results are mixed for strength and closeness.

Apart from the worse performance of linear measures during the crisis, which has already been analyzed, it is evident that asymmetric networks' superiority increases significantly during the crisis for most the cases [strength/closeness: return-variance-risk adjusted return , Eigenvector: return-beta-variance-(systemic) risk adjusted return].

In order for a comparison between nonlinear symmetric and asymmetric networks to be possible, the investing horizon should also be taken into consideration. With the exception of variance (strength/closeness), asymmetric networks perform better than nonlinear symmetric for more than 150 holding days during the crisis. On the other hand, symmetric networks seem to be performing better in terms of return and (systemic) risk adjusted return for a smaller investing horizon during the same period. It should be also noted that asymmetric networks perform better in terms of both beta and variance when it comes to a smaller investing horizon during the crisis.

Table 6: Frequency table of best performance per criterion. Additional division in max horizon (holding days: 151-250) and min horizon (51-150)

WHOLE PERIOD-max horizon				CRISIS-max horizon			
Strength	Linear	NL Symmetric	NL Asymmetric	**Strength**	Linear	NL Symmetric	NL Asymmetric
Return	100,00%	0,00%	0,00%	Return	36,00%	0,00%	64,00%
Beta	0,00%	0,00%	100,00%	Beta	0,00%	0,00%	100,00%
Variance	0,00%	100,00%	0,00%	Variance	0,00%	100,00%	0,00%
Risk adj. return	56,00%	44,00%	0,00%	Risk adj. return	0,00%	2,00%	98,00%
Systemic risk adj.return	2,00%	32,00%	66,00%	Systemic risk adj.return	0,00%	32,00%	68,00%
Eigenvector	Linear	NL Symmetric	NL Asymmetric	**Eigenvector**	Linear	NL Symmetric	NL Asymmetric
Return	100,00%	0,00%	0,00%	Return	100,00%	0,00%	0,00%
Beta	0,00%	93,00%	7,00%	Beta	0,00%	0,00%	100,00%
Variance	0,00%	63,00%	37,00%	Variance	0,00%	0,00%	100,00%
Risk adj. return	100,00%	0,00%	0,00%	Risk adj. return	1,00%	28,00%	71,00%
Systemic risk adj.return	72,00%	0,00%	28,00%	Systemic risk adj.return	12,00%	34,00%	54,00%
Closeness	Linear	NL Symmetric	NL Asymmetric	**Closeness**	Linear	NL Symmetric	NL Asymmetric
Return	100,00%	0,00%	0,00%	Return	37,00%	0,00%	63,00%
Beta	0,00%	0,00%	100,00%	Beta	0,00%	0,00%	100,00%
Variance	0,00%	100,00%	0,00%	Variance	0,00%	100,00%	0,00%
Risk adj. return	60,00%	40,00%	0,00%	Risk adj. return	0,00%	2,00%	98,00%
Systemic risk adj.return	5,00%	29,00%	66,00%	Systemic risk adj.return	0,00%	32,00%	68,00%

WHOLE PERIOD-min horizon				CRISIS-min horizon			
Strength	Linear	NL Symmetric	NL Asymmetric	**Strength**	Linear	NL Symmetric	NL Asymmetric
Return	80,00%	1,00%	19,00%	Return	49,00%	43,00%	8,00%
Beta	0,00%	0,00%	100,00%	Beta	0,00%	21,00%	79,00%
Variance	0,00%	98,00%	2,00%	Variance	0,00%	14,00%	86,00%
Risk adj. return	46,00%	43,00%	11,00%	Risk adj. return	0,00%	71,00%	29,00%
Systemic risk adj.return	5,00%	26,00%	69,00%	Systemic risk adj.return	0,00%	86,00%	14,00%
Eigenvector	Linear	NL Symmetric	NL Asymmetric	**Eigenvector**	Linear	NL Symmetric	NL Asymmetric
Return	100,00%	0,00%	0,00%	Return	54,00%	10,00%	36,00%
Beta	0,00%	0,00%	100,00%	Beta	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	100,00%	Variance	0,00%	0,00%	100,00%
Risk adj. return	100,00%	0,00%	0,00%	Risk adj. return	51,00%	35,00%	14,00%
Systemic risk adj.return	99,00%	0,00%	1,00%	Systemic risk adj.return	42,00%	33,00%	25,00%
Closeness	Linear	NL Symmetric	NL Asymmetric	**Closeness**	Linear	NL Symmetric	NL Asymmetric
Return	86,00%	0,00%	14,00%	Return	54,00%	38,00%	8,00%
Beta	0,00%	0,00%	100,00%	Beta	0,00%	21,00%	79,00%
Variance	0,00%	98,00%	2,00%	Variance	0,00%	14,00%	86,00%
Risk adj. return	48,00%	43,00%	9,00%	Risk adj. return	3,00%	68,00%	29,00%
Systemic risk adj.return	5,00%	26,00%	69,00%	Systemic risk adj.return	0,00%	86,00%	14,00%

Table 7: Comparing the performance of non-linear symmetric networks. The percentages do not have sum equal to 1, the rest of the respective best performances have been achieved by the rest networks

NL Symmetric WHOLE PERIOD		
Strength	**Net 3**	**Net 5**
Return	0,50%	0,00%
Beta	0,00%	0,00%
Variance	0,00%	99,00%
Risk adj. return	43,50%	0,00%
Systemic risk adj.return	29,00%	0,00%
Eigenvector	**Net 3**	**Net 5**
Return	0,00%	0,00%
Beta	46,50%	0,00%
Variance	0,00%	31,50%
Risk adj. return	0,00%	0,00%
Systemic risk adj.return	0,00%	0,00%
Closeness	**Net 3**	**Net 5**
Return	0,00%	0,00%
Beta	0,00%	0,00%
Variance	0,00%	99,00%
Risk adj. return	41,50%	0,00%
Systemic risk adj.return	27,50%	0,00%

NL Symmetric WHOLE PERIOD-MAX HORIZON		
Strength	**Net 3**	**Net 5**
Return	0,00%	0,00%
Beta	0,00%	0,00%
Variance	0,00%	100,00%
Risk adj. return	44,00%	0,00%
Systemic risk adj.return	32,00%	0,00%
Eigenvector	**Net 3**	**Net 5**
Return	0,00%	0,00%
Beta	93,00%	0,00%
Variance	0,00%	63,00%
Risk adj. return	0,00%	0,00%
Systemic risk adj.return	0,00%	0,00%
Closeness	**Net 3**	**Net 5**
Return	0,00%	0,00%
Beta	0,00%	0,00%
Variance	0,00%	100,00%
Risk adj. return	40,00%	0,00%
Systemic risk adj.return	29,00%	0,00%

NL Symmetric WHOLE PERIOD-MIN HORIZON		
Strength	**Net 3**	**Net 5**
Return	1,00%	0,00%
Beta	0,00%	0,00%
Variance	0,00%	98,00%
Risk adj. return	43,00%	0,00%
Systemic risk adj.return	26,00%	0,00%
Eigenvector	**Net 3**	**Net 5**
Return	0,00%	0,00%
Beta	0,00%	0,00%
Variance	0,00%	0,00%
Risk adj. return	0,00%	0,00%
Systemic risk adj.return	0,00%	0,00%
Closeness	**Net 3**	**Net 5**
Return	0,00%	0,00%
Beta	0,00%	0,00%
Variance	0,00%	98,00%
Risk adj. return	43,00%	0,00%
Systemic risk adj.return	26,00%	0,00%

NL Symmetric CRISIS		
Strength	**Net 3**	**Net 5**
Return	1,00%	20,50%
Beta	0,00%	10,50%
Variance	0,00%	57,00%
Risk adj. return	36,50%	0,00%
Systemic risk adj.return	59,00%	0,00%
Eigenvector	**Net 3**	**Net 5**
Return	0,00%	5,00%
Beta	0,00%	0,00%
Variance	0,00%	0,00%
Risk adj. return	27,50%	4,00%
Systemic risk adj.return	18,50%	15,00%
Closeness	**Net 3**	**Net 5**
Return	1,00%	18,00%
Beta	0,00%	10,50%
Variance	0,00%	57,00%
Risk adj. return	35,00%	0,00%
Systemic risk adj.return	59,00%	0,00%

NL Symmetric CRISIS-MAX HORIZON		
Strength	**Net 3**	**Net 5**
Return	0,00%	0,00%
Beta	0,00%	0,00%
Variance	0,00%	100,00%
Risk adj. return	2,00%	0,00%
Systemic risk adj.return	32,00%	0,00%
Eigenvector	**Net 3**	**Net 5**
Return	0,00%	0,00%
Beta	0,00%	0,00%
Variance	0,00%	0,00%
Risk adj. return	20,00%	8,00%
Systemic risk adj.return	17,00%	17,00%
Closeness	**Net 3**	**Net 5**
Return	0,00%	0,00%
Beta	0,00%	0,00%
Variance	0,00%	100,00%
Risk adj. return	2,00%	0,00%
Systemic risk adj.return	32,00%	0,00%

NL Symmetric CRISIS-MIN HORIZON		
Strength	**Net 3**	**Net 5**
Return	2,00%	41,00%
Beta	0,00%	21,00%
Variance	0,00%	14,00%
Risk adj. return	71,00%	0,00%
Systemic risk adj.return	86,00%	0,00%
Eigenvector	**Net 3**	**Net 5**
Return	0,00%	10,00%
Beta	0,00%	0,00%
Variance	0,00%	0,00%
Risk adj. return	35,00%	0,00%
Systemic risk adj.return	20,00%	13,00%
Closeness	**Net 3**	**Net 5**
Return	2,00%	36,00%
Beta	0,00%	21,00%
Variance	0,00%	14,00%
Risk adj. return	68,00%	0,00%
Systemic risk adj.return	86,00%	0,00%

Comparing Net 3 and Net 5, it should be pointed out that both of them are usually among the best performers in terms of return, risk and adjusted to risk return. More accurately, they usually refrain from placing at the bottom. However, it should be noted that Net 3 performs better during the whole period while Net 5 does so during the crisis. Regarding strength and closeness, Net 3 is among the top options for (systemic) risk adjusted return and beta, with Net 5 being the next option having a significant difference from the top. Net 5 is the best option for building a portfolio of low variance. On the other hand, during the crisis Net 5 is one of the two best options for beta/variance, with Net 3 being the next option. Regarding eigenvector, the main point to be made is that Net 5 does not perform well at all in terms or return/adjusted to risk return during the whole period while Net 3's performance is sufficient. During the crisis, however, Net 5 is the second best performing one, with Net 3 being the third best option.

Table 8: Comparing the performance of non-linear asymmetric networks. The percentages do not have sum equal to 1, the rest of the respective best performances have been achieved by the rest networks

NL Asymmetric- WHOLE PERIOD				
Strength	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	0,00%	9,50%	0,00%
Beta	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	1,00%	0,00%
Risk adj. return	0,00%	0,00%	0,00%	5,50%
Systemic risk adj.return	0,00%	0,00%	0,00%	67,50%
Eigenvector	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	0,00%	0,00%	0,00%
Beta	0,00%	0,00%	53,50%	0,00%
Variance	0,00%	0,00%	0,00%	68,50%
Risk adj. return	0,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	0,00%	14,50%	0,00%
Closeness	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	0,00%	7,00%	0,00%
Beta	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	1,00%	0,00%
Risk adj. return	0,00%	0,00%	0,00%	4,50%
Systemic risk adj.return	0,00%	0,00%	0,00%	67,50%

NL Asymmetric- WHOLE PERIOD- MAX HORIZON				
Strength	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	0,00%	0,00%	0,00%
Beta	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	0,00%	0,00%
Risk adj. return	0,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	0,00%	0,00%	66,00%
Eigenvector	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	0,00%	0,00%	0,00%
Beta	0,00%	0,00%	7,00%	0,00%
Variance	0,00%	0,00%	0,00%	37,00%
Risk adj. return	0,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	0,00%	28,00%	0,00%
Closeness	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	0,00%	0,00%	0,00%
Beta	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	0,00%	0,00%
Risk adj. return	0,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	0,00%	0,00%	66,00%

NL Asymmetric- WHOLE PERIOD- MIN HORIZON				
Strength	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	0,00%	19,00%	0,00%
Beta	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	2,00%	0,00%
Risk adj. return	0,00%	0,00%	0,00%	11,00%
Systemic risk adj.return	0,00%	0,00%	0,00%	69,00%
Eigenvector	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	0,00%	0,00%	0,00%
Beta	0,00%	0,00%	100,00%	0,00%
Variance	0,00%	0,00%	0,00%	100,00%
Risk adj. return	0,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	0,00%	1,00%	0,00%
Closeness	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	0,00%	14,00%	0,00%
Beta	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	2,00%	0,00%
Risk adj. return	0,00%	0,00%	0,00%	9,00%
Systemic risk adj.return	0,00%	0,00%	0,00%	69,00%

NL Asymmetric- CRISIS				
Strength	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	32,00%	4,00%	0,00%
Beta	0,00%	0,00%	89,50%	0,00%
Variance	0,00%	0,00%	43,00%	0,00%
Risk adj. return	5,50%	58,00%	0,00%	0,00%
Systemic risk adj.return	1,00%	32,50%	7,50%	0,00%
Eigenvector	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	0,00%	0,00%	18,00%
Beta	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	0,00%	100,00%
Risk adj. return	0,00%	0,00%	0,00%	42,50%
Systemic risk adj.return	0,00%	0,00%	0,00%	39,50%
Closeness	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	31,50%	4,00%	0,00%
Beta	0,00%	0,00%	89,50%	0,00%
Variance	0,00%	0,00%	43,00%	0,00%
Risk adj. return	5,50%	58,00%	0,00%	0,00%
Systemic risk adj.return	1,00%	32,50%	7,50%	0,00%

NL Asymmetric- CRISIS- MAX HORIZON				
Strength	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	64,00%	0,00%	0,00%
Beta	0,00%	0,00%	100,00%	0,00%
Variance	0,00%	0,00%	0,00%	0,00%
Risk adj. return	0,00%	98,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	65,00%	3,00%	0,00%
Eigenvector	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	0,00%	0,00%	0,00%
Beta	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	0,00%	100,00%
Risk adj. return	0,00%	0,00%	0,00%	71,00%
Systemic risk adj.return	0,00%	0,00%	0,00%	54,00%
Closeness	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	63,00%	0,00%	0,00%
Beta	0,00%	0,00%	100,00%	0,00%
Variance	0,00%	0,00%	0,00%	0,00%
Risk adj. return	0,00%	98,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	65,00%	3,00%	0,00%

NL Asymmetric- CRISIS- MIN HORIZON				
Strength	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	0,00%	8,00%	0,00%
Beta	0,00%	0,00%	79,00%	0,00%
Variance	0,00%	0,00%	86,00%	0,00%
Risk adj. return	11,00%	18,00%	0,00%	0,00%
Systemic risk adj.return	2,00%	0,00%	12,00%	0,00%
Eigenvector	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	0,00%	0,00%	36,00%
Beta	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	0,00%	100,00%
Risk adj. return	0,00%	0,00%	0,00%	14,00%
Systemic risk adj.return	0,00%	0,00%	0,00%	25,00%
Closeness	Net 4 in	Net 4 out	Net 6 in	Net 6 out
Return	0,00%	0,00%	8,00%	0,00%
Beta	0,00%	0,00%	79,00%	0,00%
Variance	0,00%	0,00%	86,00%	0,00%
Risk adj. return	11,00%	18,00%	0,00%	0,00%
Systemic risk adj.return	2,00%	0,00%	12,00%	0,00%

Regarding asymmetric networks, Net 4 does not make it to the top performers for any of the tested holding days during the whole testing period. However, Net 4 (out) shows up a regularly good performer during the crisis [strength/closeness: return-(systemic) risk adjusted return]. Nevertheless, it should be noted that Net 6 clearly outperforms Net 4, judging from both the whole as well as the crisis period. During the whole period, Net 6 (out) is the best beta performer for all holding days, the best systemic risk adjusted return performer for most of the days, while its performance in risk adjusted return and variance is also among the top. During the crisis, Net 6 is by far the best beta performer for strength/eigenvector/closeness, while it is also the best variance performer for eigenvector. On this other hand, Net 4's risk performance during the crisis is very poor and often comparable to the one of Net 1 & Net 2. Their seeming superiority in adjusted to (systemic) risk return during the crisis is due to the return.

However, returns in such a crisis period are by default negligible; risk is of the utmost importance at such times and after all, Net 4 does not manage to have such a performance in adjusted to (systemic) risk return during the whole period. Net 6's consistency leads therefore to the conclusion that it is more suitable than Net 4 in order for portfolios with low risk or relatively high adjusted to risk return to be constructed.

EPILOGUE-CONCLUSIONS

In a nutshell, the main conclusions to be drawn from this thesis are the following:

- Stocks with lower strength, eigenvector, and closeness centrality and higher eccentricity scores form better performing portfolios in terms of return, risk (total and systemic), and adjusted risk return.
- Eccentricity performs worse in comparison to strength, eigenvector and closeness. No conclusion can be drawn about betweenness centrality.
- Networks identified from linear correlation measures lead to better performing portfolios in terms of return
- Networks identified from nonlinear measures lead to portfolios containing less systematic and unsystematic risk, while networks identified from the best performing nonlinear measures perform equally or sometimes better than the ones identified from linear ones.
- During periods of crisis, the better performance of nonlinear measures in contrast to linear measures in terms of building of portfolios of less total and systematic risk is emphatic.
- During periods of crisis, the importance of asymmetric networks is increased in order for better performance to be ensured.
- For strength/closeness, asymmetric networks build portfolios of less systematic risk and higher adjusted to systematic risk return.
- Net 6 (Information Dependence) is the best of the asymmetric networks tested. Net 3(Normalized Mutual Information) performs decently both during the whole period and the crisis while Net 5(Information Interdependence) performs better than Net 3 during the crisis.

The application of the same methodology to further datasets would be very useful in order for the possibility of the extraction of more generic conclusions to be examined. Furthermore, apart from network centralities, network communities could also be exploited in portfolio selection and especially in asset allocation. Instead of allocating wealth among different asset categories or sector, which is common practice, it would be very fruitful to check if an allocation between network communities can lead to better performance and risk mitigation.

APPENDICES

Appendix A. Images

Appendix A.1.1- Whole Testing Period

Strength

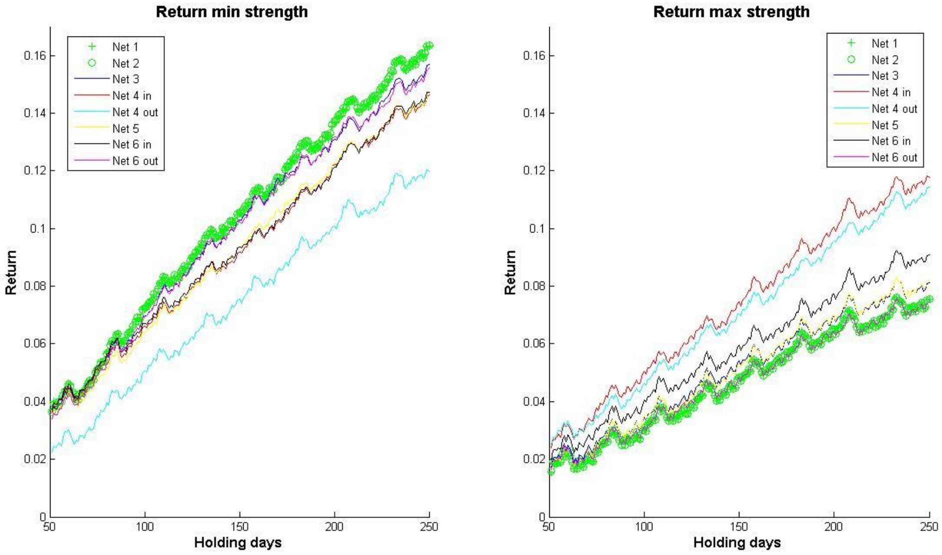

Image 27: Strength-Return

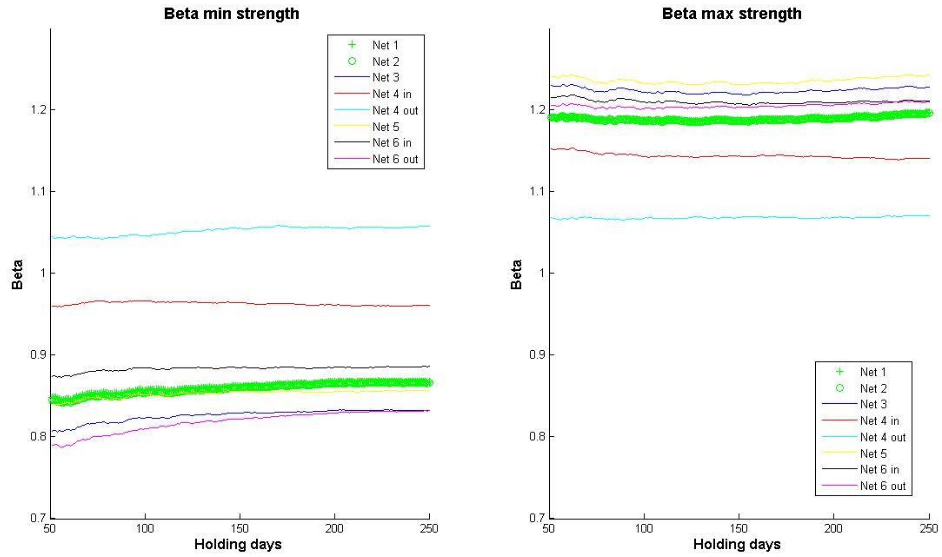

Image 28: Strength-Beta

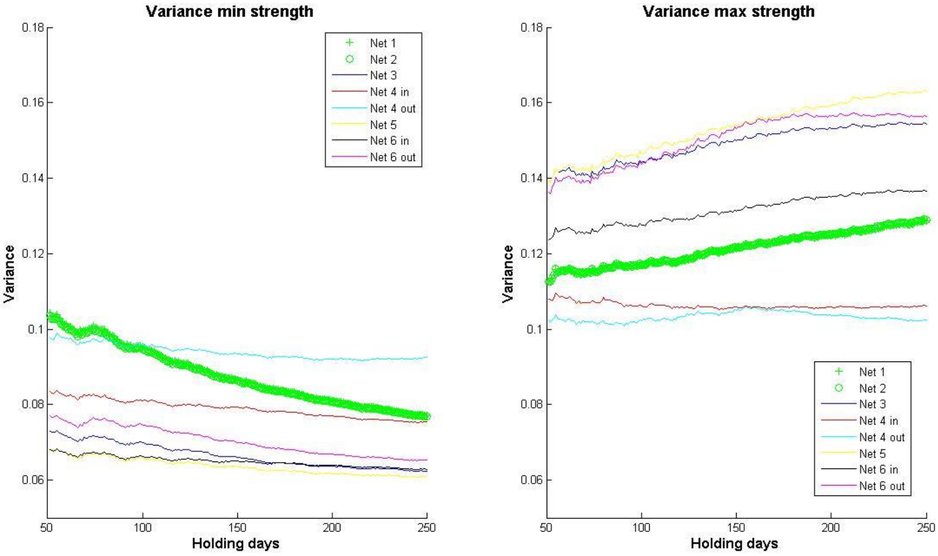

Image 29: Strength-Variance

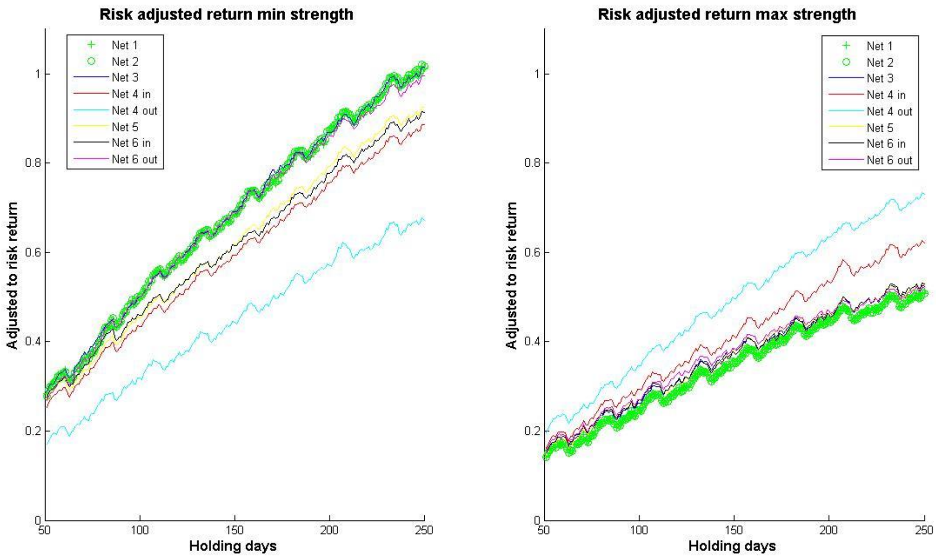

Image 30: Strength-Adjusted to risk return

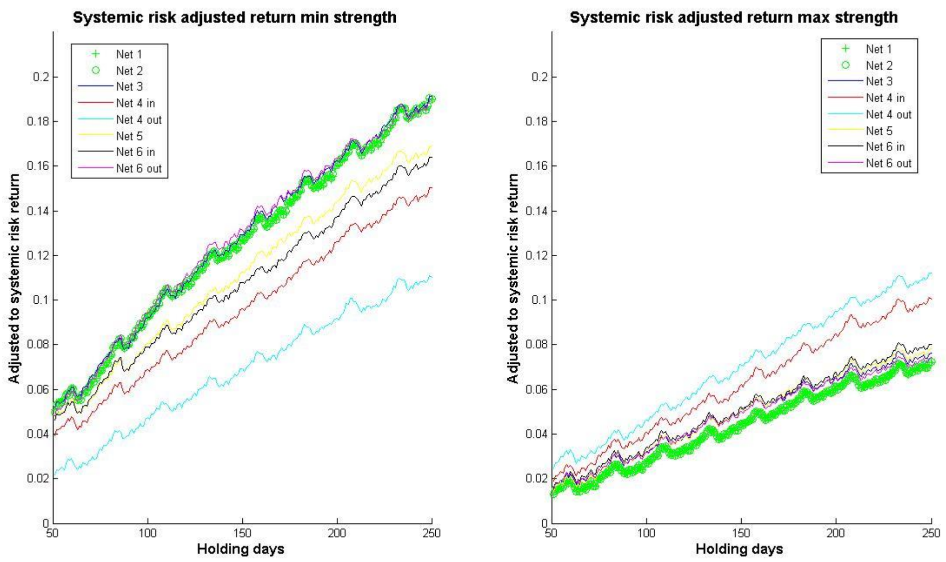

Image 31: Strength-Adjusted to systemic risk return

Eigenvector

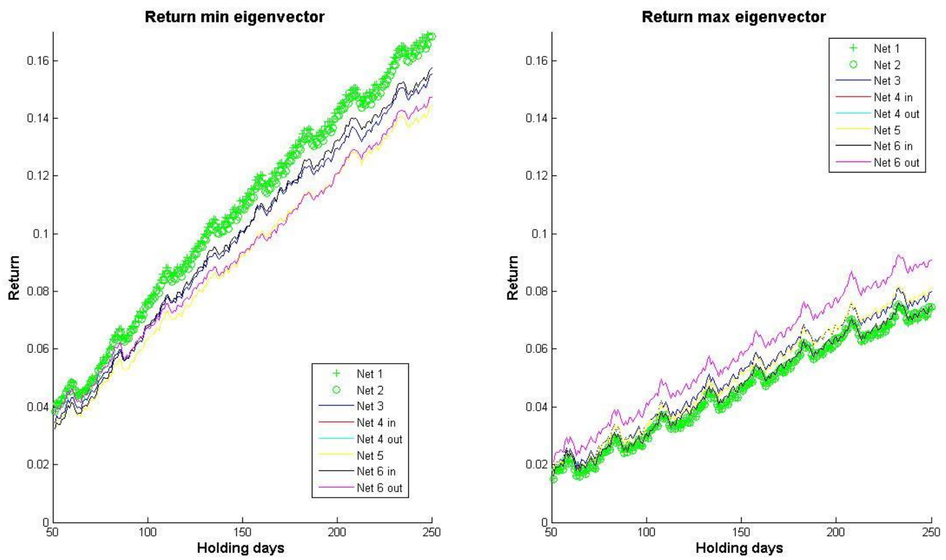

Image 32: Eigenvector-Return

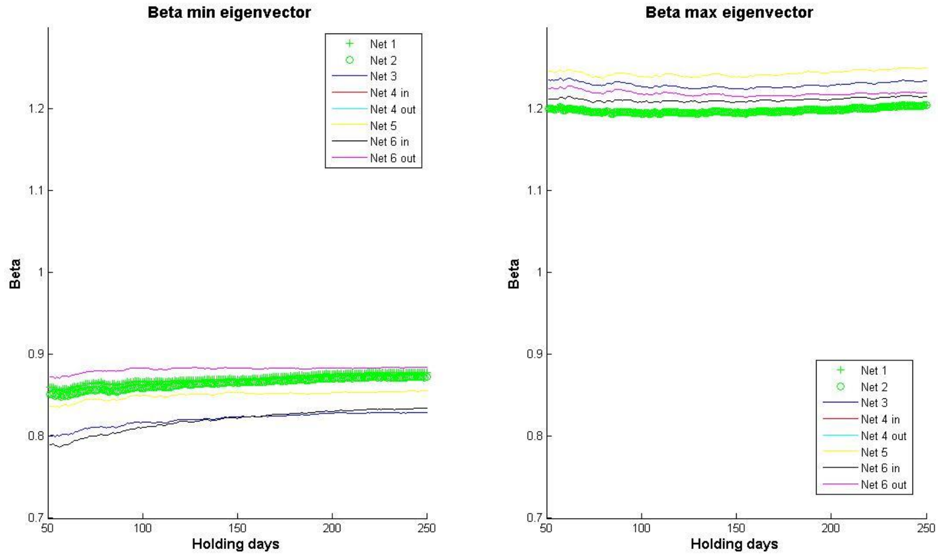

Image 33: Eigenvector Beta

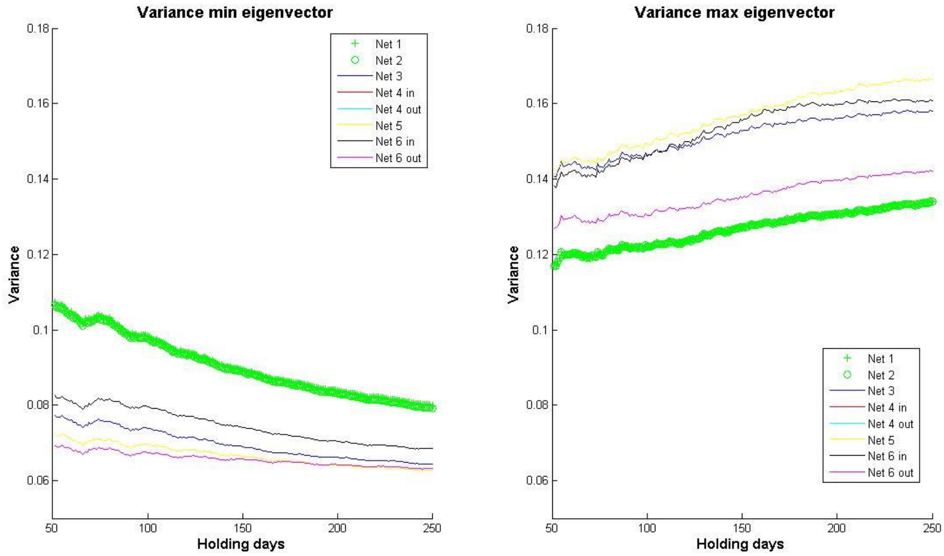

Image 34: Eigenvector Variance

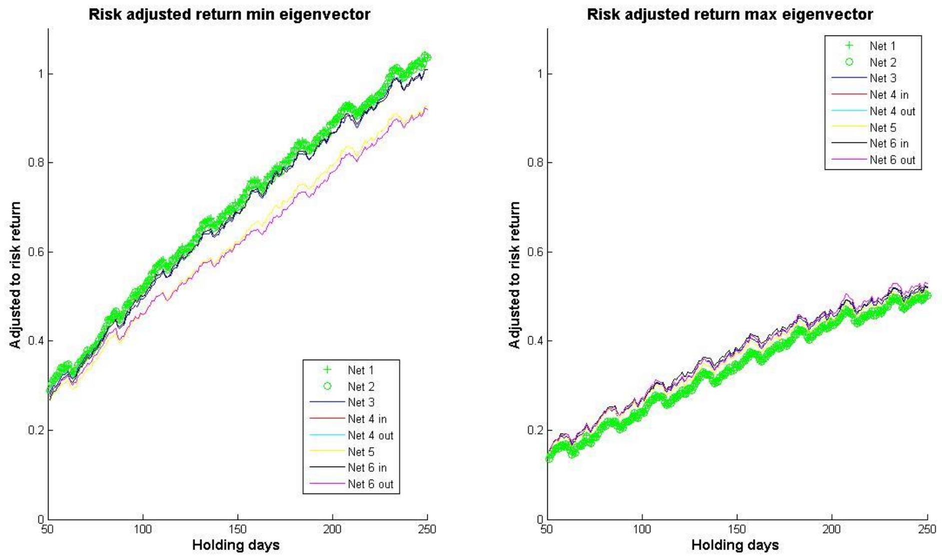

Image 35: Eigenvector Adjusted to risk return

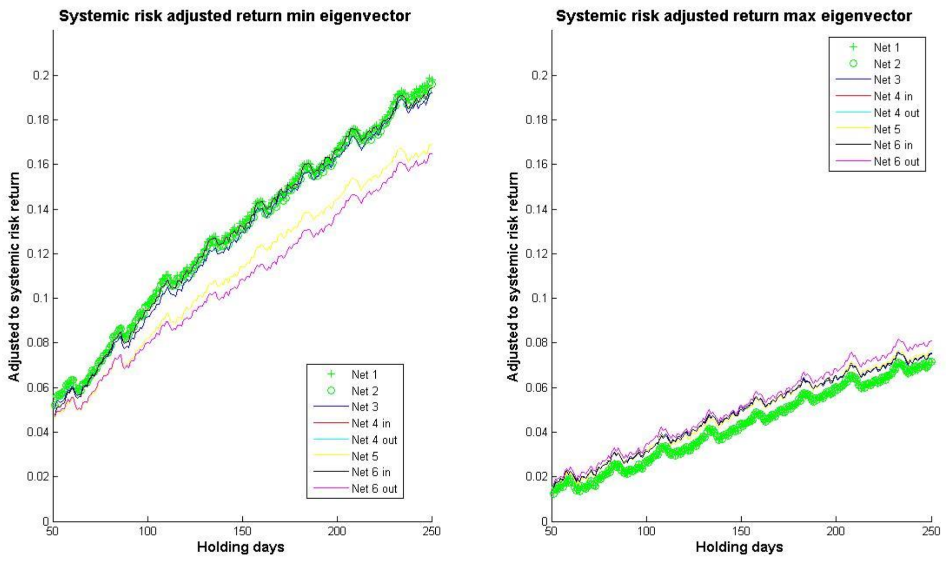

Image 36: Eigenvector Adjusted to systemic risk return

Closeness

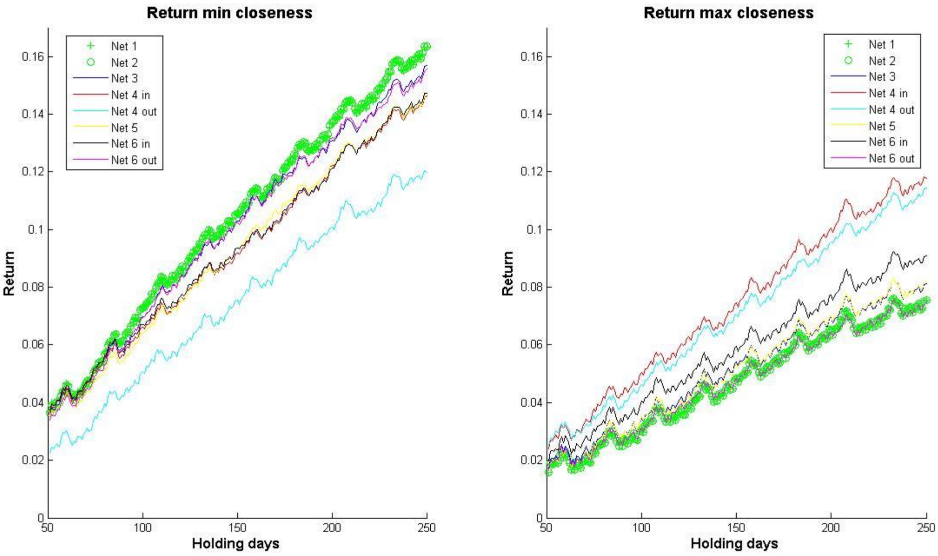

Image 37: Closeness Return

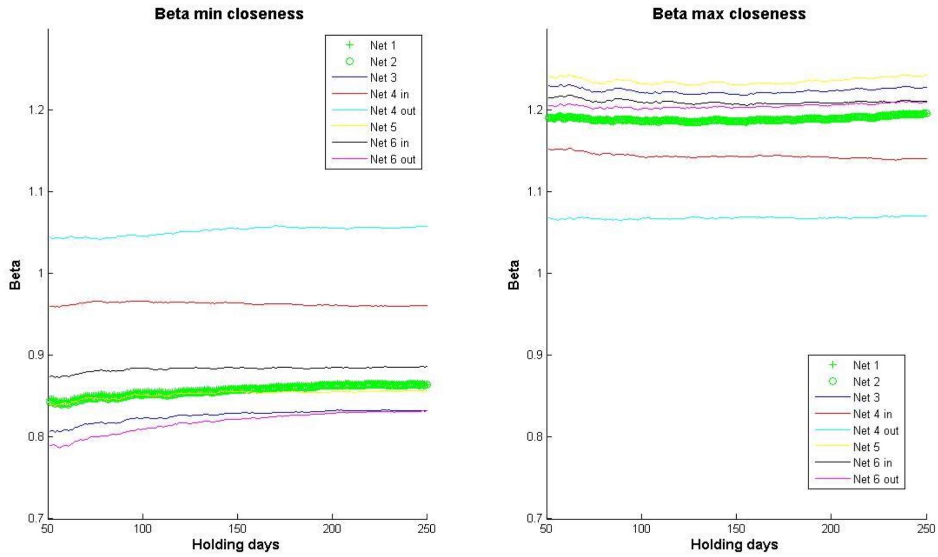

Image 38: Closeness Beta

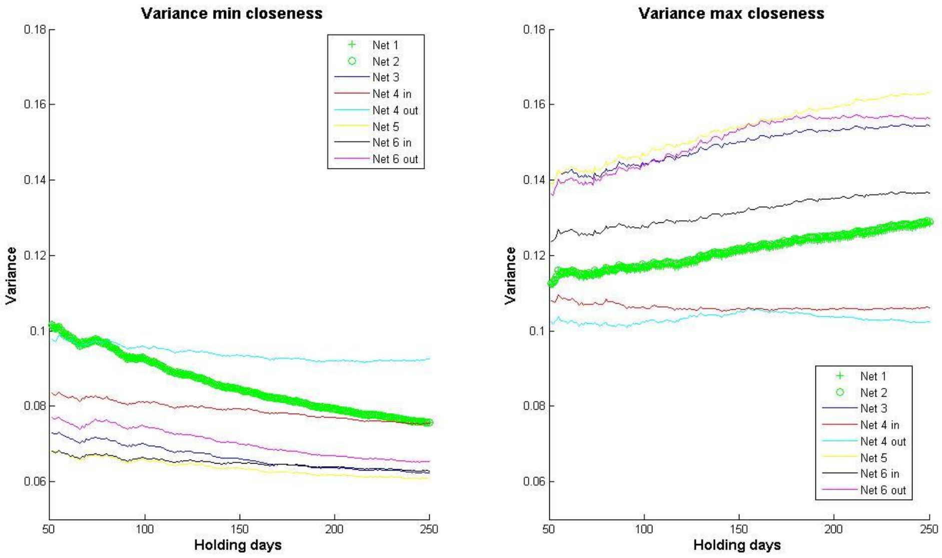

Image 39: Closeness Variance

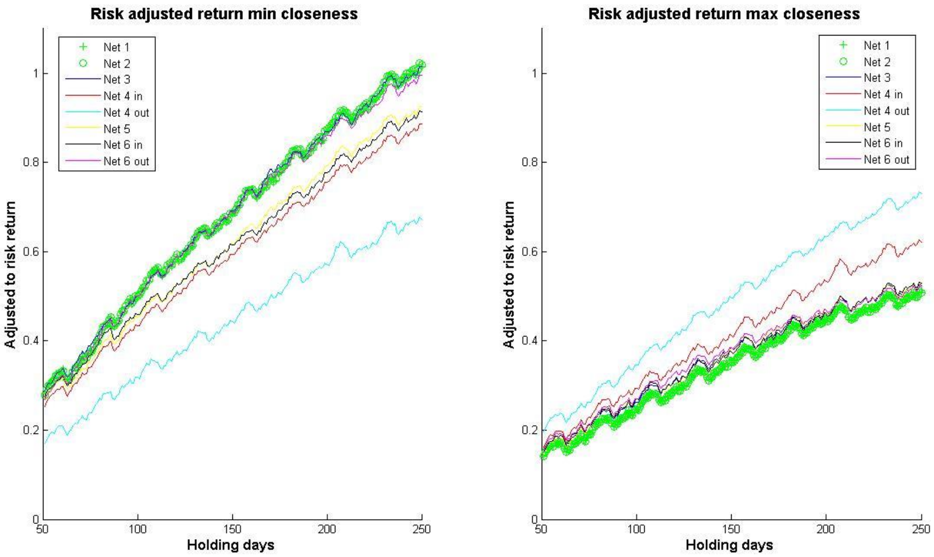

Image 39: Closeness Adjusted to risk return

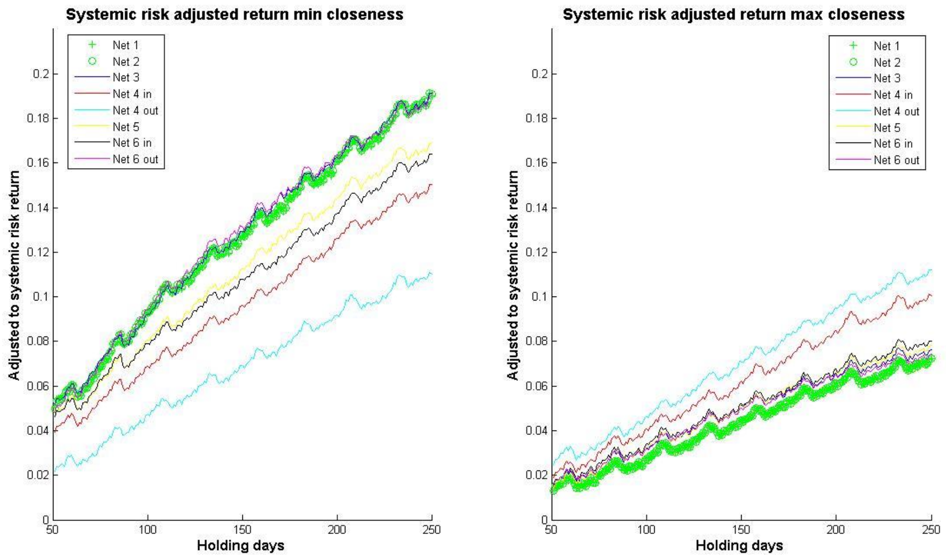

Image 40: Closeness Adjusted to systemic risk return

Betweenness

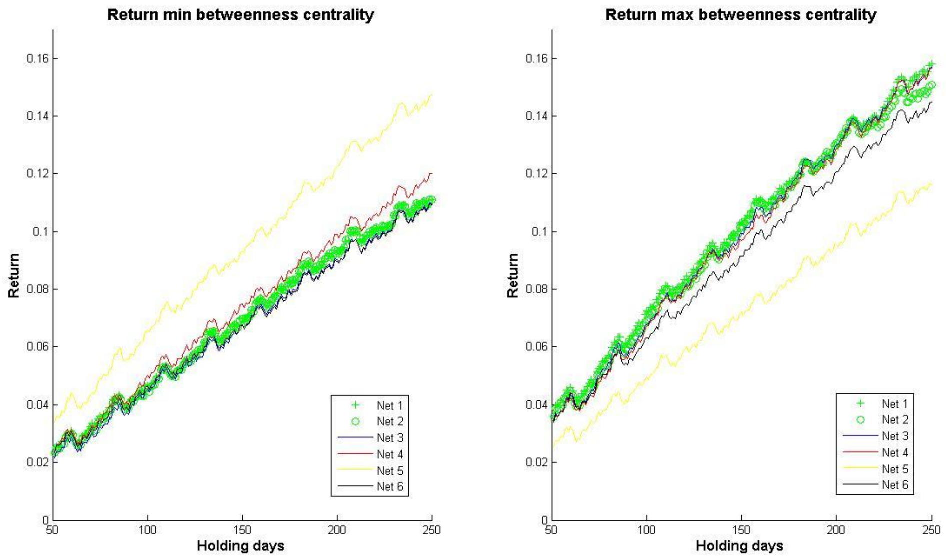

Image 41: Betweenness Return

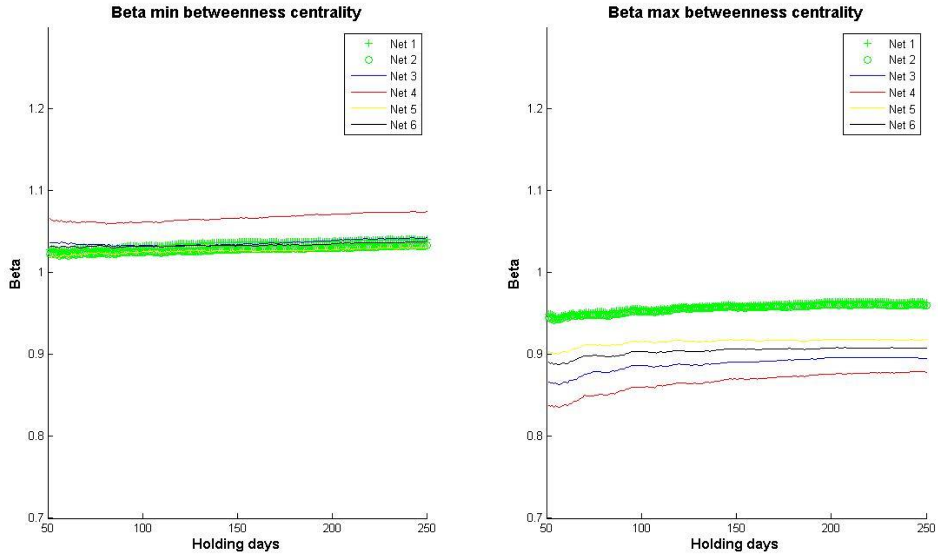

Image 42: Betweenness Beta

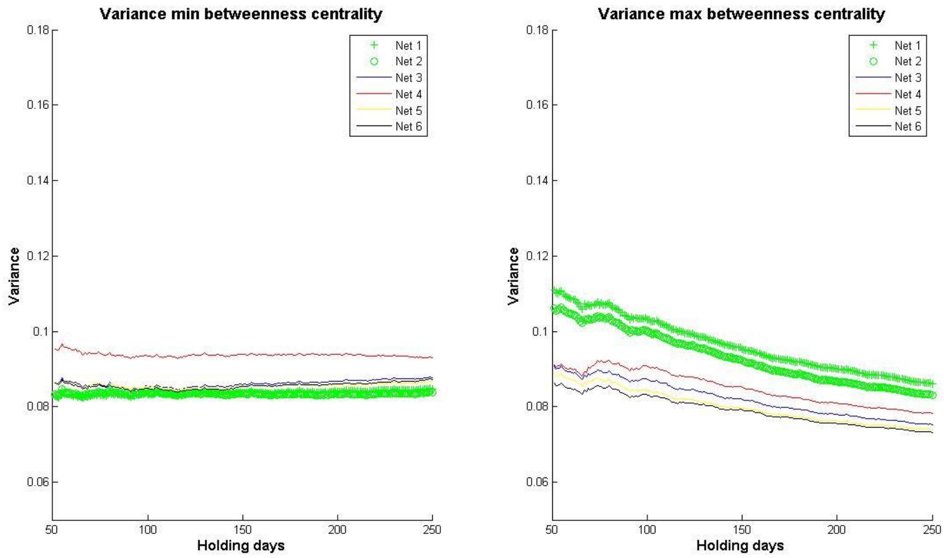

Image 43: Betweenness Variance

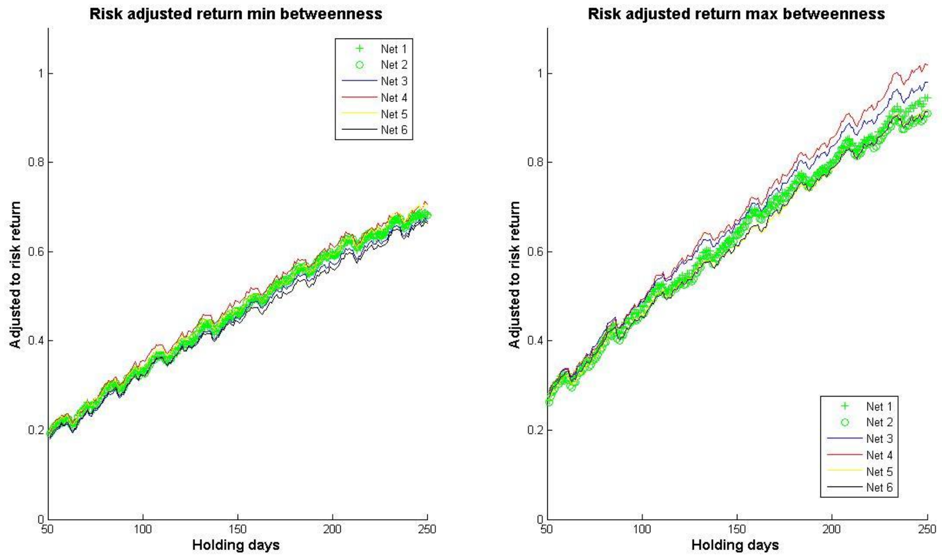

Image 44: Betweenness Adjusted to risk return

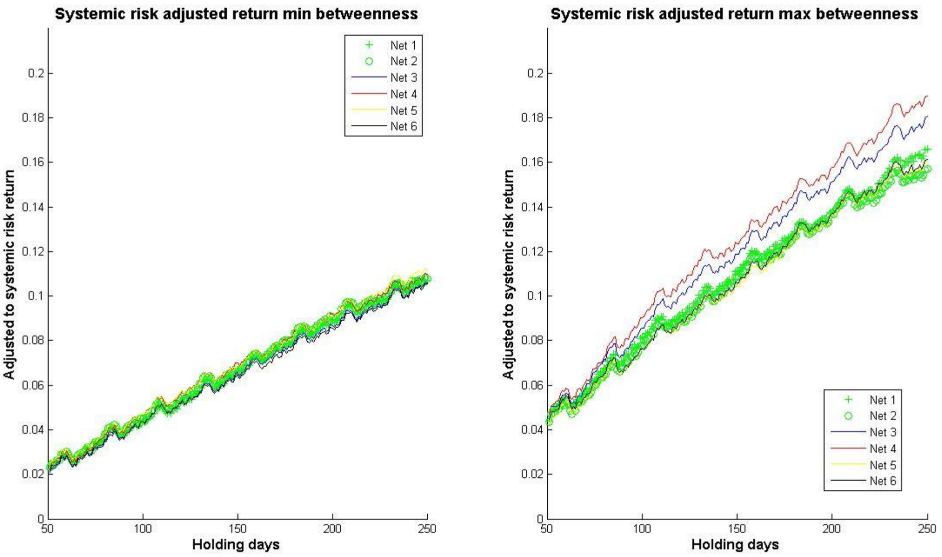

Image 45: Betweenness Adjusted to systemic risk return

Eccentricity

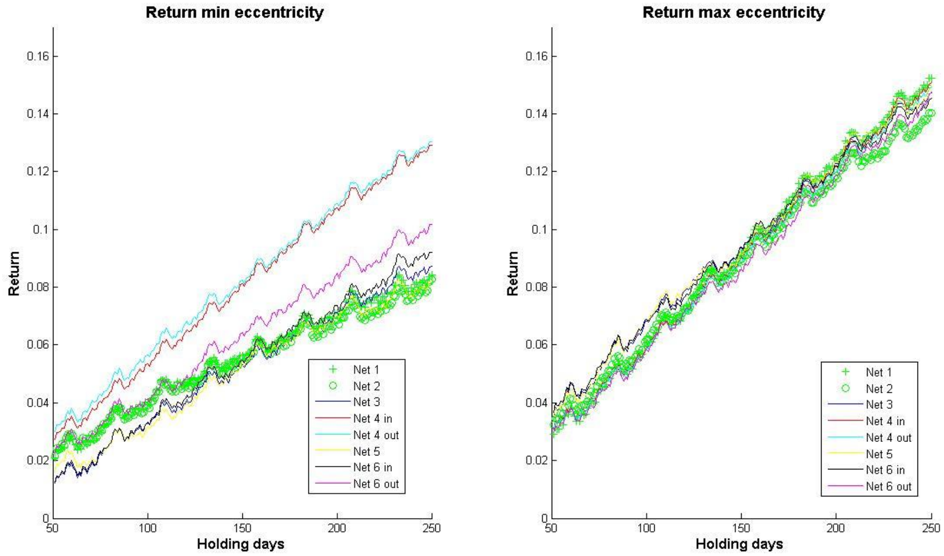

Image 46: Eccentricity Return

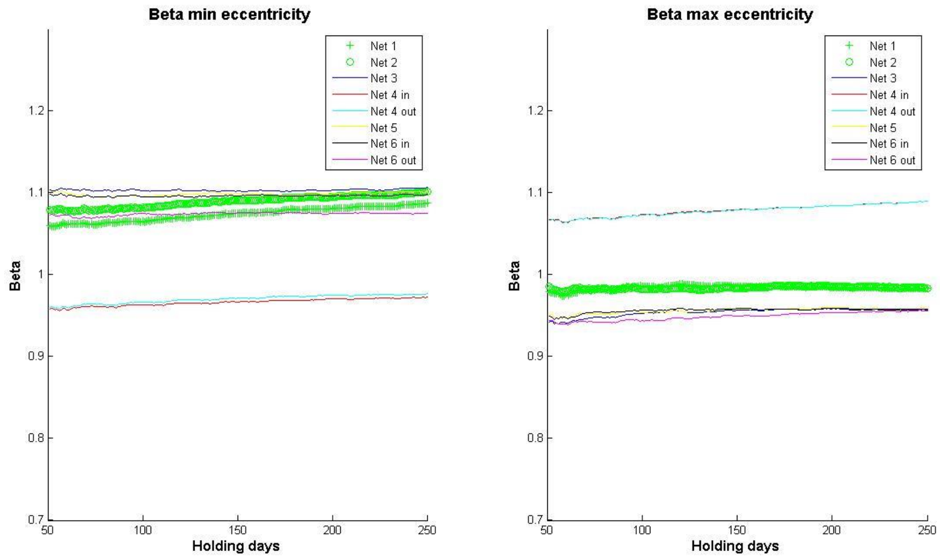

Image 47: Eccentricity Beta

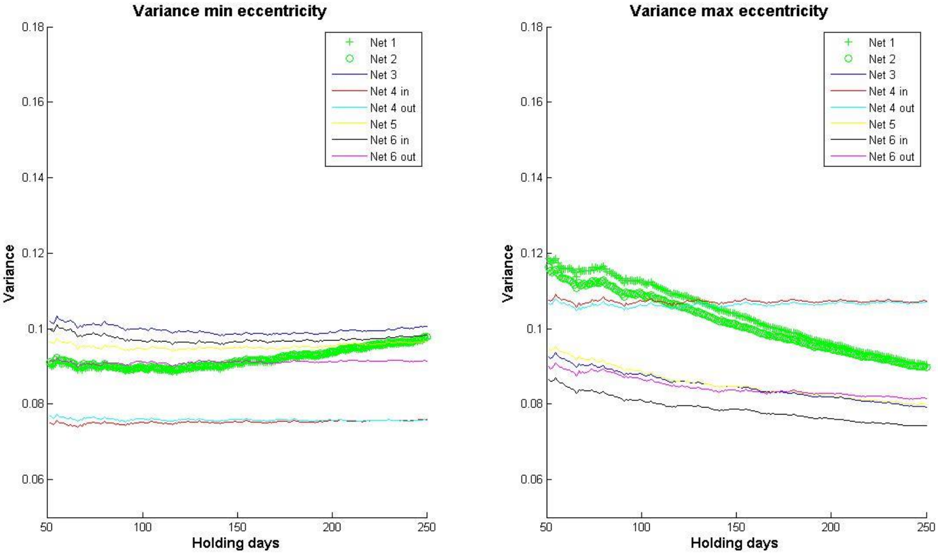

Image 48: Eccentricity Variance

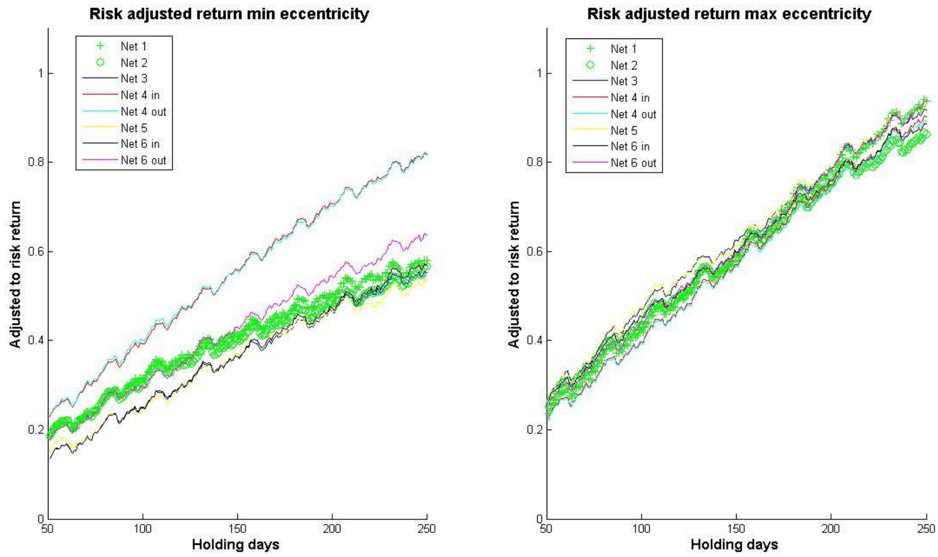

Image 49: Eccentricity Adjusted to risk return

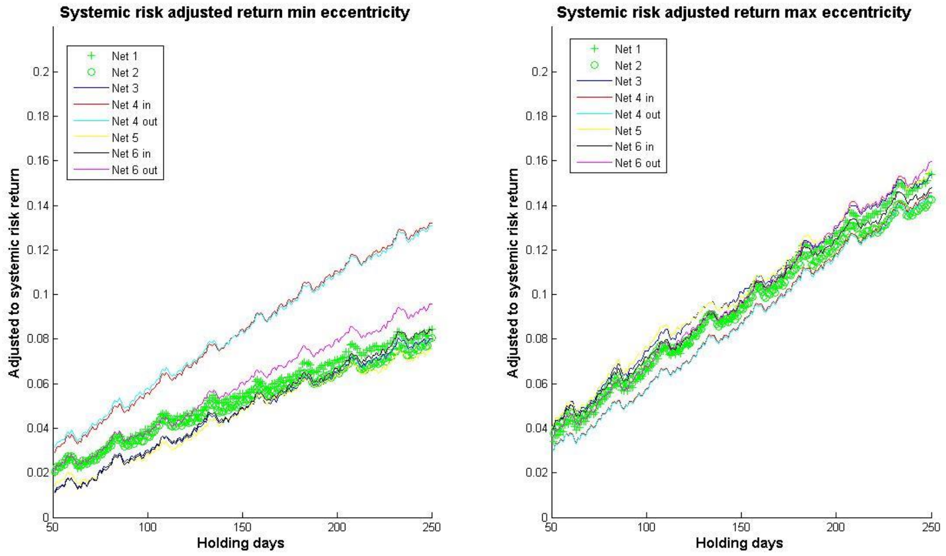

Image 50: Eccentricity Adjusted to systemic risk return

Appendix A.1.2- Whole Testing Period/Crisis axis bounds

The images of Appendix A.1.2 refer to the same values as Appendix A.1.1. However, they are plotted now with same axis bounds as the crisis images in Appendix A.2 in order for a comparison between the whole testing period and the crisis period to be facilitated.

Strength

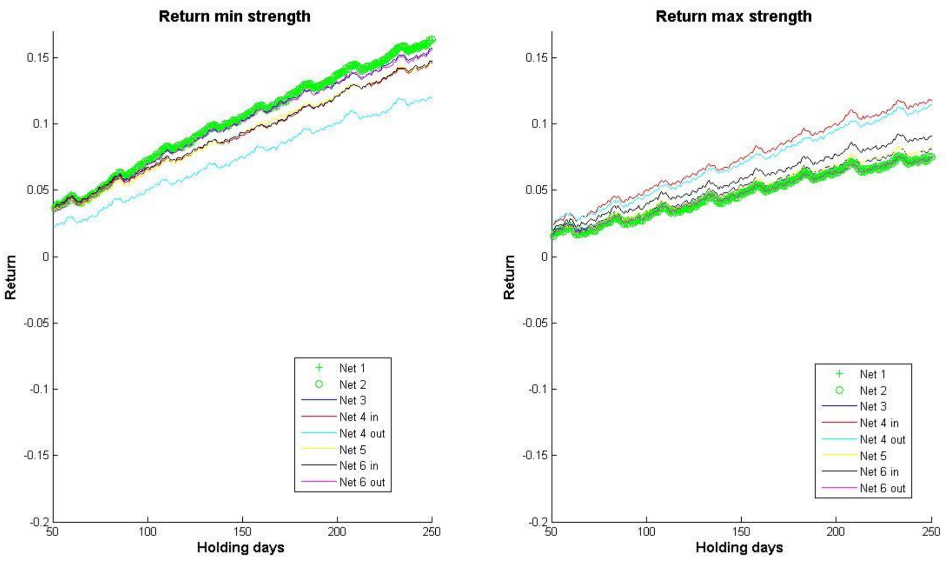

Image 51: Strength return

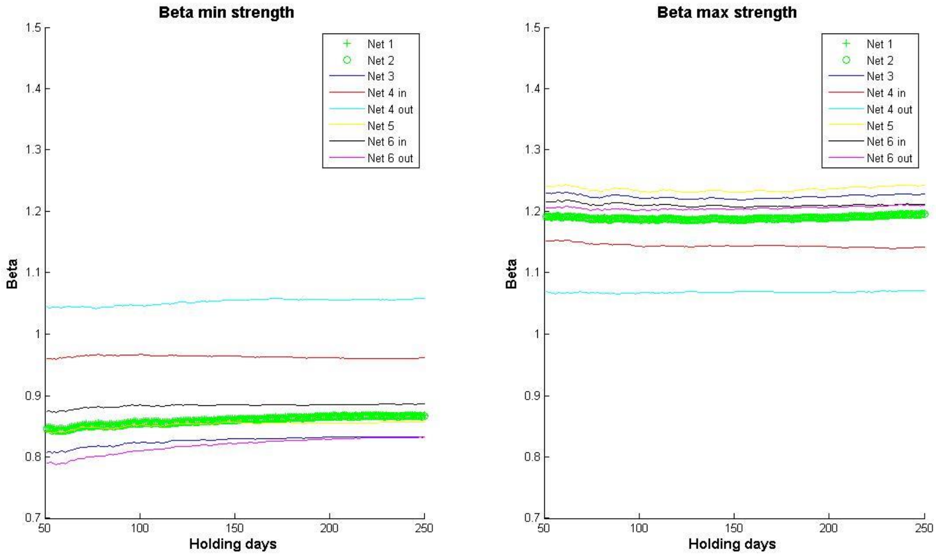

Image 52: Strength Beta

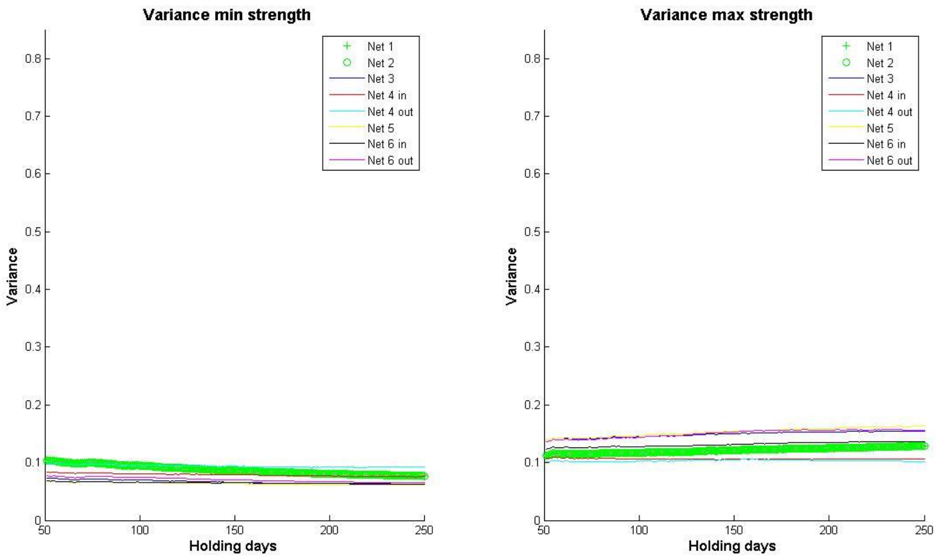

Image 53: Strength Variance

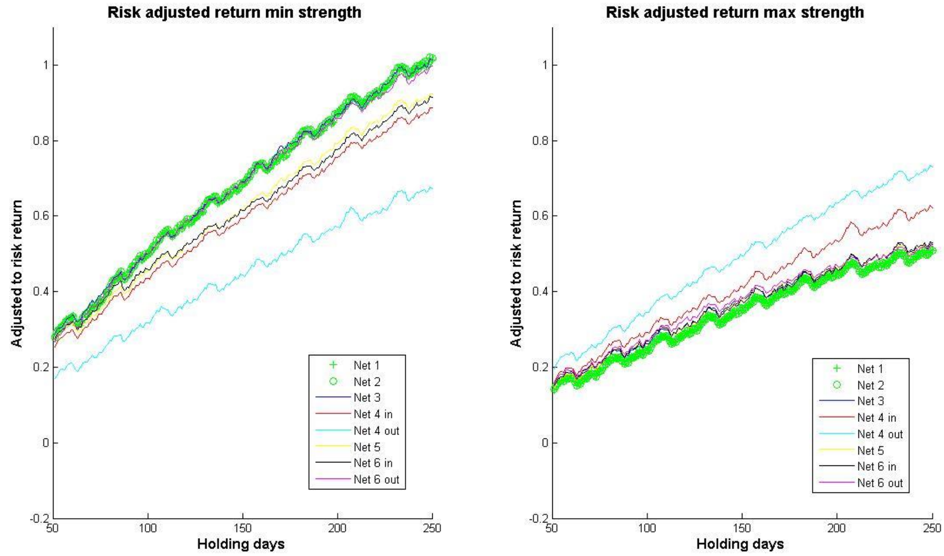

Image 54: Strength Adjusted to risk return

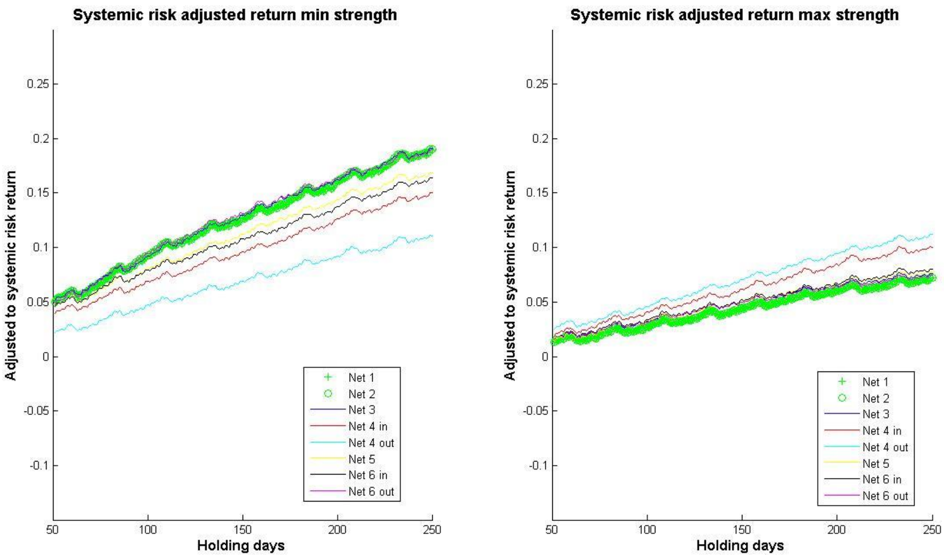

Image 55: Strength Adjusted to systemic risk return

Eigenvector

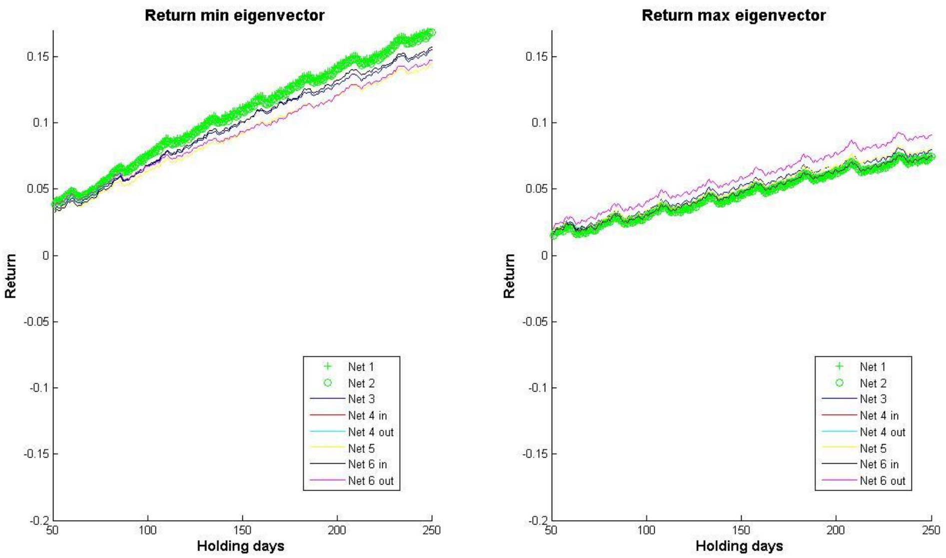

Image 56: Eigenvector Return

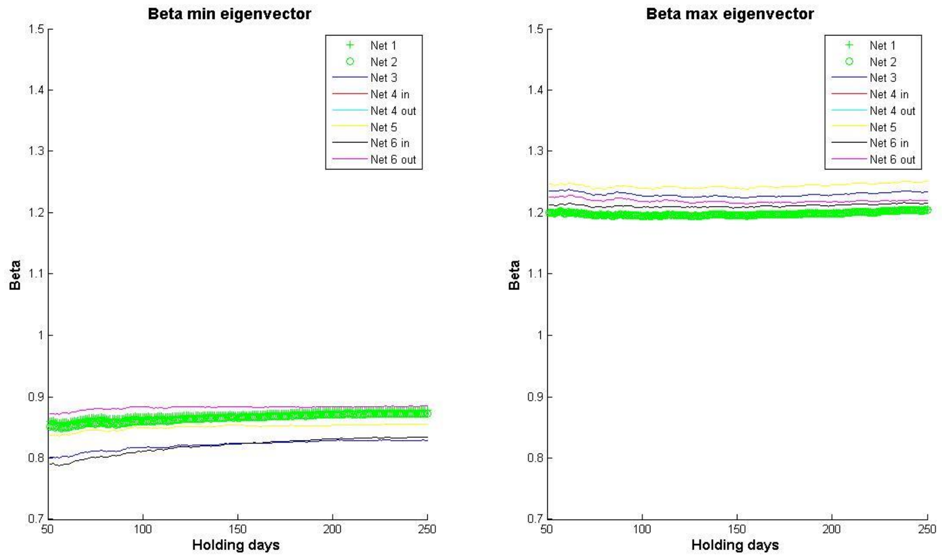

Image 57: Eigenvector Beta

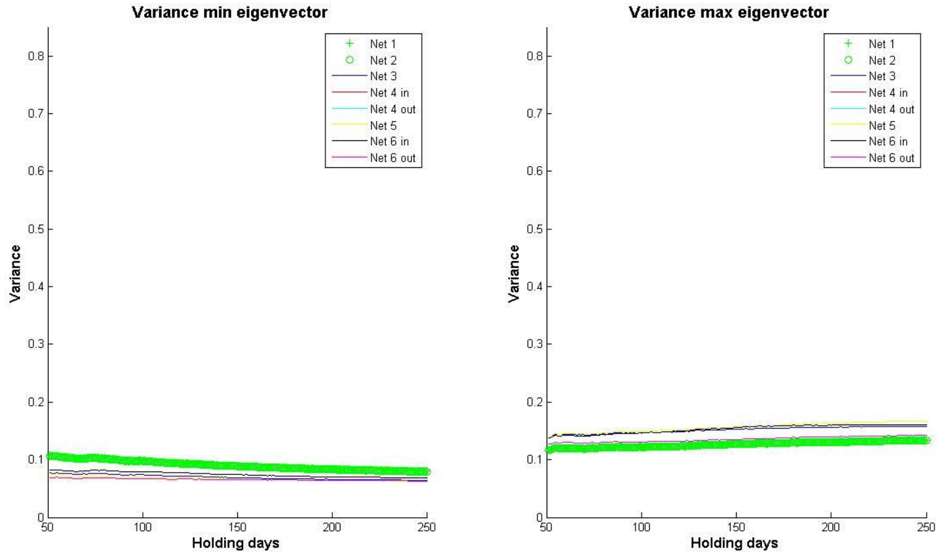

Image 58: Eigenvector Variance

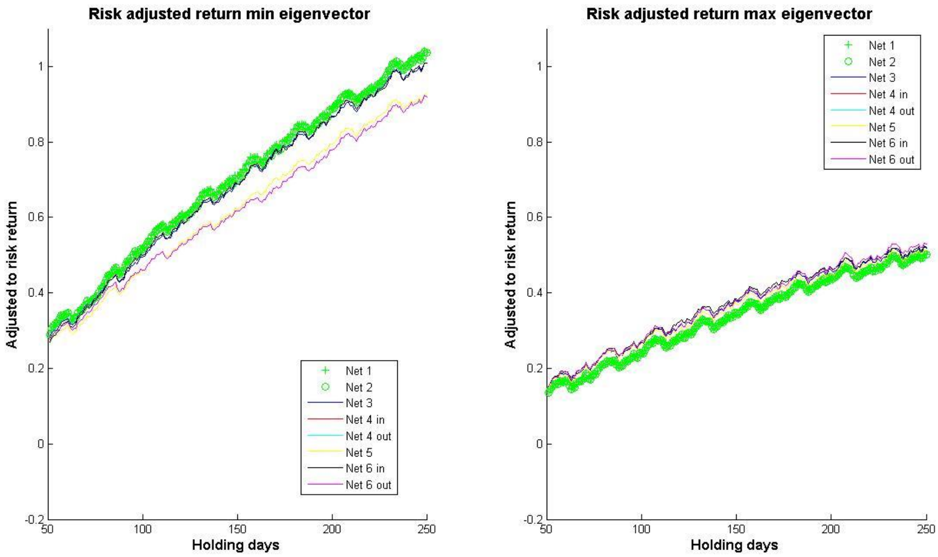

Image 59: Eigenvector Adjusted to risk return

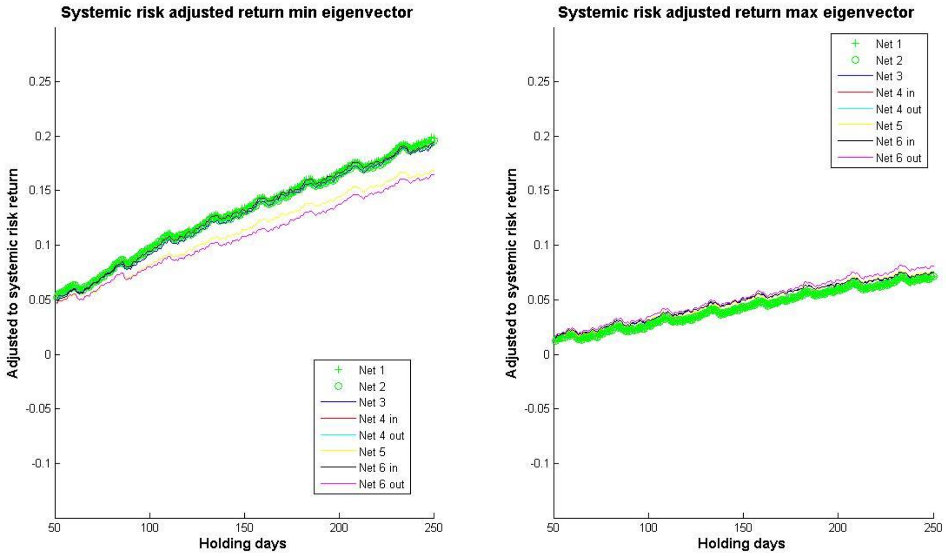

Image 60: Eigenvector Adjusted to systemic risk return

Closeness

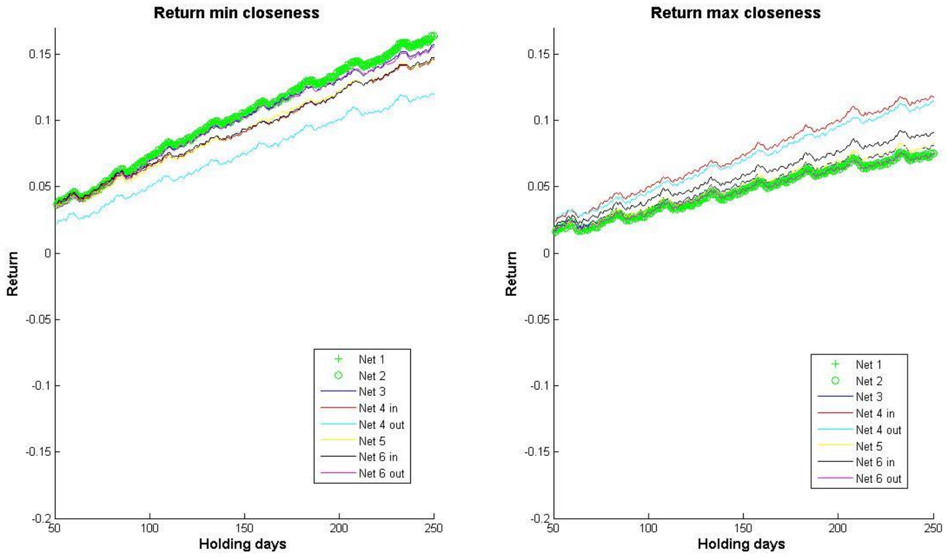

Image 61: Closeness return

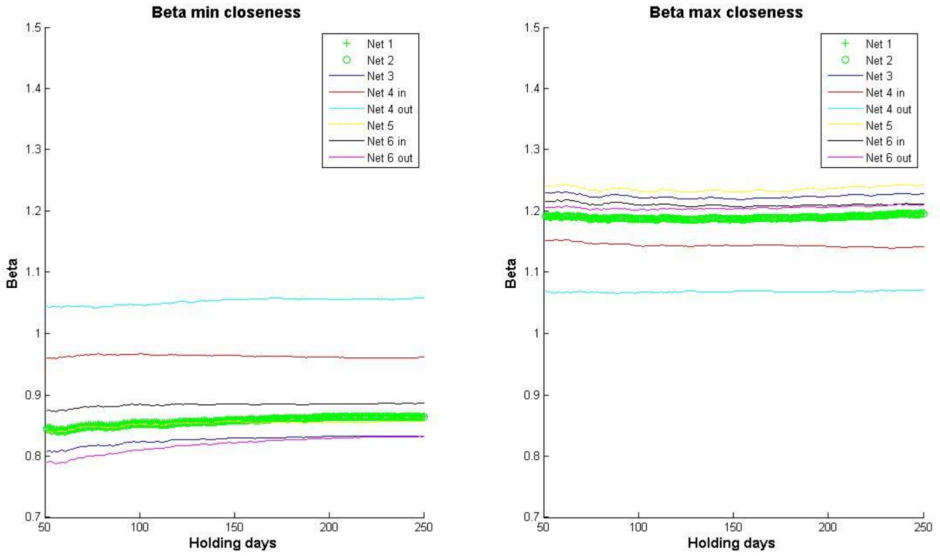

Image 62: Closeness beta

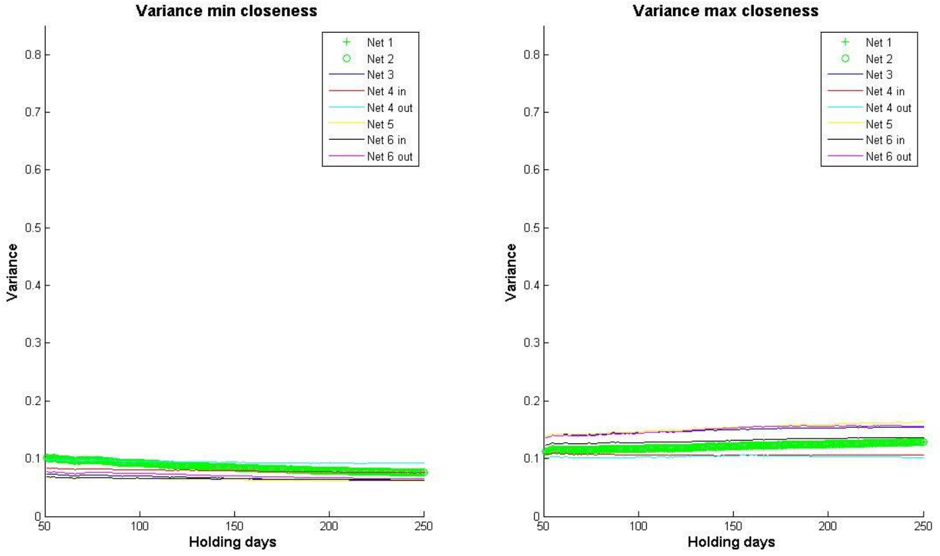

Image 63: Closeness variance

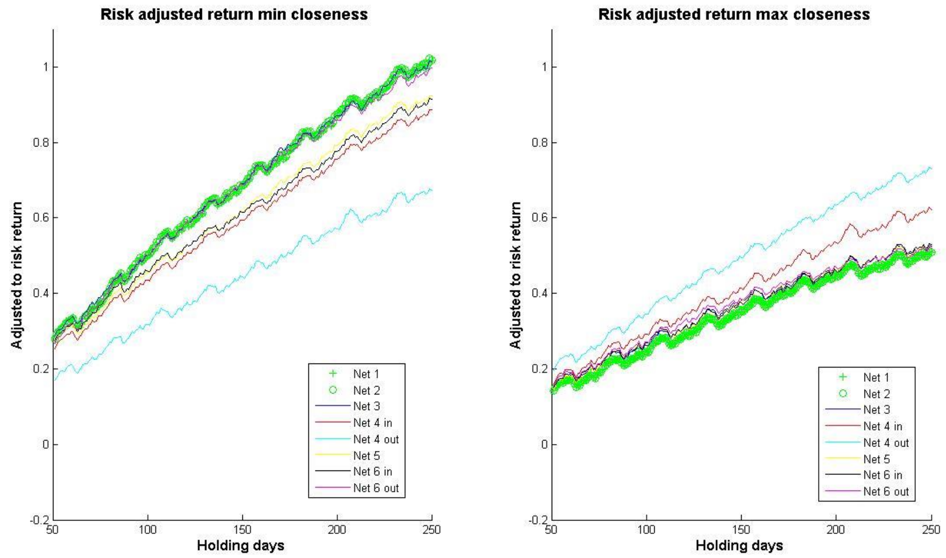

Image 64: Closeness Adjusted to risk return

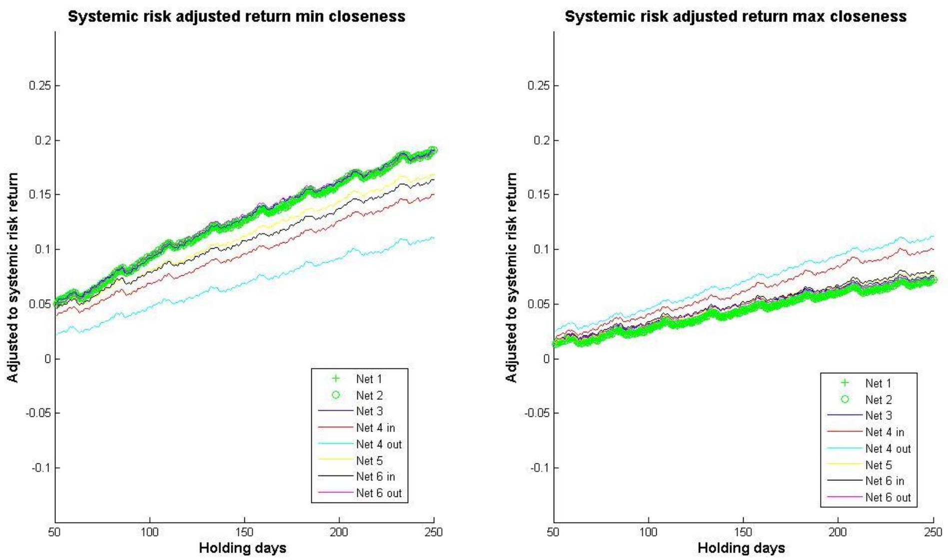

Image 64: Closeness Adjusted to systemic risk return

Betweenness

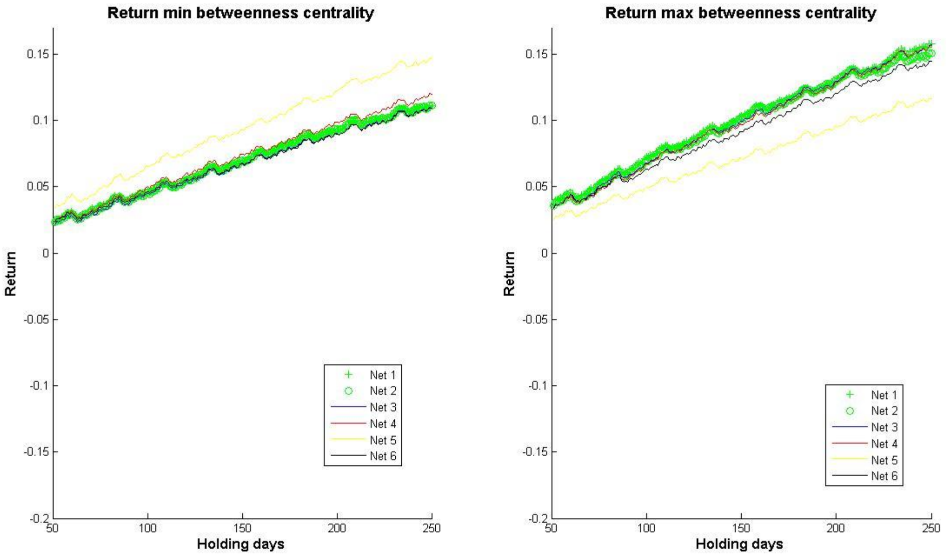

Image 65: Betweenness Return

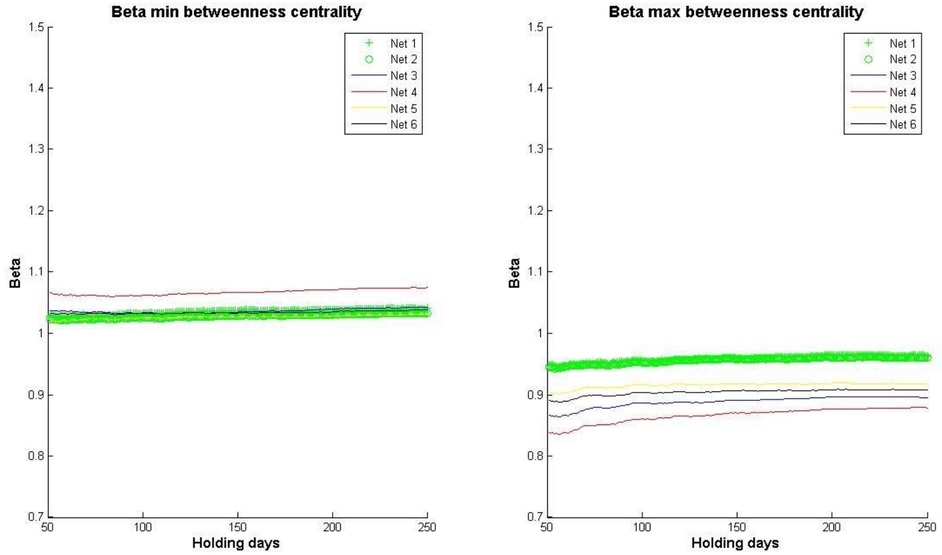

Image 66: Betweenness beta

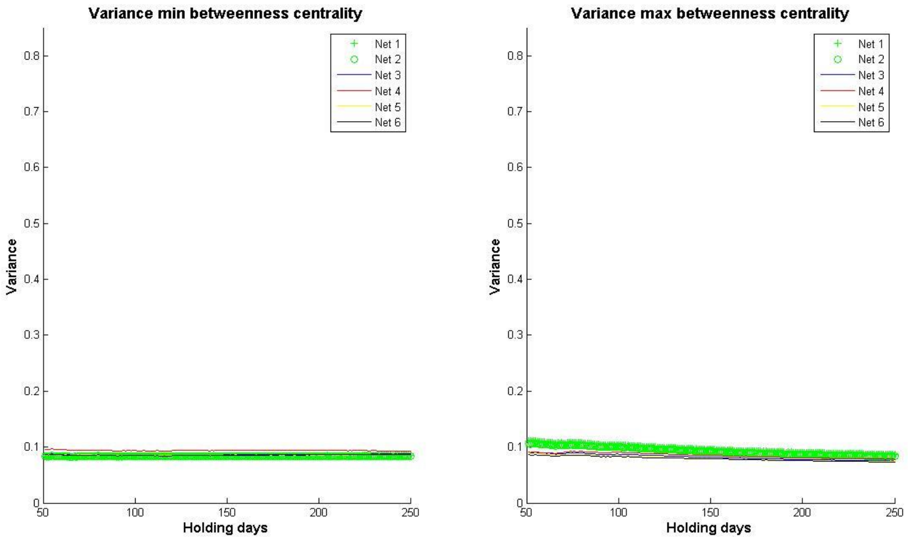

Image 67: Betweenness variance

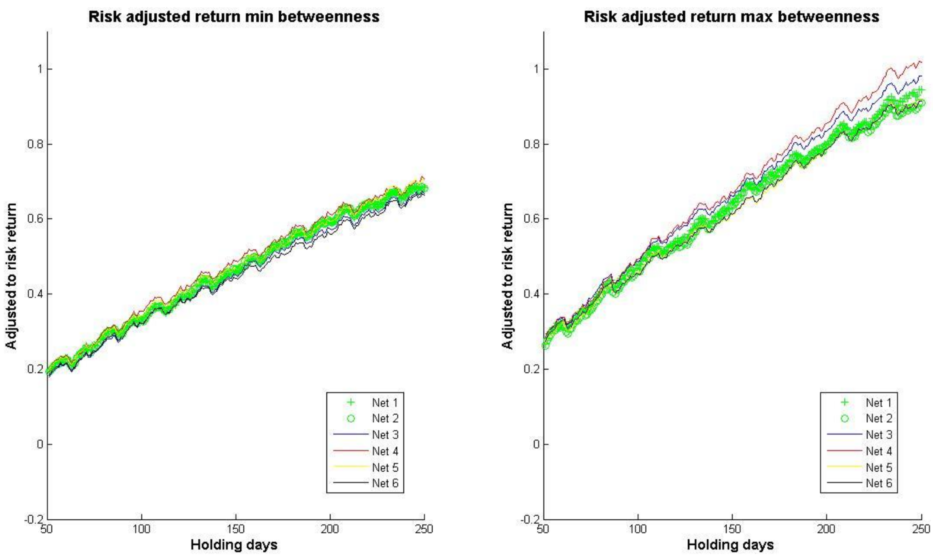

Image 68: Betweenness Adjusted to risk return

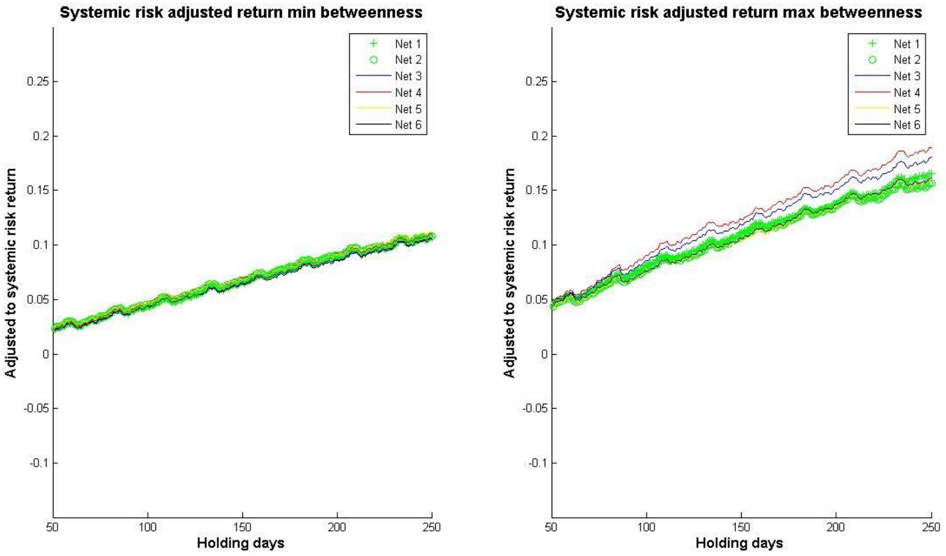

Image 69: Betweenness Adjusted to systemic risk return

Eccentricity

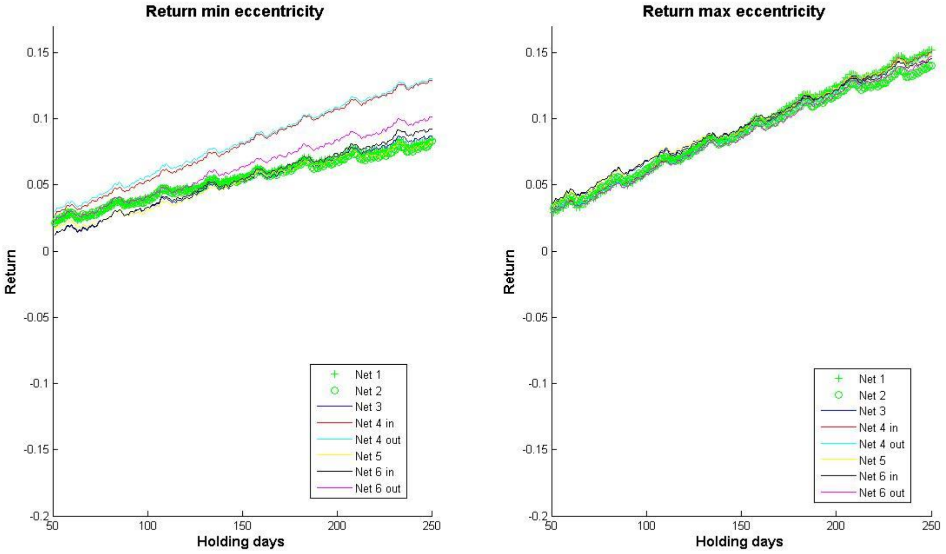

Image 70: Eccentricity return

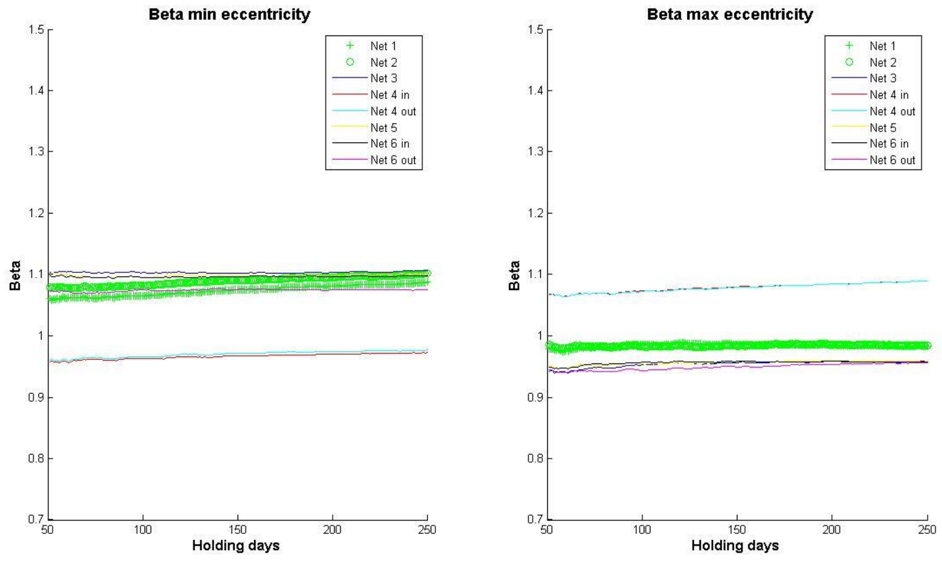

Image 71: Eccentricity beta

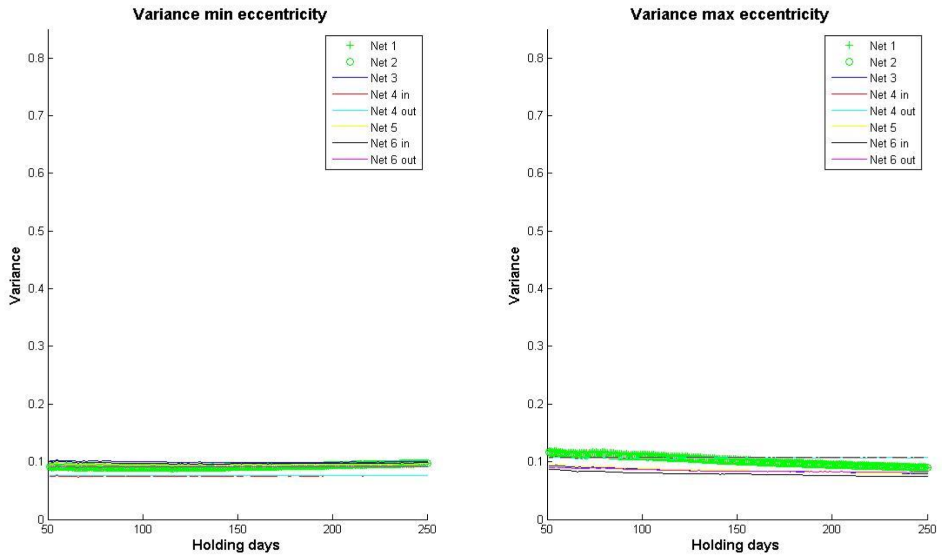

Image 72: Eccentricity variance

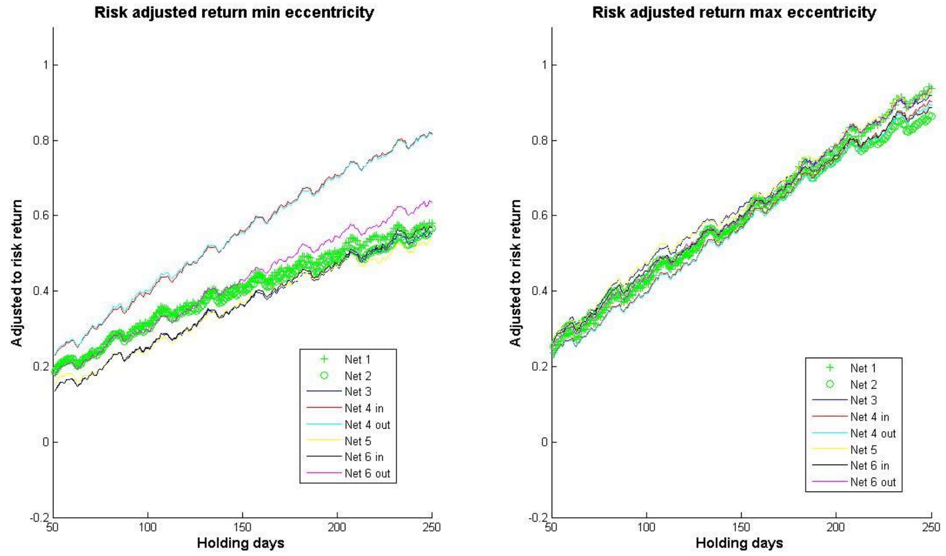

Image 73: Eccentricity Adjusted to risk return

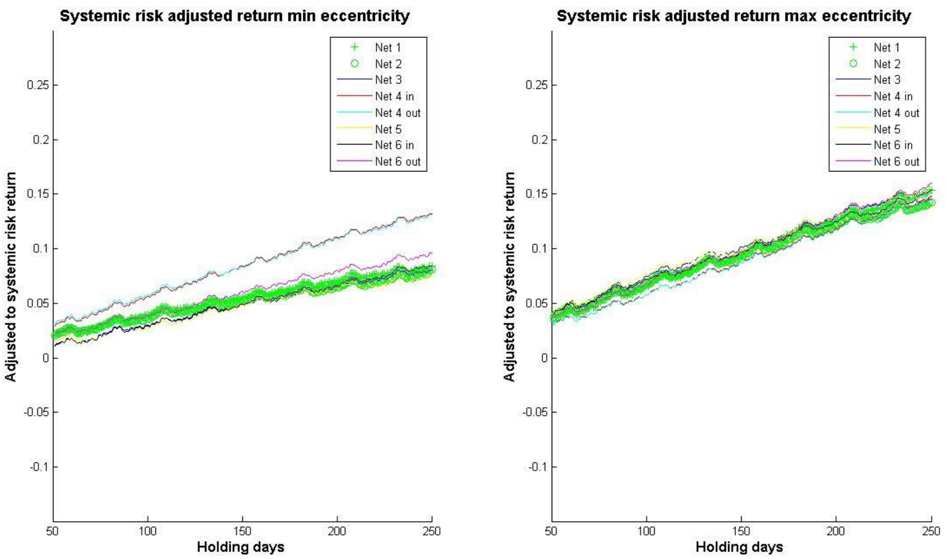

Image 74: Eccentricity Adjusted to systemic risk return

Appendix A.2- Crisis Period

Strength

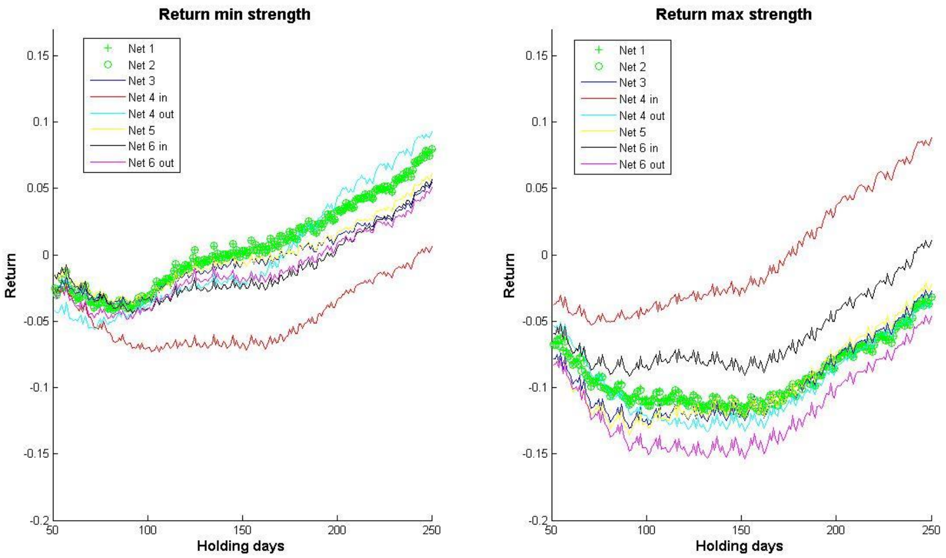

Image 75: Strength return during the crisis

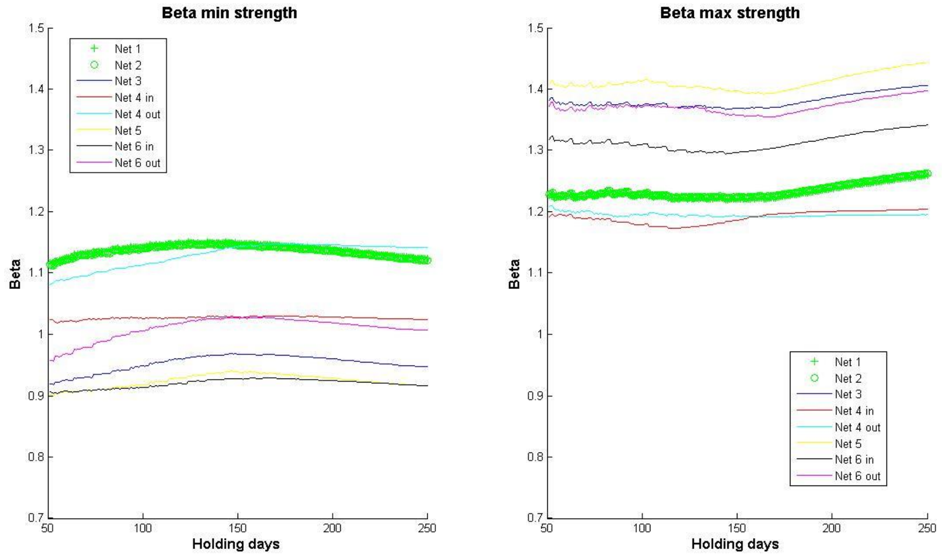

Image 76: Strength beta during the crisis

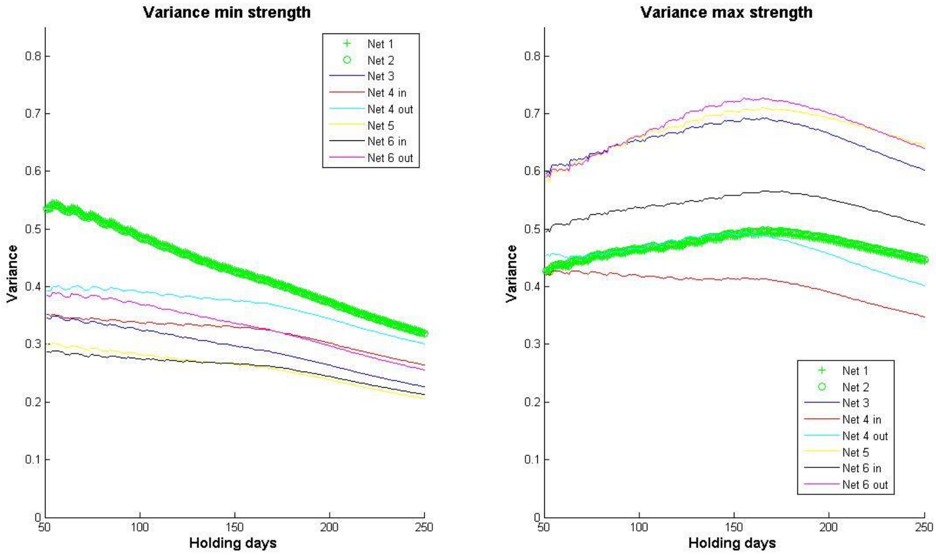

Image 77: Strength variance during the crisis

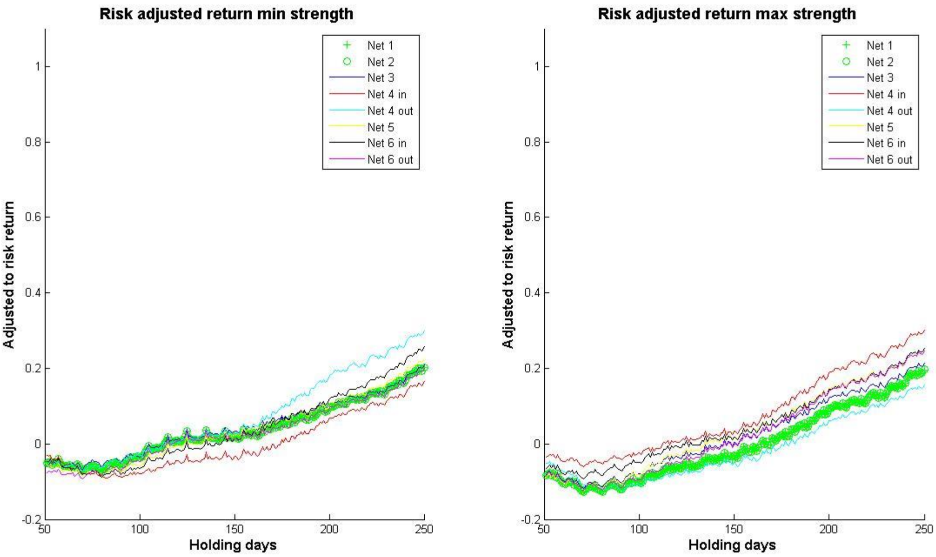

Image 78: Strength Adjusted to risk return during the crisis

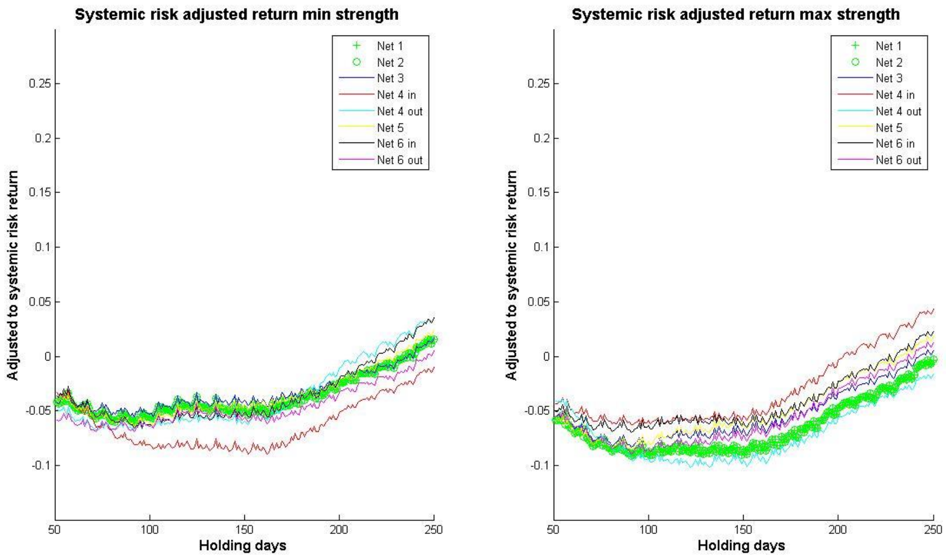

Image 79: Strength Adjusted to systemic risk return during the crisis

Eigenvector

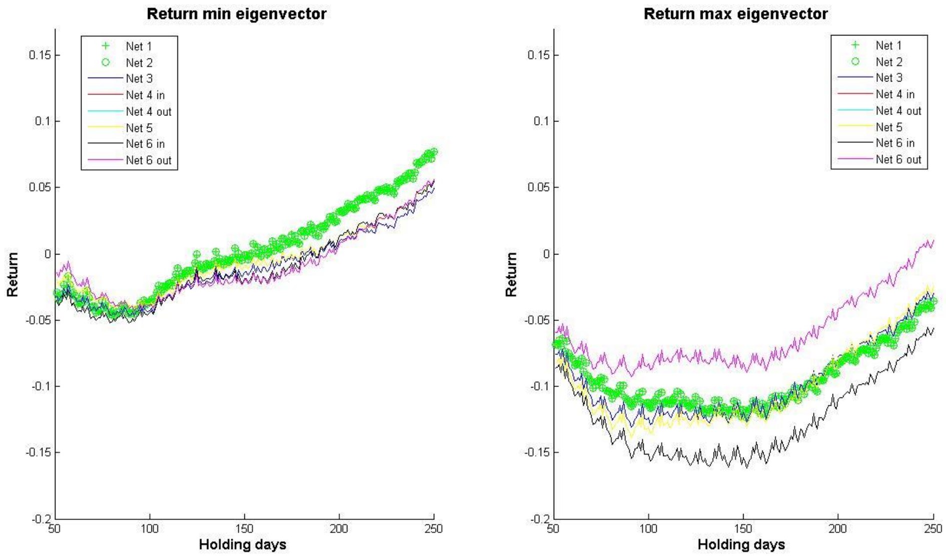

Image 80: Eigenvector return during the crisis

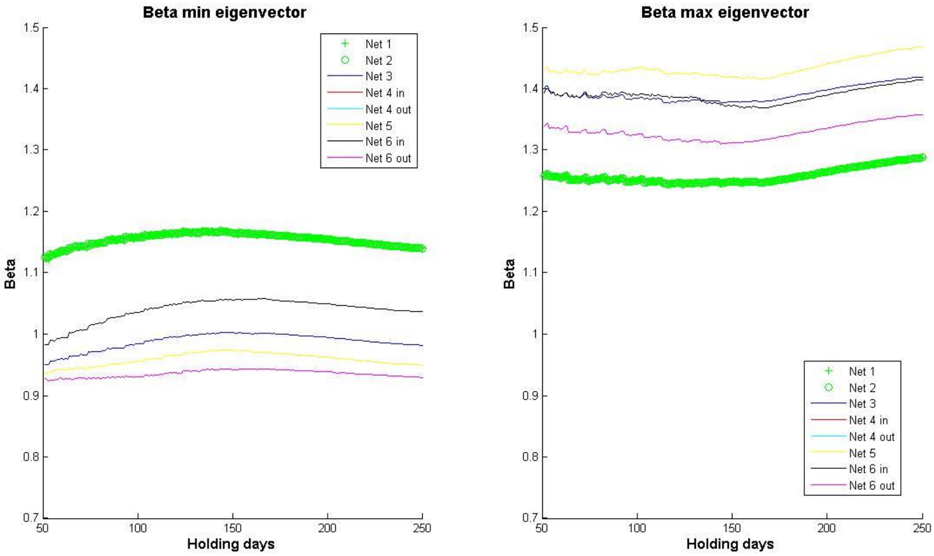

Image 81: Eigenvector beta during the crisis

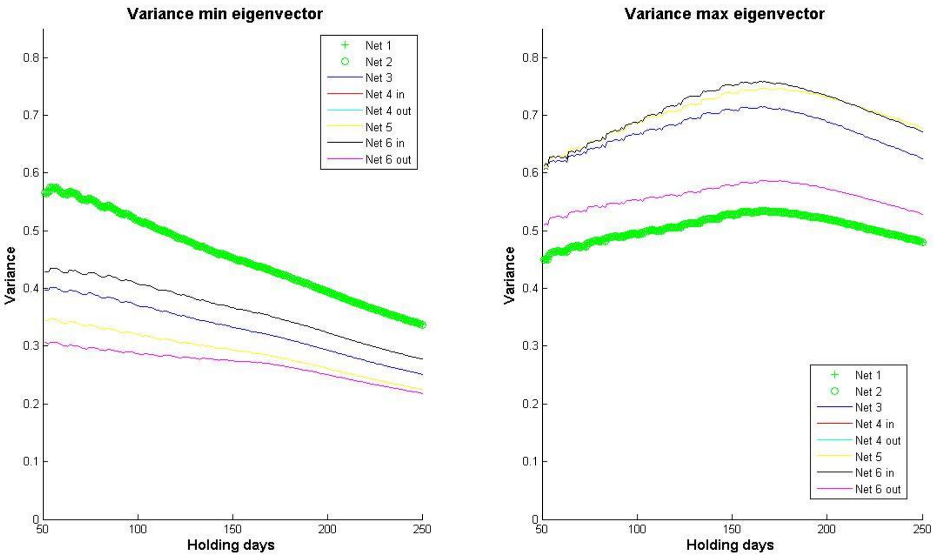

Image 82: Eigenvector variance during the crisis

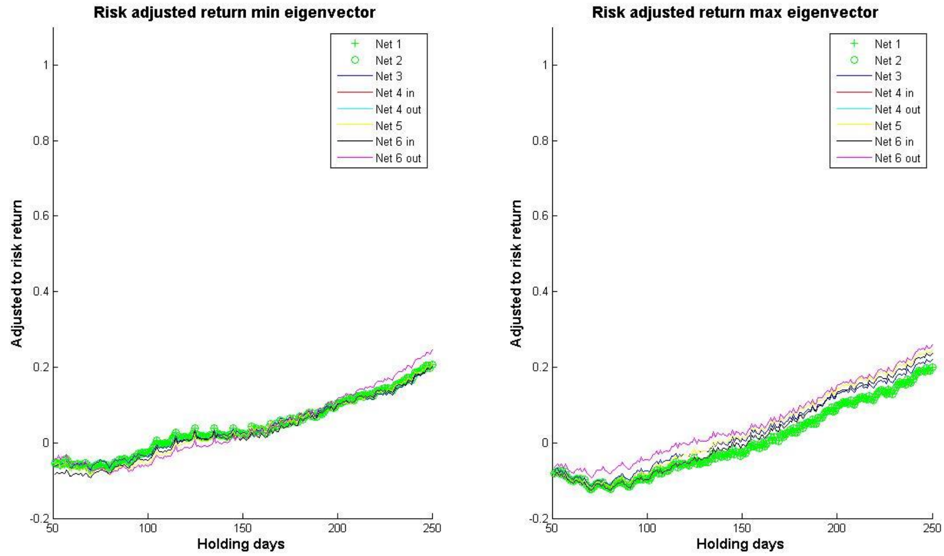

Image 83: Eigenvector Adjusted to risk return during the crisis

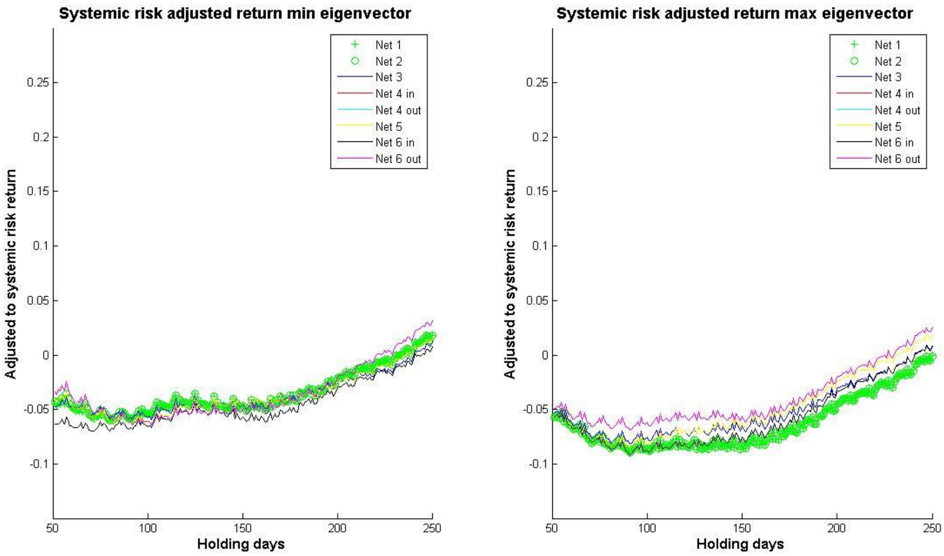

Image 84: Eigenvector Adjusted to systemic risk return during the crisis

Closeness

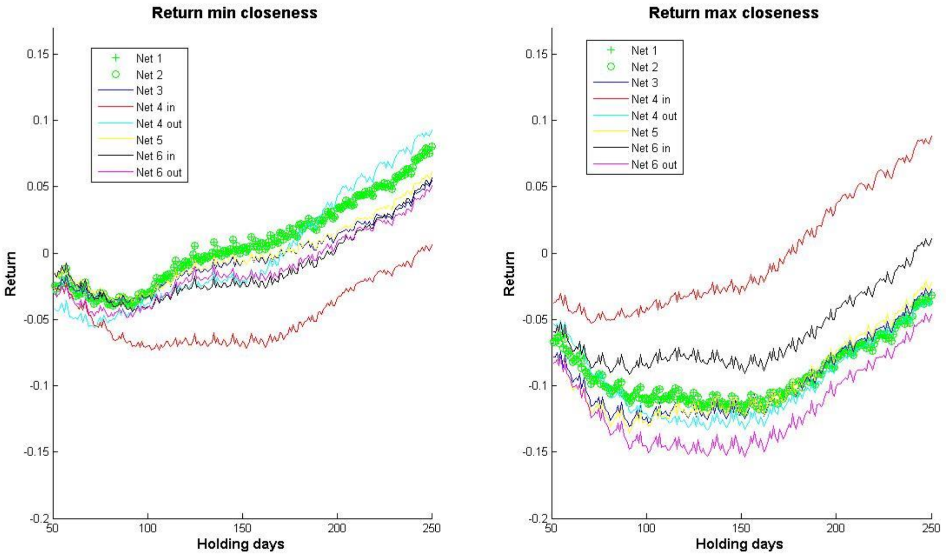

Image 85: Closeness return during the crisis

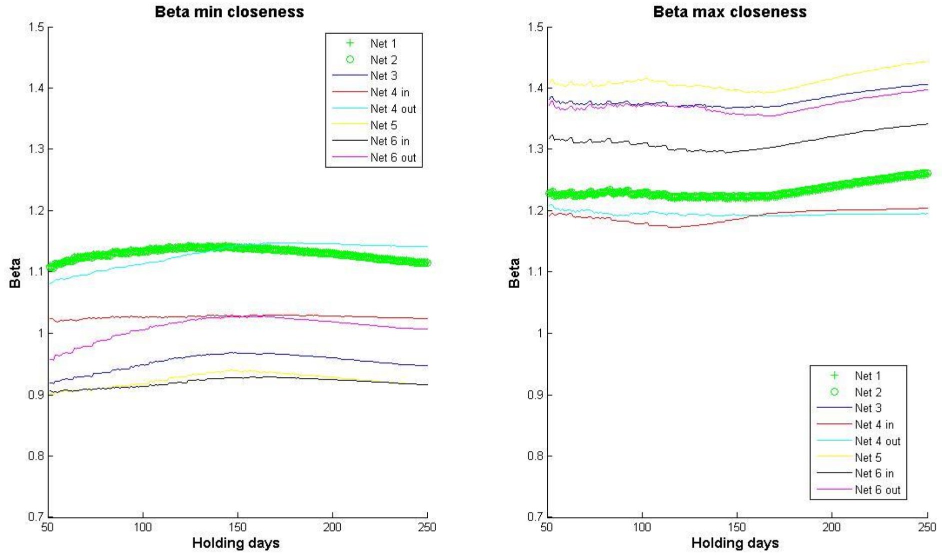

Image 86: Closeness beta during the crisis

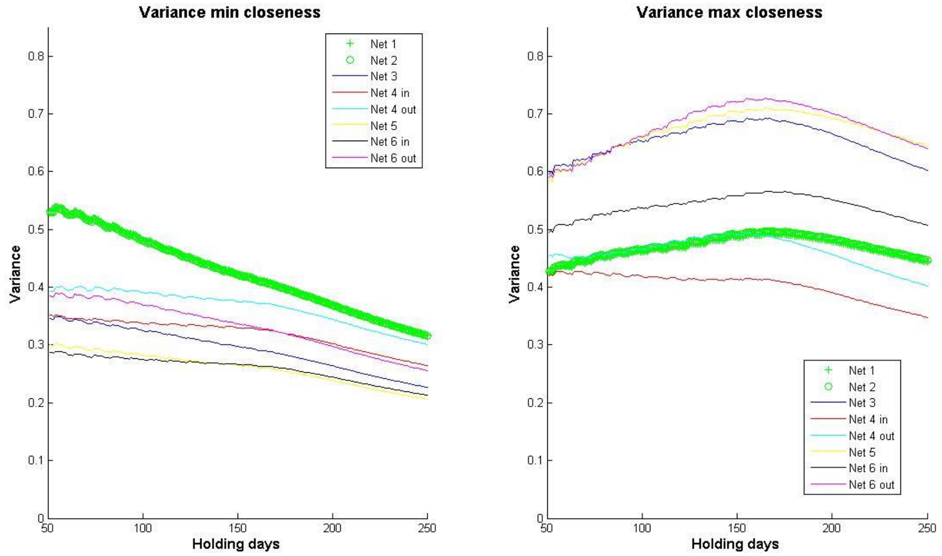

Image 87: Closeness variance during the crisis

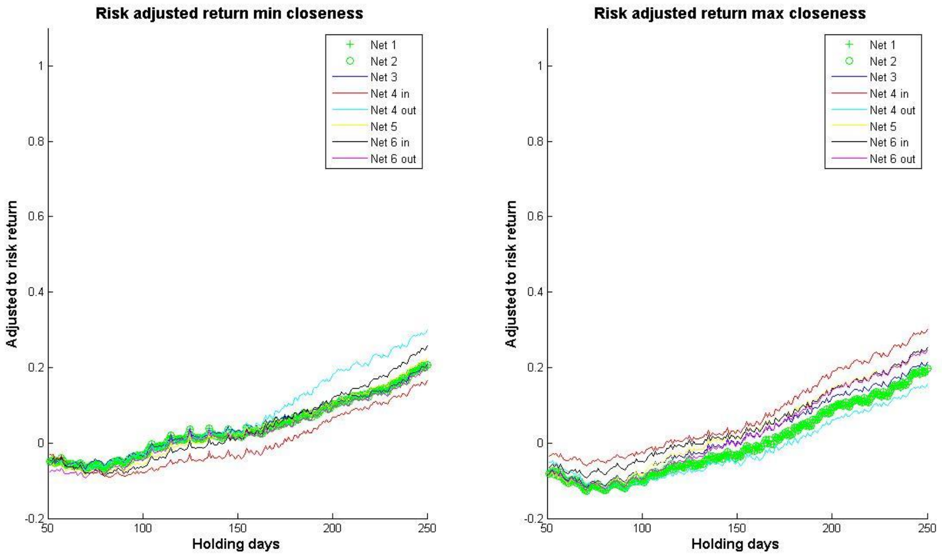

Image 88: Closeness Adjusted to risk return during the crisis

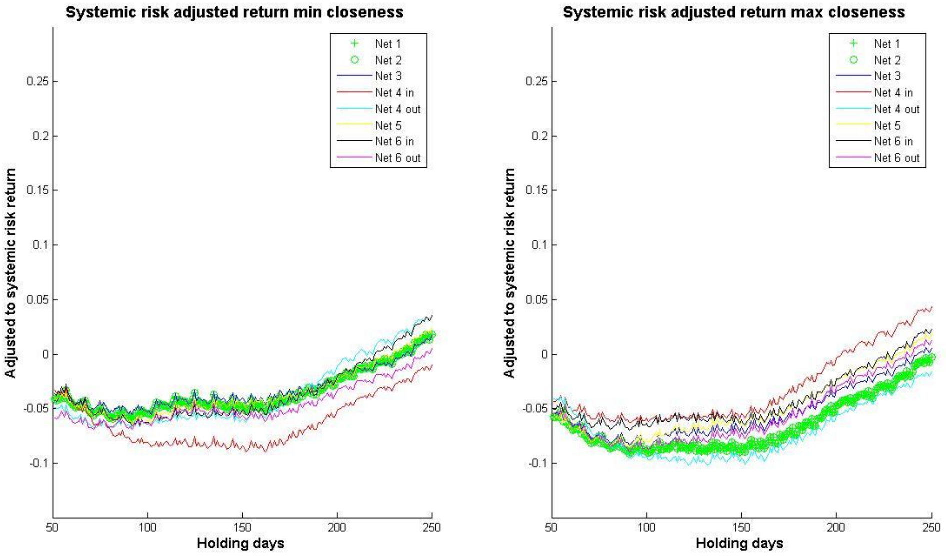

Image 89: Closeness Adjusted to systemic risk return during the crisis

Betweenness

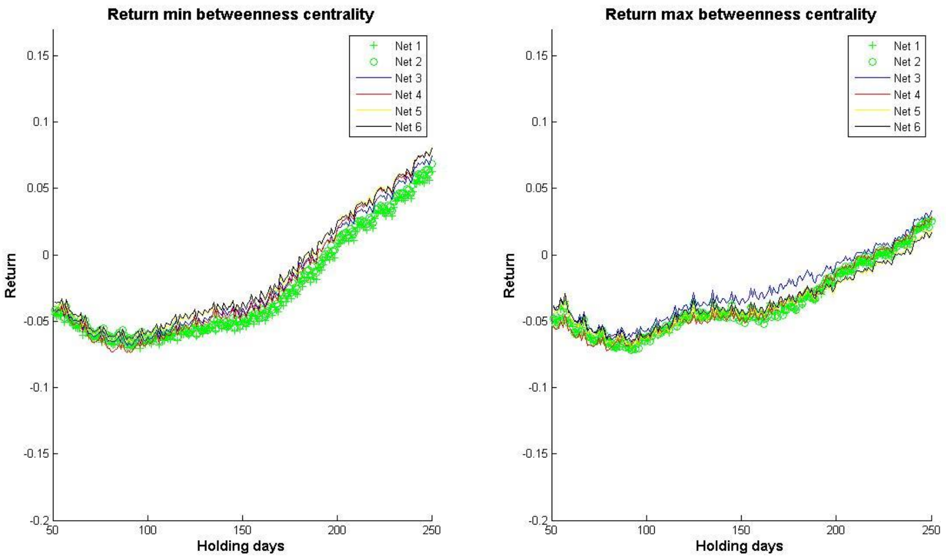

Image 90: Betweenness return during the crisis

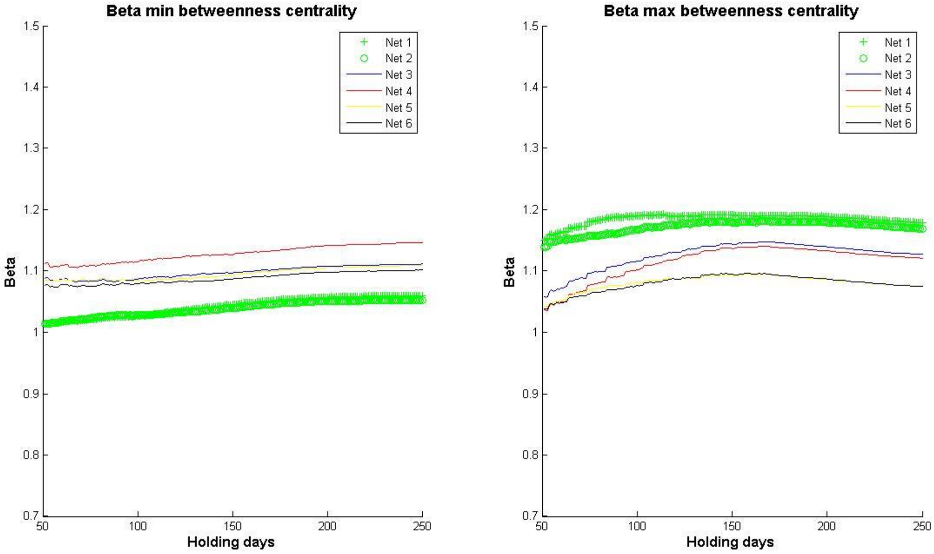

Image 91: Betweenness beta during the crisis

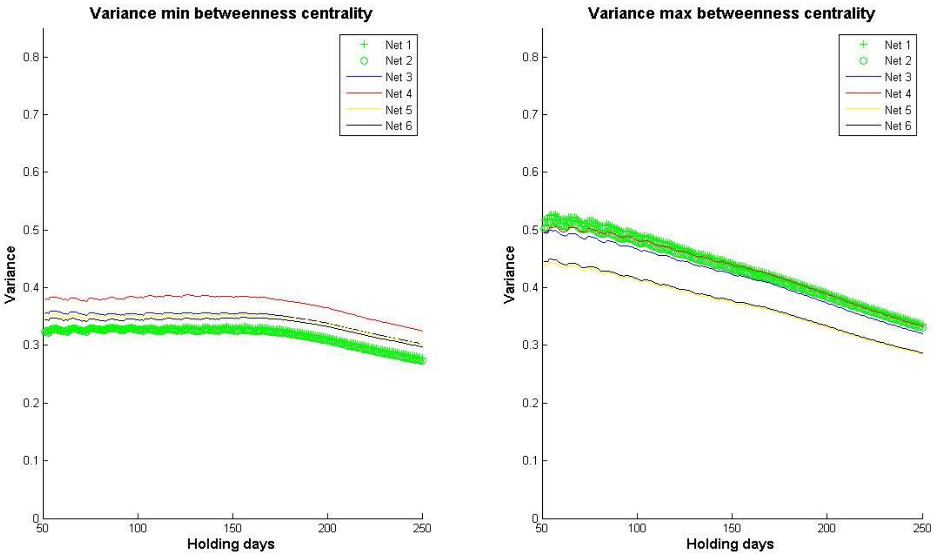

Image 92: Betweenness variance during the crisis

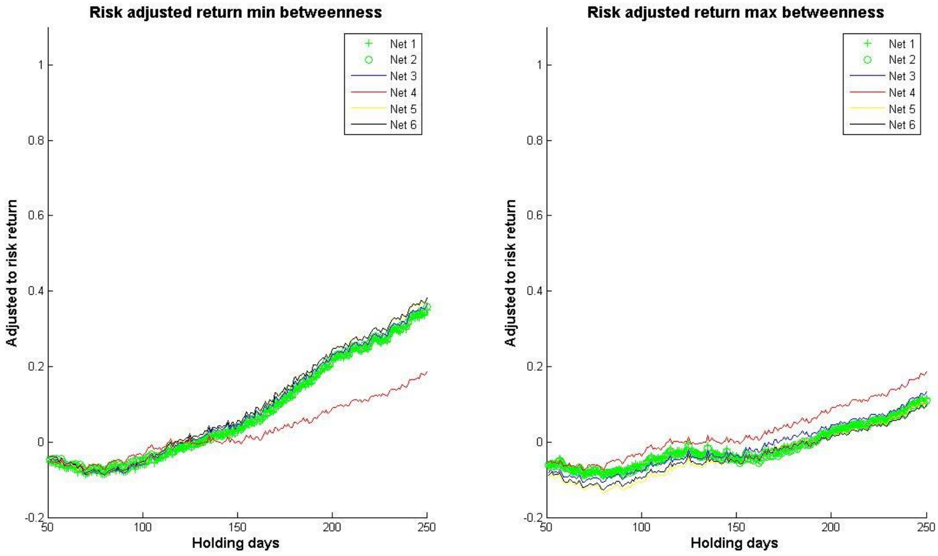

Image 93: Betweenness Adjusted to risk return during the crisis

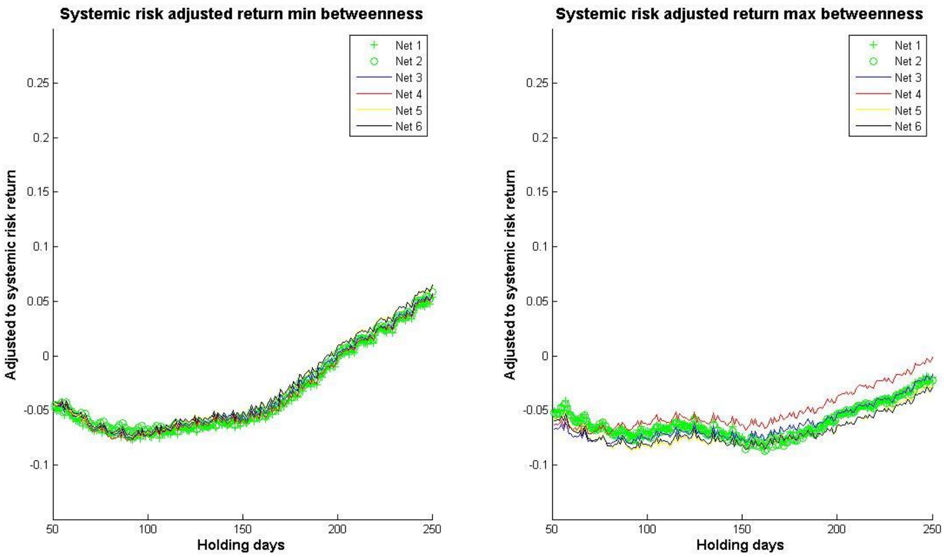

Image 94: Betweenness Adjusted to systemic risk return during the crisis

Eccentricity

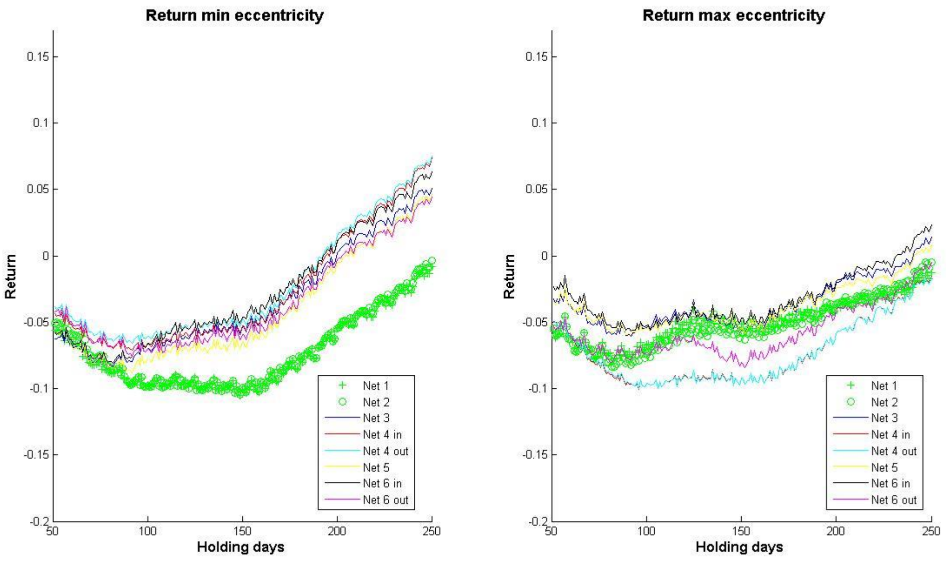

Image 95: Eccentricity return during the crisis

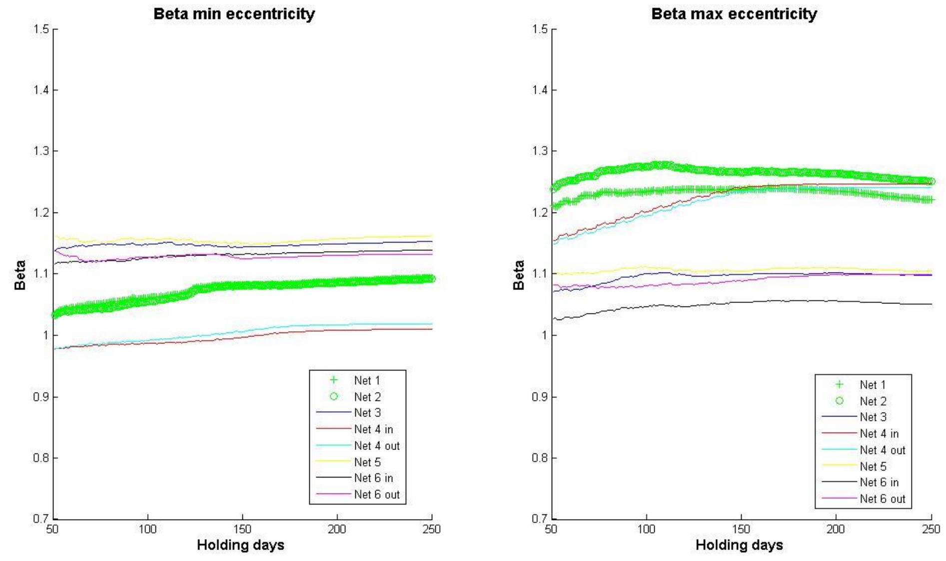

Image 96: Eccentricity beta during the crisis

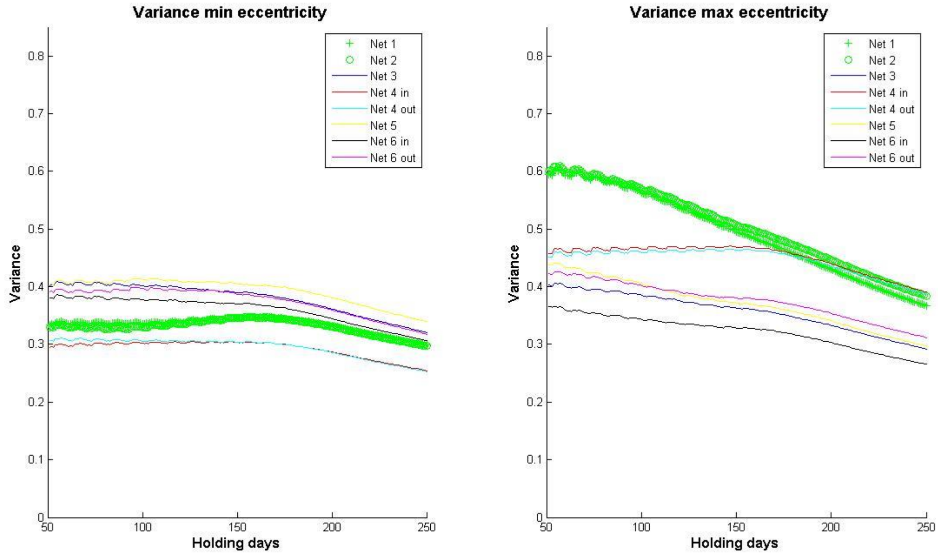

Image 97: Eccentricity variance during the crisis

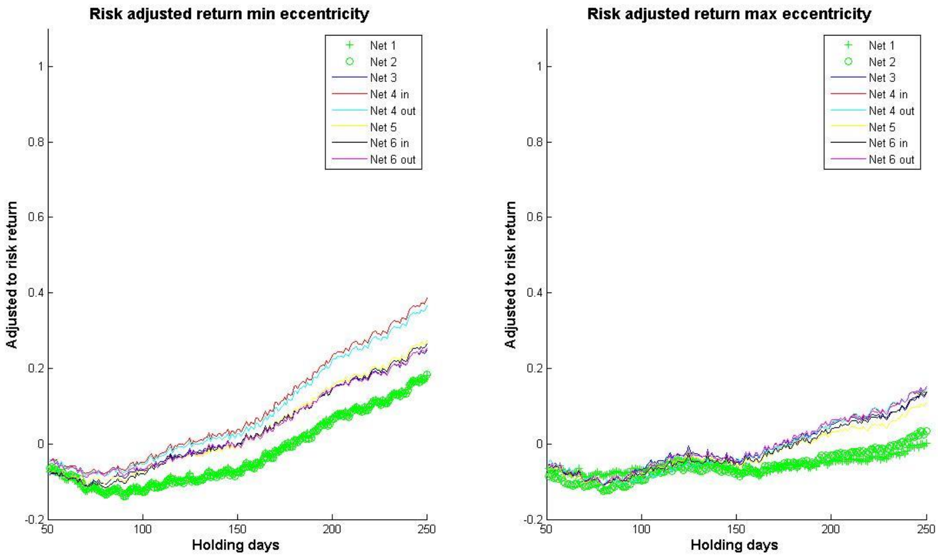

Image 97: Eccentricity Adjusted to risk return during the crisis

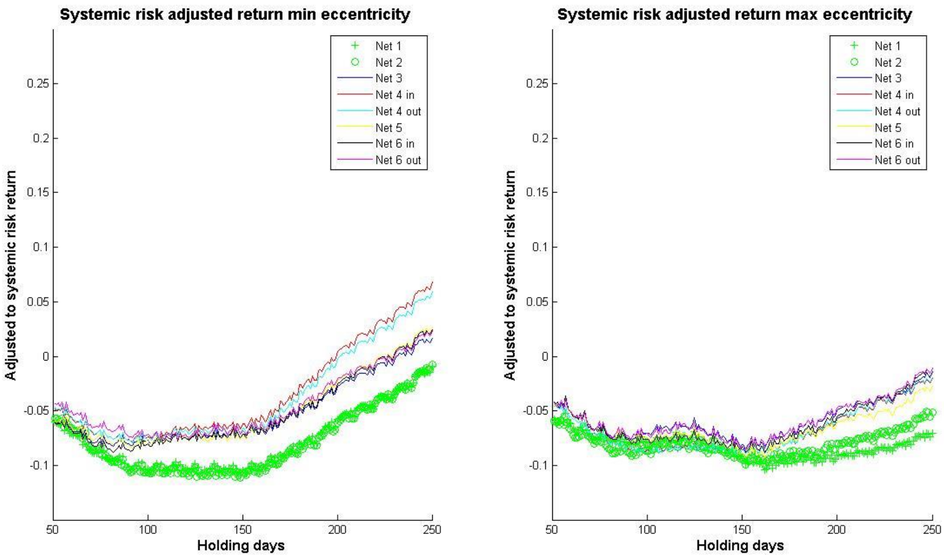

Image 98: Eccentricity Adjusted to systemic risk return during the crisis

Appendix B. Frequency Tables

Table 9: Frequency Table of best performances per performance criterion: whole testing period (Oct 2002-Dec 2012), holding days: 51-250

WHOLE PERIOD								
Strength	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	8,50%	81,50%	0,50%	0,00%	0,00%	0,00%	9,50%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	99,00%	1,00%	0,00%
Risk adj. return	2,00%	49,00%	43,50%	0,00%	0,00%	0,00%	0,00%	5,50%
Systemic risk adj.return	2,50%	1,00%	29,00%	0,00%	0,00%	0,00%	0,00%	67,50%
Eigenvector	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	100,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Beta	0,00%	0,00%	46,50%	0,00%	0,00%	0,00%	53,50%	0,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	31,50%	0,00%	68,50%
Risk adj. return	79,50%	20,50%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	76,50%	9,00%	0,00%	0,00%	0,00%	0,00%	14,50%	0,00%
Closeness	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	12,00%	81,00%	0,00%	0,00%	0,00%	0,00%	7,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	99,00%	1,00%	0,00%
Risk adj. return	2,00%	52,00%	41,50%	0,00%	0,00%	0,00%	0,00%	4,50%
Systemic risk adj.return	2,50%	2,50%	27,50%	0,00%	0,00%	0,00%	0,00%	67,50%
Betweenness	**Net 1**	**Net 2**	**Net 3**	**Net 4**	**Net 5**	**Net 6**		
Return	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%		
Beta	0,00%	18,00%	0,00%	0,00%	82,00%	0,00%		
Variance	36,50%	63,50%	0,00%	0,00%	0,00%	0,00%		
Risk adj. return	0,50%	0,00%	0,00%	88,50%	11,00%	0,00%		
Systemic risk adj.return	0,00%	0,00%	0,00%	30,00%	70,00%	0,00%		
Eccentricity	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	30,00%	0,00%	9,50%	0,00%	0,00%	24,50%	36,00%	0,00%
Beta	0,00%	0,00%	2,50%	0,00%	0,00%	0,00%	0,00%	97,50%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%
Risk adj. return	7,50%	0,00%	25,50%	0,00%	0,00%	64,50%	0,00%	2,50%
Systemic risk adj.return	0,00%	0,00%	19,50%	0,00%	0,00%	60,50%	0,00%	20,00%

Table 10: Frequency Table of best performances per performance criterion: whole testing period(Oct 2002-Dec 2012), holding days: 151-250 (max horizon)

WHOLE PERIOD-MAX HORIZON								
Strength	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	2,00%	98,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%	0,00%
Risk adj. return	0,00%	56,00%	44,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	2,00%	32,00%	0,00%	0,00%	0,00%	0,00%	66,00%
Eigenvector	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	100,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Beta	0,00%	0,00%	93,00%	0,00%	0,00%	0,00%	7,00%	0,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	63,00%	0,00%	37,00%
Risk adj. return	96,00%	4,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	72,00%	0,00%	0,00%	0,00%	0,00%	0,00%	28,00%	0,00%
Closeness	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	1,00%	99,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%	0,00%
Risk adj. return	0,00%	60,00%	40,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	5,00%	29,00%	0,00%	0,00%	0,00%	0,00%	66,00%
Betweenness	**Net 1**	**Net 2**	**Net 3**	**Net 4**	**Net 5**	**Net 6**		
Return	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%		
Beta	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%		
Variance	0,00%	100,00%	0,00%	0,00%	0,00%	0,00%		
Risk adj. return	0,00%	0,00%	0,00%	87,00%	13,00%	0,00%		
Systemic risk adj.return	0,00%	0,00%	0,00%	36,00%	64,00%	0,00%		
Eccentricity	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	60,00%	0,00%	2,00%	0,00%	0,00%	19,00%	19,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%
Risk adj. return	15,00%	0,00%	14,00%	0,00%	0,00%	66,00%	0,00%	5,00%
Systemic risk adj.return	0,00%	0,00%	10,00%	0,00%	0,00%	50,00%	0,00%	40,00%

Table 11: Frequency Table of best performances per performance criterion: whole testing period(Oct 2002-Dec 2012), holding days: 51-150 (min horizon)

WHOLE PERIOD-MIN HORIZON								
Strength	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	15,00%	65,00%	1,00%	0,00%	0,00%	0,00%	19,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	98,00%	2,00%	0,00%
Risk adj. return	4,00%	42,00%	43,00%	0,00%	0,00%	0,00%	0,00%	11,00%
Systemic risk adj.return	5,00%	0,00%	26,00%	0,00%	0,00%	0,00%	0,00%	69,00%
Eigenvector	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	100,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%
Risk adj. return	63,00%	37,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	81,00%	18,00%	0,00%	0,00%	0,00%	0,00%	1,00%	0,00%
Closeness	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	23,00%	63,00%	0,00%	0,00%	0,00%	0,00%	14,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	98,00%	2,00%	0,00%
Risk adj. return	4,00%	44,00%	43,00%	0,00%	0,00%	0,00%	0,00%	9,00%
Systemic risk adj.return	5,00%	0,00%	26,00%	0,00%	0,00%	0,00%	0,00%	69,00%
Betweenness	**Net 1**	**Net 2**	**Net 3**	**Net 4**	**Net 5**	**Net 6**		
Return	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%		
Beta	0,00%	36,00%	0,00%	0,00%	64,00%	0,00%		
Variance	73,00%	27,00%	0,00%	0,00%	0,00%	0,00%		
Risk adj. return	1,00%	0,00%	0,00%	90,00%	9,00%	0,00%		
Systemic risk adj.return	0,00%	0,00%	0,00%	24,00%	76,00%	0,00%		
Eccentricity	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	0,00%	0,00%	17,00%	0,00%	0,00%	30,00%	53,00%	0,00%
Beta	0,00%	0,00%	5,00%	0,00%	0,00%	0,00%	0,00%	95,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%
Risk adj. return	0,00%	0,00%	37,00%	0,00%	0,00%	63,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	0,00%	29,00%	0,00%	0,00%	71,00%	0,00%	0,00%

Table 12: Frequency Table of best performances per performance criterion: financial crisis period(Aug 2007-Mar 2009), holding days: 51-250

CRISIS								
Strength	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	3,00%	39,50%	1,00%	0,00%	32,00%	20,50%	4,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	10,50%	89,50%	0,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	57,00%	43,00%	0,00%
Risk adj. return	0,00%	0,00%	36,50%	5,50%	58,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	0,00%	59,00%	1,00%	32,50%	0,00%	7,50%	0,00%
Eigenvector	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	29,50%	47,50%	0,00%	0,00%	0,00%	5,00%	0,00%	18,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%
Risk adj. return	14,00%	12,00%	27,50%	0,00%	0,00%	4,00%	0,00%	42,50%
Systemic risk adj.return	9,00%	18,00%	18,50%	0,00%	0,00%	15,00%	0,00%	39,50%
Closeness	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	0,00%	45,50%	1,00%	0,00%	31,50%	18,00%	4,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	10,50%	89,50%	0,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	57,00%	43,00%	0,00%
Risk adj. return	0,00%	1,50%	35,00%	5,50%	58,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	0,00%	59,00%	1,00%	32,50%	0,00%	7,50%	0,00%
Betweenness	**Net 1**	**Net 2**	**Net 3**	**Net 4**	**Net 5**	**Net 6**		
Return	0,00%	0,50%	0,00%	0,50%	26,00%	73,00%		
Beta	25,00%	75,00%	0,00%	0,00%	0,00%	0,00%		
Variance	6,50%	93,50%	0,00%	0,00%	0,00%	0,00%		
Risk adj. return	0,00%	1,50%	0,00%	25,50%	16,00%	57,00%		
Systemic risk adj.return	0,00%	17,00%	0,00%	0,00%	37,50%	45,50%		
Eccentricity	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	0,00%	0,00%	13,50%	0,00%	0,00%	2,50%	84,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%
Risk adj. return	17,00%	0,00%	24,00%	5,00%	16,50%	0,00%	0,00%	37,50%
Systemic risk adj.return	0,00%	0,00%	39,00%	0,00%	0,00%	1,00%	8,50%	51,50%

Table 13: Frequency Table of best performances per performance criterion: financial crisis period(Aug 2007-Mar 2009), holding days: 151-250 (max horizon)

CRISIS-MAX HORIZON								
Strength	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	6,00%	30,00%	0,00%	0,00%	64,00%	0,00%	0,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%	0,00%
Risk adj. return	0,00%	0,00%	2,00%	0,00%	98,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	0,00%	32,00%	0,00%	65,00%	0,00%	3,00%	0,00%
Eigenvector	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	48,00%	52,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%
Risk adj. return	0,00%	1,00%	20,00%	0,00%	0,00%	8,00%	0,00%	71,00%
Systemic risk adj.return	12,00%	0,00%	17,00%	0,00%	0,00%	17,00%	0,00%	54,00%
Closeness	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	0,00%	37,00%	0,00%	0,00%	63,00%	0,00%	0,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%	0,00%
Risk adj. return	0,00%	0,00%	2,00%	0,00%	98,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	0,00%	32,00%	0,00%	65,00%	0,00%	3,00%	0,00%
Betweenness	**Net 1**	**Net 2**	**Net 3**	**Net 4**	**Net 5**	**Net 6**		
Return	0,00%	0,00%	0,00%	1,00%	52,00%	47,00%		
Beta	0,00%	100,00%	0,00%	0,00%	0,00%	0,00%		
Variance	0,00%	100,00%	0,00%	0,00%	0,00%	0,00%		
Risk adj. return	0,00%	0,00%	0,00%	0,00%	28,00%	72,00%		
Systemic risk adj.return	0,00%	0,00%	0,00%	0,00%	47,00%	53,00%		
Eccentricity	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%
Risk adj. return	0,00%	0,00%	4,00%	2,00%	27,00%	0,00%	0,00%	67,00%
Systemic risk adj.return	0,00%	0,00%	31,00%	0,00%	0,00%	0,00%	1,00%	68,00%

Table 14: Frequency Table of best performances per performance criterion: financial crisis period(Aug 2007-Mar 2009), holding days: 51-250 (min horizon)

CRISIS-MIN HORIZON								
Strength	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	0,00%	49,00%	2,00%	0,00%	0,00%	41,00%	8,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	21,00%	79,00%	0,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	14,00%	86,00%	0,00%
Risk adj. return	0,00%	0,00%	71,00%	11,00%	18,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	0,00%	86,00%	2,00%	0,00%	0,00%	12,00%	0,00%
Eigenvector	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	11,00%	43,00%	0,00%	0,00%	0,00%	10,00%	0,00%	36,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%
Risk adj. return	28,00%	23,00%	35,00%	0,00%	0,00%	0,00%	0,00%	14,00%
Systemic risk adj.return	6,00%	36,00%	20,00%	0,00%	0,00%	13,00%	0,00%	25,00%
Closeness	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	0,00%	54,00%	2,00%	0,00%	0,00%	36,00%	8,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	21,00%	79,00%	0,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	14,00%	86,00%	0,00%
Risk adj. return	0,00%	3,00%	68,00%	11,00%	18,00%	0,00%	0,00%	0,00%
Systemic risk adj.return	0,00%	0,00%	86,00%	2,00%	0,00%	0,00%	12,00%	0,00%
Betweenness	**Net 1**	**Net 2**	**Net 3**	**Net 4**	**Net 5**	**Net 6**		
Return	0,00%	1,00%	0,00%	0,00%	0,00%	99,00%		
Beta	50,00%	50,00%	0,00%	0,00%	0,00%	0,00%		
Variance	13,00%	87,00%	0,00%	0,00%	0,00%	0,00%		
Risk adj. return	0,00%	3,00%	0,00%	51,00%	4,00%	42,00%		
Systemic risk adj.return	0,00%	34,00%	0,00%	0,00%	28,00%	38,00%		
Eccentricity	**Net 1**	**Net 2**	**Net 3**	**Net 4 in**	**Net 4 out**	**Net 5**	**Net 6 in**	**Net 6 out**
Return	0,00%	0,00%	27,00%	0,00%	0,00%	5,00%	68,00%	0,00%
Beta	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%
Variance	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%
Risk adj. return	34,00%	0,00%	44,00%	8,00%	6,00%	0,00%	0,00%	8,00%
Systemic risk adj.return	0,00%	0,00%	47,00%	0,00%	0,00%	2,00%	16,00%	35,00%

Appendix C. -Stocks of the dataset

Table 15: Stocks included in the dataset

ABBOTT LABORATORIES	TARGET	LOWE'S COMPANIES	PRUDENTIAL FINL.
ADOBE SYSTEMS	DEERE	DOMINION RESOURCES	EDISON INTL.
E I DU PONT DE NEMOURS	MORGAN STANLEY	MCDONALDS	SOUTHERN
BOSTON PROPERTIES	WALT DISNEY	MARSH & MCLENNAN	BB&T
ALLSTATE	DOW CHEMICAL	METLIFE	AT&T
HONEYWELL INTL.	DTE ENERGY	CVS HEALTH	CHEVRON
AMGEN	EBAY	MICROSOFT	STATE STREET
HESS	BANK OF AMERICA	3M	STARBUCKS
AMERICAN EXPRESS	CITIGROUP	NATIONAL OILWELL VARCO	PUBLIC STORAGE
AFLAC	ECOLAB	NEWMONT MINING	STRYKER
AMERICAN INTL.GP.	EMERSON ELECTRIC	NIKE 'B'	SUNTRUST BANKS
ANADARKO PETROLEUM	EOG RES.	NOBLE ENERGY	SYMANTEC
ALEXION PHARMS.	EQUITY RESD.TST.PROPS. SHBI	NORFOLK SOUTHERN	SYSCO
VALERO ENERGY	ESTEE LAUDER COS.'A'	NISOURCE	TEXAS INSTRUMENTS
APACHE	EXXON MOBIL	COACH	THERMO FISHER SCIENTIFIC
APPLE	NEXTERA ENERGY	NORTHROP GRUMMAN	MARATHON OIL
ARCHER-DANLS.-MIDL.	MACY'S	WELLS FARGO & CO	UNION PACIFIC
AUTOMATIC DATA PROC.	FRANKLIN RESOURCES	MONSANTO	UNITED TECHNOLOGIES
BAKER HUGHES	FREEPORT-MCMORAN	CAPITAL ONE FINL.	UNITEDHEALTH GROUP
BERKSHIRE HATHAWAY 'B'	GAP	OCCIDENTAL PTL.	VORNADO REALTY TRUST
BAXTER INTL.	GENERAL DYNAMICS	ORACLE	WAL MART STORES
BECTON DICKINSON	GENERAL MILLS	PACCAR	WASTE MANAGEMENT
VERIZON COMMUNICATIONS	GILEAD SCIENCES	EXELON	WEYERHAEUSER
FIRSTENERGY	MCKESSON	PPL	WHOLE FOODS MARKET
BRISTOL MYERS SQUIBB	GENERAL ELECTRIC	PEPSICO	WILLIAMS
ONEOK	HALLIBURTON	PFIZER	YAHOO
SEMPRA EN.	GOLDMAN SACHS GP.	CONOCOPHILLIPS	TJX
FEDEX	HERSHEY	PG&E	MOLSON COORS BREWING 'B'
BROWN-FORMAN 'B'	REYNOLDS AMERICAN	ALTRIA GROUP	CBS 'B'
CSX	HOME DEPOT	PNC FINL.SVS.GP.	BANK OF NEW YORK MELLON
CONSTELLATION BRANDS 'A'	BIOGEN	AETNA	CHUBB
CARDINAL HEALTH	ILLINOIS TOOL WORKS	PPG INDUSTRIES	TRANSOCEAN
CATERPILLAR	INTUIT	PRAXAIR	PROLOGIS
CELGENE	INTEL	COSTCO WHOLESALE	ACCENTURE CLASS A
CENTURYLINK	INTERNATIONAL PAPER	T ROWE PRICE GROUP	RALPH LAUREN CL.A
JP MORGAN CHASE & CO.	JOHNSON & JOHNSON	PROCTER & GAMBLE	MOTOROLA SOLUTIONS
CIGNA	DEVON ENERGY	QUALCOMM	AON CLASS A
CISCO SYSTEMS	KELLOGG	REGENERON PHARMS.	INGERSOLL-RAND
COCA COLA	KIMBERLY-CLARK	US BANCORP	TIME WARNER
COLGATE-PALM.	BLACKROCK	MERCK & COMPANY	AMERICAN TOWER
CONSOLIDATED EDISON	ELI LILLY	PRICELINE GROUP	EXPRESS SCRIPTS HOLDING
CORNING	UNITED PARCEL SER.'B'	SCHLUMBERGER	DUKE ENERGY
CUMMINS	LOCKHEED MARTIN	CHARLES SCHWAB	MONDELEZ INTERNATIONAL CL.A
DANAHER	LOEWS	SHERWIN-WILLIAMS	EATON
COGNIZANT TECH.SLTN.'A'	CARNIVAL	SIMON PROPERTY GROUP	
MEDTRONIC	HP	JOHNSON CONTROLS INTL.	
WALGREENS BOOTS ALLIANCE	ALLERGAN	CROWN CASTLE INTL.	

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