Landform evolution using Discrete Markov chains

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Περίληψη

Σε αυτό το άρθρο παρουσιάζεται ένα μοντέλο εξέλιξης του γήινου αναγλύφου βασισμένο στη θεωρία των πιθανοτήτων. Οι κύριες παράμετροι που λαμβάνονται υπ' όψιν είναι η διάβρωση, η ανύψωση της περιοχής και ο χρόνος.

Έτσι το ανάγλυφο μπορεί να ταξινομηθεί σε μερικά περιγραφικά στάδια εξέλιξης, τα οποία θεωρούνται διάφορες καταστάσεις (states) ενός συστήματος.

Η ανάλυση του παραπάνω συστήματος γίνεται χρησιμοποιώντας τις διακριτές αλυσίδες Markov. Οι εξαγόμενοι πίνακες μας δίνουν όλα τα στοιχεία για την εξέλιξη του συστήματος στον χρόνο.

Abstract

In this paper I present a model for a long term landform evolution based on a probability theory. The main factors which were taken into account were the erosion, the magnitude of uplift and the time. Based on these factors the landforms can be classified into numerous descriptive stages, which are treated as the stages of a system. The analysis of the above system is achieved by using discrete Markov chains. The resulting matrices give a clear picture of the system evolution.

Introduction

The most influential long-term landscape models are certainly those proposed by Davis (1698) and Penck (1924). Nowadays our knowledge about the factors which are responsible for the landform development is much better than in the past.

We dispose a vast data concerning exogenic or endogenic processes, but even now the extreme interrelations between the different factors and process render probability models more appropriate to formulate.

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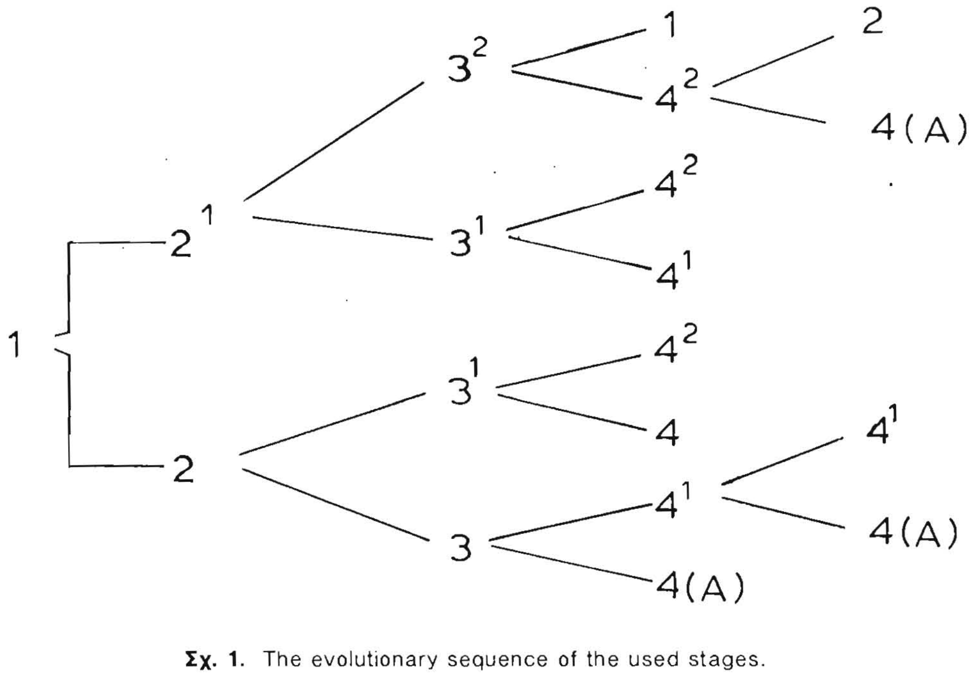
A Markov chain Model

In this paper we suppose that the landforms can be classified into numerous descriptive stages mainly of erosion, the magnitude of the uplift and subsidence, and the time. Supposedly the climate has to be humid-temperate, but with few modifications the proposed model can be applied to different climatic conditions. Some geometric stages are shown in Fig. 1. I assume an evolutionary sequence of stages (Fig. 2), this means that from an initial relatively flat relief (stage 1) there are two possibilities. First a normal evolution without an uplift to stage 2 with probability P(t) and second, a contemporaneous uplift and the passage to stage 2^1, with probability q(t), (Fig. 3).



Σχ. 1. The evolutionary sequence of the used stages.

In this scheme (Fig.1, 2) the numbers 1, 2, 3 and 4 represent normal evolution of the Landform, the upper index (ex. 21) shows the periods of uplift. Finite Markov chains have been studied by many authors as FRECHET (1938), CHUNG (1967), FELLER (1968), IOSIFESCU (1979), BHAT (1972).

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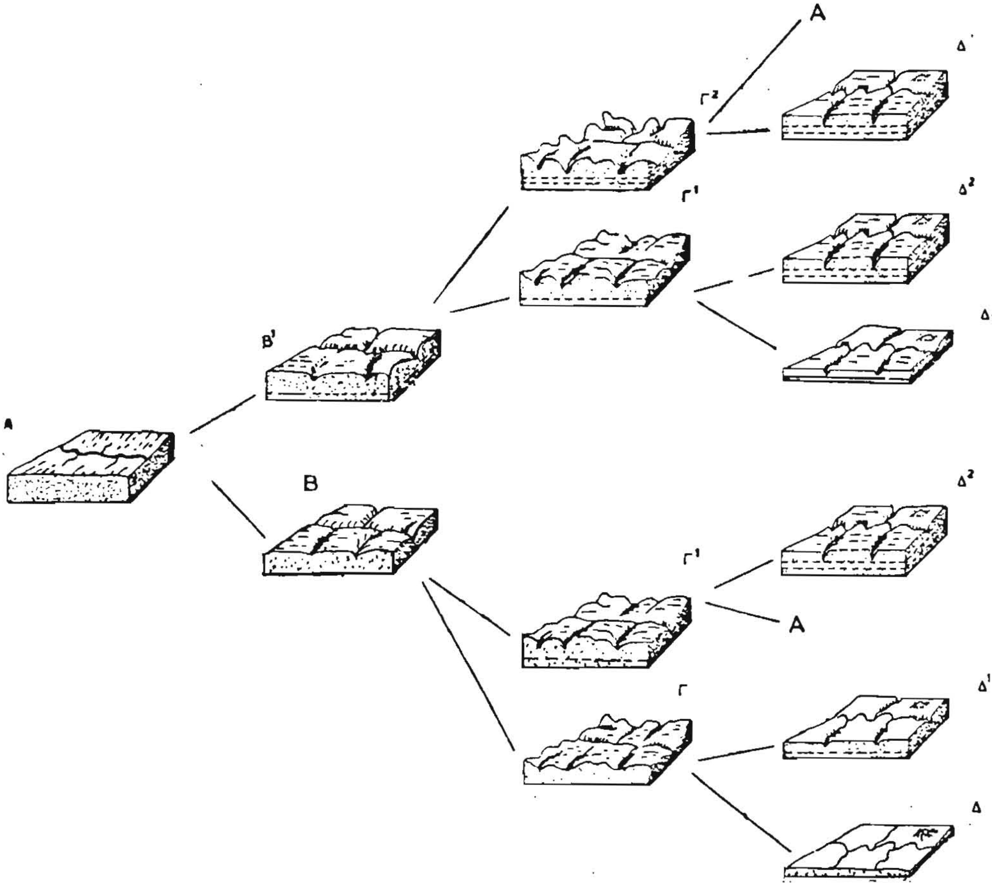

Σχ. 2. The Landform development of the geomorphological stages.

To apply discrete Markov chains to Landform development we firstly define the states (stages) of the system. Fig. 1 contains all the stages and their mutual transitions. It must be noted that we assumed that there is an upper limit for the uplift movement (reflective state), meaning that the landform can not grow infinitely. The second step is to analyze this process and to formulate the transition matrix and put it in its canonical form.

$$A = [P_{ij}] \qquad\qquad A = \begin{bmatrix} I & O \\ R & Q \end{bmatrix}$$

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	4	1	2	2^1	3	3^1	3^2	4^1	4^2
4	1								
1			p	q					
2					p	q			
2^1						p	q		
3	p							q	
3^1								p	q
3^2		q							p
4^1	p							q	
4^2	p		q						

Σχ. 3. The transition matrix of probabilities.

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$p = 0{,}6 \qquad q = 0{,}4$

$$I-Q = \begin{bmatrix} 1 & -0{,}6 & -0{,}4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -0{,}6 & -0{,}4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -0{,}6 & -0{,}4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -0{,}4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -0{,}6 & -0{,}4 \\ -0{,}4 & 0 & 0 & 0 & 0 & 1 & 0 & -0{,}6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0{,}6 & 0 \\ 0 & -0{,}4 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad M_1 = \begin{bmatrix} 4.67127 \\ 3.16239 \\ 4.43458 \\ 1.66667 \\ 2.90598 \\ 4.22748 \\ 1.66667 \\ 2.26946 \end{bmatrix} \qquad L_1 = \begin{bmatrix} 0{,}98 \\ 0{,}99 \\ 0{,}98 \\ 0{,}99 \\ 0{,}98 \\ 0{,}98 \\ 0{,}99 \\ 0{,}96 \end{bmatrix}$$

$p = 0{,}4 \qquad q = 0{,}6$

$$I-Q = \begin{bmatrix} 1 & -0{,}4 & -0{,}6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -0{,}4 & -0{,}6 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -0{,}4 & -0{,}6 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -0{,}6 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -0{,}4 & -0{,}6 \\ -0{,}6 & 0 & 0 & 0 & 0 & 1 & 0 & -0{,}4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0{,}4 & 0 \\ 0 & -0{,}6 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad M_2 = \begin{bmatrix} 6.79717 \\ 4.54082 \\ 6.63474 \\ 2.50000 \\ 4.23469 \\ 6.56810 \\ 2.50000 \\ 3.72449 \end{bmatrix} \qquad L_2 = \begin{bmatrix} 0{,}98 \\ 0{,}99 \\ 0{,}99 \\ 0{,}99 \\ 0{,}99 \\ 0{,}99 \\ 1 \\ 0{,}99 \end{bmatrix}$$

Σχ. 4. The final matrices of the two examples

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Where, I : the identity matrix which corresponds to the unique absorbing state,
Q : the matrix of probabilities of transient states,
R : the matrix of probabilities from transients to recurrent state.

We can now proceed to find the fundamental matrix M.

Two simple examples:

The fundamental problem to apply such a model is to construct a transition probability matrix which better represents the geotectonic position of a Landform. This means to assign the appropriate values of probabilities to the above evolutionary scheme.

In the first application we assume that $P(t) = 0,6$ $q(t) = 0,4$. This example refers to an area where erosion dominates and uplift movements are less important.

The resulting fundamental matrix is: M_1 and the matrix L_1 (Fig. 4) gives the expected number of states the system spent in the transient states prior to absorption (the total denudation of the relief).

In the second example we assume that $P(t) = 0,4$ $q(t) = 0,6$. In this case we assume a high probability for uplift movements and low probabilities for erosion. Here the fundamental matrix is M_2 and the matrix giving the expected number of changes before the chain enters the absorbing state is L_2 (Fig. 4).

Conclusions

From the above two simple applications it is obvious that in the first case where erosion is more probable than uplift a rapid decay of relief will result. The matrices M1 and L1 suggest that the whole process will finish relatively rapidly, the number of changes at states are relatively small. The same is valid also for the variety of relief forms. On the contrary in the second case, where the uplift movements dominate, a repeated constructional process will be present. The results from the matrices, show that the whole system will pass from many Landform stages, before its total denudation.

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