A VARIATIONAL METHOD FOR THE DETERMINATION OF THE CORRELATION FUNCTION BETWEEN TWO NUKLEONS IN NUCLEAR MATTER

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Abstract: In this paper we summerize the basic conclusions of a new variational method for the determination of the correlation function f, between two nucleons in the «infinite» nuclear matter. The first step in this approach is to put the denominator in the expression of the energy per particle: E/N in a suitable form. By using subsequently the variational principle we obtain a non-linear integrodifferential equation for the function f. By studying the behaviour of this equation for large distances we are led to the following integral constraint:

$$2\rho \int \left[1 - f^2\left(1 - \frac{1}{4} l^2(k_F r_{12})\right)\right] d\mathbf{r}_{12} = 1, \qquad l(k_F r_{12}) = \frac{3 j_1(k_F r_{12})}{k_F r_{12}}$$

It should be pointed out that no use of arbitrary condition is made in the proposed approach. It seems therefore that this method can be the appropriate way of facing a problem, which has existed for a long time and is known as «Emery difficulty».

Finally, the results of some preliminary numerical calculations are given.

1. *Introduction*

Variational calculations of the energy per particle in nuclear matter [1] are based on a cluster expansion of the type:

$$\frac{E}{N} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \cdots \tag{1}$$

where ε_1 is the Fermi energy, ε_2 the two - body cluster terms (proportional to the density ρ), ε_3 the three - body terms e.t.c. The ε_2 and higher terms are functionals of the nucleon - nucleon correlation function f, which appears in the well-known Jastrow trial many-body wave function [1,3].

In practice, expansion (1) is usually truncated at ε_2. Such a truncated cluster expansion, however, has the deficiency that normalization is absent from it, with the effect that too much energy per particle might be obtained. In order to avoid this difficulty, known as «Emery difficulty» [2,1] a restricted variation of E/N has to be performed. This variation is either functional (with respect to f) or with respect to parameters, which are contained in the f, if a given analytic form for this function is assumed. In the calculations performed so far, quite a few types of restrictions [2,4,5] have been used, but the imposition of them has been done in an «ad hoc» manner and it is not clear which the appropriate condition is that should be satisfied by f.

In this paper we summerize the basic conclusions of a new variational method for the calculation of E/N and give some preliminary numerical results. Details of the formalism will appear elsewhere.

In the present approach a suitable expression for E/N is functionally varied and the initial (general) constraint, which is introduced, has its origin to the denominator appearing in this expression. The specific condition, which f has to satisfy, is subsequently derived by studying the Euler equation of the variational problem. No «ad hoc» constraint is imposed.

2. *Summary of the formalism*

We start from the general expression for E/N, as this is given by Aviles (see expressions (100) and (101) of ref. (3).), which we rewrite in the form:

$$\frac{E}{N} = \frac{3}{5}\frac{\hbar^2 K_F^2}{2m} + \frac{(N-1)\int \frac{\rho}{2}\left\{\left[\frac{\hbar^2}{2m}\left((\nabla f)^2 - f\nabla^2 f\right) + V(r_{12})f^2\right]G_F(r_{12}) - \frac{\hbar^2}{2m}\frac{1}{2}\boldsymbol{\nabla} f^2 \cdot \mathbf{F}(r_{12})\right\} d\mathbf{r}_{12}}{\rho\int f^2(r_{12})\, G_F(r_{12})\, d\mathbf{r}_{12}} \qquad (2)$$

This novel form follows from the definition of functions G_F and F (formulae (11) and (12) of ref. (3)).

Since we are interested in the limit $N \to \infty$, $\Omega \to \infty$, the ratio N/Ω remaining constant $= \rho$, variation of (2) with respect to f, is equivalent to variation of the numerator (divided by (N-1)) of the second term in this expression, provided that

$$\rho \int \left(f^2 G_F(r_{12}) - 1\right) d\mathbf{r}_{12} = \text{finite constant} \qquad (3)$$

The Euler equation of the variational problem is the following non - linear integrodifferential equation :

$$\left(-\frac{\hbar^2}{m}\right)\left[r_{12}^2\, G_F(r_{12})\, \frac{d^2 f}{dr_{12}^2} + \left(2\, r_{12}\, G_F(r_{12}) + r_{12}^2\, \frac{dG_F(r_{12})}{dr_{12}}\right) \frac{df}{dr_{12}}\right] +$$

$$+ \left\{ r_{12}^2 \left(V(r_{12}) + \lambda\right) G_F(r_{12}) + \frac{\hbar^2}{2\,m}\, r_{12} \left(\frac{\mathbf{r}_{12}}{r_{12}} \cdot \mathbf{F}(\mathbf{r}_{12})\right) + \right.$$

$$\left. + \left(-\frac{\hbar^2}{2m}\right) r_{12} \left[\frac{dG_F(r_{12})}{dr_{12}} + \frac{1}{2}\, r_{12} \left(\frac{d^2 G_F(r_{12})}{dr_{12}^2} - \frac{d}{dr_{12}} \left(\frac{\mathbf{r}_{12}}{r_{12}} \cdot \mathbf{F}(\mathbf{r}_{12})\right)\right)\right]\right\} f +$$

$$+ r_{12}^2 \left[\left(\frac{\hbar^2}{2m}\right)\left(\left(\frac{df}{dr_{12}}\right)^2 - f\left(\frac{d^2 f}{dr_{12}^2} + \frac{2}{r_{12}} \frac{df}{dr_{12}}\right)\right) + \left(V(r_{12}) + \lambda\right) f^2\right] \cdot \frac{1}{2} \frac{\partial G_F(r_{12})}{\partial f} +$$

$$+ r_{12}^2 \left(-\frac{\hbar^2}{2m}\right) f \frac{df}{dr_{12}} \cdot \frac{1}{2} \frac{\partial}{\partial f} \left(\frac{\mathbf{r}_{12}}{r_{12}} \cdot \mathbf{F}(\mathbf{r}_{12})\right) = 0 \qquad (4)$$

This equation must be solved with boundary conditions :

$$f(c) = 0 \qquad\qquad f(\infty) = 1 \qquad (5)$$

where c is the hard core radius of the internucleon potential.

We may note that if instead of nuclear matter, which is a many - fermion system, we consider a many - boson system, then in the resulting integrodifferential equation the terms with the function $F(\mathbf{r}_{12})$ do not appear and instead of $G_F(r_{12})$ we have the function $G(r_{12})$ given by expression (5) of ref. (3).

Taking into account the three - body terms in the cluster expansions of G_F, $\frac{\partial G_F}{\partial f}$ and the other functions appearing in equation (4), it can be shown that the following condition must be satisfied for the proper behaviour of this equation for large distances :

$$2\,\rho \int \left[1 - f^2\left(1 - \frac{1}{4}\, l^2(k_F r_{12})\right)\right] d\mathbf{r}_{12} = 1, \qquad l(k_F r_{12}) = \frac{3 j_1(k_F r_{12})}{k_F r_{12}} \qquad (6)$$

This differs from the well - known «first order normalization condition» by a factor of 2.

3. *Preliminary numerical results*

Since it is a very complicated task to solve equ. (4), we performed numerical calculations, assuming the following form for the f:

$$f(r_{12}) = \begin{cases} 0\ , & 0 \leq r_{12} \leq c \\ \{1 - \exp[-\mu_1(r_{12} - c)]\}\cdot\{1 + \nu \exp[-\mu_2(r_{12} - c)]\}, & c \leq r_{12} < \infty \end{cases} \qquad (7)$$

and we fixed the value of ν by condition (6), while we determined the values of μ_1 and μ_2 (for each value of K_F) by minimizing $E/N=\varepsilon_1+\varepsilon_2$ with respect to them. This was done in the same way as in the case of «first order normalization condition», which was shown [7] to be the appropriate one for the «impure nuclear matter problem» (e.g. of a Λ - particle in nuclear matter), and in fact the program of ref. 6) with a slight modification was used.

Our computations were performed with the hard core potential of Iwamoto and Yamada [8]:

$$V(r_{12}) = \begin{cases} \infty \ , & 0 \leqq r_{12} \leqq 0.6 \text{ fm} \\ -397.3 \exp\left[-2.627\,(r_{12}-0.6)\right], & 0.6 \leqq r_{12} < \infty \end{cases} \qquad (8)$$

,which acts only in even states. The results for - E/N are shown in fig. 1.

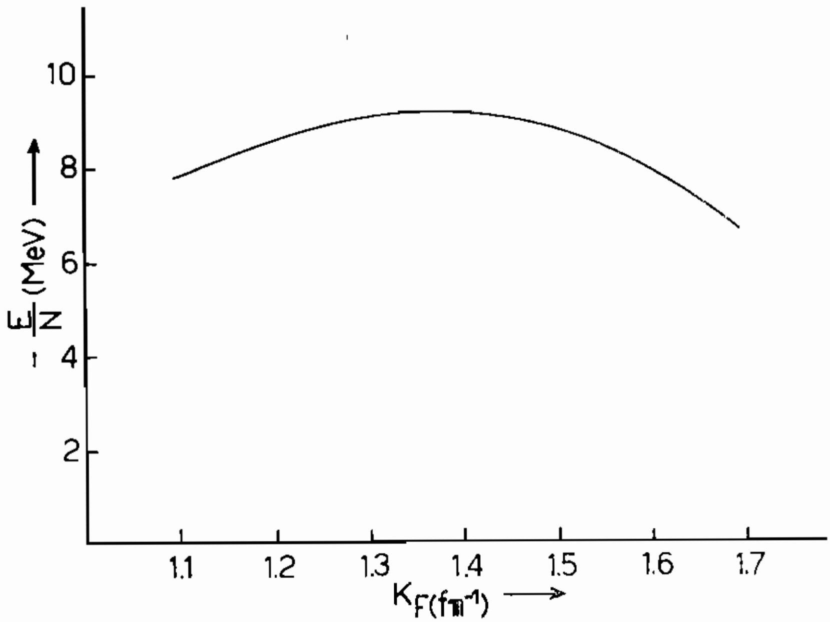

Figure 1. The binding energy per particle $\left(-\frac{E}{N}\right)$ *as function of* K_F.

The minimum of the saturation curve (E/N as function of K_F) is at $K_F = 1.376$ fm^{-1}, which is very close to the experimental value. The minimum value of E/N is:

$$\frac{E}{N} = -9.26 \text{ MeV}$$

This value is closer to the experimental one, compared to the corresponding results of certain other calculations [4,8]. The inclusion, however, of three - body terms in E/N is expected to make the difference between our results and those of the above references, smaller.

The values of μ_1, μ_2 and ν, which correspond to the minimum, are:

$$\mu_1 = 0.987 \text{ fm}^{-1}, \qquad \mu_2 = 1.425 \text{ fm}^{-1}, \qquad \nu = 3.53$$

The above results are close to those obtained with the normalization condition .

We should finally point out that the present analysis suggests that one - parameter correlation functions, which have been frequently used, are not appropriate.

I would like to thank E. Mavrommatis for computational assistance.

ΜΙΑ ΜΕΘΟΔΟΣ ΜΕΤΑΒΟΛΩΝ ΠΡΟΣ ΚΑΘΟΡΙΣΜΟΝ ΤΗΣ ΣΥΝΑΡΤΗΣΕΩΣ ΣΥΣΧΕΤΙΣΕΩΣ ΔΥΟ ΝΟΥΚΛΕΟΝΙΩΝ ΤΗΣ ΠΥΡΗΝΙΚΗΣ ΥΛΗΣ

ὑπὸ

ΜΙΧΑΗΛ ΕΛ. ΓΡΥΠΑΙΟΥ

Σπουδαστήριον Θεωρητικῆς Φυσικῆς Πανεπιστημίου Θεσσαλονίκης

Εἰς τὴν παροῦσαν ἐργασίαν, ἐκθέτομεν ἐν περιλήψει τὰ βασικὰ συμπεράσματα μιᾶς νέας μεθόδου μεταβολῶν, πρὸς καθορισμὸν τῆς συναρτήσεως συσχετίσεως $f(r_{12})$ δύο νουκλεονίων τῆς «ἀπείρου» πυρηνικῆς ὕλης. Τὸ πρῶτον βῆμα εἰς τὴν ἐν λόγω μέθοδον, εἶναι νὰ θέσωμεν ὑπὸ κατάλληλον μορφὴν τὸν παρονομαστὴν τῆς ἐκφράσεως τῆς ἐνεργείας ἀνὰ σωμάτιον E/N. Χρησιμοποιοῦντες ἐν συνεχείᾳ τὴν ἀρχὴν τῶν μεταβολῶν, λαμβάνομεν μίαν μὴ - γραμμικὴν ὁλοκληροδιαφορικὴν ἐξίσωσιν διὰ τὴν συνάρτησιν f. Μελετῶντες τὴν συμπεριφορὰν τῆς ἐν λόγῳ ἐξισώσεως διὰ μεγάλας ἀποστάσεις, ὁδηγούμεθα εἰς τὸν ἑξῆς ὁλοκληρωτικὸν περιορισμὸν διὰ τὴν f :

$$2\rho \int \left[1 - f^2\left(1 - \frac{1}{4}\, l^2(k_F r_{12})\right)\right] dr_{12} = 1, \qquad l(k_F r_{12}) = \frac{3 j_1(k_F r_{12})}{k_F r_{12}}$$

Θὰ πρέπῃ νὰ ἀναφερθῇ, ὅτι εἰς τὴν προτεινομένην μέθοδον, δὲν γίνεται χρῆσις αὐθαιρέτου συνθήκης. Ὡς ἐκ τούτου ἡ μέθοδος αὕτη δύναται ὡς φαίνεται νὰ ἀποτελέσῃ τὸν ἐνδεδειγμένον τρόπον ἀντιμετωπίσεως, τοῦ ἐπὶ μακρὸν ὑφισταμένου προβλήματος, γνωστοῦ ὡς «Emery difficulty».

Εἰς τὸ τέλος τῆς ἐργασίας δίδομεν τὰ ἀποτελέσματα ὡρισμένων προκαταρκτικῶν ἀριθμητικῶν ὑπολογισμῶν.

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