Sci. Annals, Fac. Phys. & Mathem., Univ. Thessaloniki, **13**, 145 (1973)

RADIATION PATTERNS OF ANTENNAS OF ANY SHAPE

By

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(Received 27.6.1973)

Abstract: *A method of evaluating radiation patterns of antennas of any shape by approximating the intensities of the electromagnetic field $\vec{E}$ and $\vec{H}$ is studied.*

The approximation is made by numerical integration of the components A_θ and A_φ of the vector potential, $\vec{A}$.

Methods are proposed to calculate the coordinates of any point of the antenna as a function of its length L, electrically and photographically.

I. INTRODUCTION

I. I. The distribution in space of the radiation of an antenna is described by the antenna patterns.

The antenna patterns, depending upon the physical quantity to be described, are given in the form of the following equations.

$$\vec{E} = \vec{E}(r,\theta,\varphi,t)\ , \qquad \vec{H} = \vec{H}(r,\theta,\varphi,t) \qquad \text{and} \qquad \vec{P} = \vec{P}(r,\theta,\varphi,t) \tag{1}$$

where: $\vec{E}$ is the intensity of the electric field

$\vec{H}$ is the intensity of the magnetic field, $\vec{P}$ the Poynting vector. Given $r = \text{const.}$ and $t = t_0$ equations (1) become:

$$\vec{E} = \vec{E}(\theta,\varphi)\ , \quad \vec{H} = \vec{H}(\theta,\varphi) \qquad \text{and} \qquad \vec{P} = \vec{P}(\theta,\varphi) \tag{2}$$

$$\text{or} \quad \begin{array}{lll} \vec{E}_{\varphi=\pi/2} = \vec{E}(\theta)\ , & \vec{H}_{\theta=\pi/2} = \vec{H}(\theta) & \vec{P}_{\varphi=\pi/2} = \vec{P}(\theta) \\ \vec{E}_{\theta=\pi/2} = \vec{E}(\varphi)\ , & \vec{H}_{\theta=\pi/2} = \vec{H}(\varphi) & \vec{P}_{\theta=\pi/2} = \vec{P}(\varphi) \end{array} \tag{3}$$

The equations which will express $\vec{E}$, $\vec{H}$, $\vec{P}$ will then have the form of equations (3).

1. 2. The intensities of the electromagnetic field are given by the following Maxwell's equations.

$$\vec{E} = -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t} \quad \text{and} \quad \vec{H} = \frac{1}{\mu_0}\vec{\nabla}\times\vec{A} \tag{4}$$

Given that the quantities $\vec{A}$, V, are sinusoidal functions of time, equations (4) become.

$$\vec{E} = -\vec{\nabla}V - j\omega\vec{A} \quad \text{and} \quad \vec{H} = \frac{1}{\mu_0}\vec{\nabla}\times\vec{A} \tag{5}$$

From equations (5) we can therefore conclude that it is possible to find $\vec{E}$ and $\vec{H}$ should the potential of the electromagnetic field be known in the Minkowsky space. The potential in the case of Minkowsky space is given by the equations:

$$V = \frac{1}{4\pi\varepsilon_0}\int_\tau \frac{\rho(t-r/c)}{r}\,d\tau \quad \text{and} \quad \vec{A} = \frac{1}{4\pi\varepsilon_0 c^2}\int_\tau \frac{\vec{J}(t-r/c)}{r}\,d\tau \tag{6}$$

which in the case of the charge and current densities being sinusoidal functions, so that (5) holds, become:

$$V = \frac{1}{4\pi\varepsilon_0}\int_\tau \frac{\rho e^{-j2\pi\lambda/r}}{r}\,d\tau \quad \text{and} \quad \vec{A} = \frac{1}{4\pi\varepsilon_0 c^2}\int_\tau \frac{\vec{J}e^{-j2\pi\lambda/r}}{r}\,d\tau \tag{7}$$

2. STATE OF THE PROBLEM

2. 1. In the case of an antenna we can assume that the elements from which it consists have the same cross section, so that the volume element $d\tau$ be written as: $d\tau = sdl$ (s = cross section)

Given that $\vec{J}\,.\,dt = I\,.\,d\vec{l}$

equations (7) become:

$$V = \frac{s}{4\pi\varepsilon_0}\int_l \frac{\rho e^{-j2\pi\lambda/r}}{r}\,dl \quad \text{and} \quad \vec{A} = \frac{s}{4\pi\varepsilon_0 c^2}\int_l \frac{Ie^{-j2\pi\lambda/r}}{r}\,d\vec{l} \tag{8}$$

In polar coordinates equations (5) have the form:

$$E_r = -j\omega A_r - \frac{\partial V}{\partial r} \qquad H_r = \frac{1}{\mu_0}\cdot\frac{1}{r^2\sin\theta}\left\{\frac{\partial}{\partial\theta}(r\sin\theta A_\varphi) - \frac{\partial}{\partial\varphi}(rA_\theta)\right\}$$

$$E_\theta = -j\omega A_\theta - \frac{1}{r}\frac{\partial V}{\partial \theta} \qquad H_\theta = \frac{1}{\mu_0}\cdot r\left\{\frac{\partial}{\partial \varphi}(A_r) - \frac{\partial}{\partial r}(r\sin\theta A_\varphi)\right\} \quad (9)$$

$$E_\varphi = -j\omega A_\varphi - \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi} \quad H_\varphi = \frac{1}{\mu_0}\cdot r\sin\theta\left\{\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}(A_r)\right\}$$

Since as it results from equations (6), (7), (8) the quantities V and $\vec{A}$ are inversely proportional to the distance r of the receiver point from the antenna, we can omit terms including A_r, A_θ, A_φ, V multiplied by 1/r, considering them as second order differentials.

Hence, equations (9) become:

$$\begin{aligned} E_r &= 0 & H_r &= 0 \\ E_0 &= -j\omega A_\theta & H_\theta &= -\frac{1}{\mu_0} j2\pi/\lambda A_\varphi \qquad (10) \\ E_\varphi &= -j\omega A_\varphi & H_\varphi &= \frac{1}{\mu_0} j2\pi/\lambda A_\theta \end{aligned}$$

if, of course, due to the above assumption, it is taken into consideration that:

$$V = -\frac{1}{j\omega\mu_0\varepsilon}\operatorname{div}\vec{A} = -\frac{1}{j\omega\mu_0\varepsilon}\left(\frac{2}{r}A_r + \right.$$

$$\left. + \frac{\partial A_r}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\right) \simeq \frac{2\pi}{\lambda}\frac{1}{\omega\mu_0}A_r$$

so that:

$$\vec{E} = -j\omega\{A_\theta\vec{\theta}_0 + A_\varphi\vec{\varphi}_0\} \qquad \vec{H} = -\frac{1}{\mu_0} j\frac{2\pi}{\lambda}\{A_\varphi\vec{\theta}_0 - A_\theta\vec{\varphi}_0\}$$

and $$\vec{P} = \frac{1}{2}\vec{E}\times\vec{H}^* = \frac{1}{2}\omega\frac{2\pi/\lambda}{\mu_0}\{A_\theta A_\theta^* + A_\varphi A_\varphi^*\}\vec{r}_0 = P_r\vec{r}_0 \qquad (11)$$

From equations (11) results that the problem of the evaluation of $\vec{E}$, $\vec{H}$, $\vec{P}$ reduces to finding A_θ, A_φ.

2. 2. In fig (1) an antenna of arbitrary shape is considered, and we want to evaluate A_θ and A_φ at the point $P(X_0, Y_0, Z_0)$.

We have:

$$\vec{dl} = (dx\sin\theta\cos\varphi + dy\sin\theta\sin\varphi + dz\cos\theta)\vec{R}_0 + \{dx\cos\theta\cos\varphi +$$

$$+ dy\cos\theta\sin\varphi - dz\sin\theta\}\vec{\theta}_0 + \{-dx\sin\varphi + dy\cos\varphi\}\vec{\varphi}_0;$$

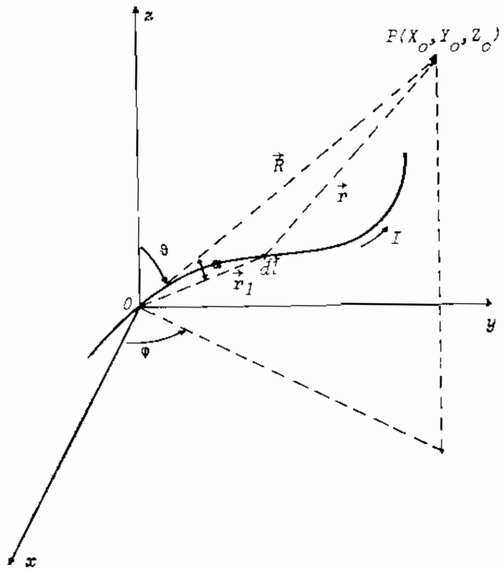


Fig. 1

on the other hand: $R.r_1 = X_0.x + Y_0.y + Z_0.z = Rr_1$ cosa from which we get

$$r_1 \cos a = X_0.x + Y_0.y + Z_0.y/R = x \sin\theta \ \cos\varphi + \\ + y \sin\theta \sin\varphi + z \cos\theta \qquad (13)$$

Moreover, it is obvious that $r \cong R - r_1 \cos a$ for $r_1 \langle\langle r, R$; then from (8) we get

$$A_\theta = \frac{1}{4\pi\varepsilon_0 c^2} \int I \frac{e^{-j2\pi r/\lambda}}{r} dl_\theta \cong \frac{1}{4\pi\varepsilon_0 c^2} \int I \frac{e^{-j2\pi(R-r_1 \cos a)/\lambda}}{R-r_1\cos a} dl_\theta \cong$$

$$\cong \frac{1}{4\pi\varepsilon_0 c^2} \int I \frac{e^{-j2\pi(R-r_1 \cos a)/\lambda}}{R} dl_\theta \qquad (14)$$

which combined with (12) and (13) gives

$$A_\theta = \frac{e^{-j2\pi R/\lambda}}{4\pi\varepsilon_0 c^2} \cdot \frac{1}{R} \int I \, e^{-j2\pi/\lambda(x\sin\theta\cos\varphi + y\sin\theta\sin\varphi + z\cos\theta)} \ (dx\cos\theta\cos\varphi + \\ + dy\cos\theta\sin\varphi - dz\sin\theta) \qquad (15)$$

Similarly we get for A_φ:

$$A_\varphi = \frac{e^{-j2\pi R/\lambda}}{4\pi\varepsilon_0 c^2} \cdot \frac{1}{R} \int I\, e^{-j2\pi/\lambda(x\sin\theta\cos\varphi + y\sin\theta\sin\varphi + z\cos\theta)} \; (-dx\sin\varphi + dy\cos\varphi) \qquad (16)$$

3. *Integration*

The problem consists of evaluating the above two integrals which generally could be evaluated by numerical integration.

Thus, current intensity as a periodic function of the antenna length is given by

$$I = I_0 \sin 2\pi/\lambda \; (L_0 - L_i)$$

x, y, z being functions of the length L.

The integration is made by Simpson's methods for each one of A_θ and A_φ in the cases where $\varphi = \pi/2$, $\theta = \pi/2$ correspondingly, using a computer which is programmed to evaluate E_θ, E_φ.

4. *Methods of evaluation of point series x,y,z,L*

4. 1. In the case of an antenna with geometrically defined shape, regarding x,y,z,L there is no problem.

4. 2. In the case of an antenna of arbitrary shape, two methods are proposed:

I. i) We use some wire covered by white plastic, on which black strips in every 0,5 cm are placed.

ii) The model of an antenna is made and photographed from two mutually perpendicular positions.

The two photos determine directly x,y,z,L.

II. i) Wire plastic covered, is wound by chromnickel wire.

ii) The antenna's model is constructed and placed as in fig. 2.

iii) A sliding contact, D, slides on the antenna and on a XY plotter we get the (R_1,L) (R_2,L) (R_3,L) curves which can be converted to X,Y,Z,L. The sliding contact D, excludes the circuits (1) and (0), the points 1,2,3 being metallic rings through which the chromnickel wire passes, supported by a spring.

```
  SAXALOS   IOANNHS
   DIMENSION X(100),Y(100),Z(100),S(100)
   READ 50,SO,N,H
   SO=SO*2.*3.1415926
   H=H*2.*3.1415926
   READ 40,(X(I),Y(I),Z(I),S(I),I=1,N)
   DO 300 I=1,N
   X(I)=X(I)*2.*3.1415926
   Y(I)=Y(I)*2.*3.1415926
   S(I)=S(I)*2.*3.1415926
300 Z(I)=Z(I)*2.*3.1415926
   L=N-1
   M=0
   SSIN1=SINF(SO-S(1))
   SSINN=SINF(SO-S(N))
   DO 100K=1,36
   IF(M)2,1,2
 1 F=0
   M=1
   GO TO 3
 2 F=F+3.1415926/18.
 3 MA=0
   MB=0
 4 IF(MA)5,6,5
 6 FCOS1=COSF(X(1)*COSF(F)+Y(1)*SINF(F))
   FCOSN=COSF(X(N)*COSF(F)+Y(N)*SINF(F))
   FSIN1=SQRTF(1.-FCOS1**2)
   FSINN=SQRTF(1.-FCOSN**2)
 7 IF(MB)9,8,9
 9 FDIV1=((X(2)-X(1))*SINF(F)-(Y(2)-Y(1))*COSF(F))/(S(2)-S(
   FDIVN=(X(N)*SINF(F)-Y(N)*COSF(F))/S(N)
   GO TO 10
 8 FDIV1=(Z(2)-Z(1))/(S(2)-S(1))
   FDIVN=Z(N)/S(N)
   GO TO 10
 5 FCOS1=COSF(Y(1)*SINF(F)+Z(1)*COSF(F))
   FCOSN=COSF(Y(N)*SINF(F)+Z(N)*COSF(F))
   FSIN1=SQRTF(1.-FCOS1**2)
   FSINN=SQRTF(1.-FCOSN**2)
11 IF(MB)13,12,13
12 FDIV1=(X(2)-X(1))/(S(2)-S(1))
   FDIVN=X(N)/S(N)
   GO TO 10
13 FDIV1=((Y(2)-Y(1))*COSF(F)-(Z(2)-Z(1))*SINF(F))/(S(2)-S(
   FDIVN=(Y(N)*COSF(F)-Z(N)*SINF(F))/S(N)
10 FJ1=SSIN1*FCOS1*FDIV1+SSINN*FCOSN*FDIVN
   FJ2=SSIN1*FSIN1*FDIV1+SSINN*FSINN*FDIVN
   DO 200 I=2,L
   SSIN=SINF(SO-S(I))
   IF(MA)14,15,14
```

```
15 FCOS=COSF(X(I)*COSF(F)+Y(I)*SINF(F))
   FSIN=SQRTF(1.-FCOS**2)
   IF(MB)17,16,17
17 FDIV=((X(I+1)-X(I))*SINF(F)-(Y(I+1)-Y(I))*COSF(F))/
   GO TO 20                                         (S(I+1)-S(I))
16 FDIV=(Z(I+1)-Z(I))/(S(I+1)-S(I))
   GO TO 20
14 FCOS=COSF(Y(I)*SINF(F)+Z(I)*COSF(F))
   FSIN=SQRTF(1.-FCOS**2)
   IF(MB)19,18,19
18 FDIV=(X(I+1)-X(I))/(S(I+1)-S(I))
   GO TO 20
19 FDIV=((Y(I+1)-Y(I))*COSF(F)-(Z(I+1)-Z(I))*SINF(F))/
20 FI=I                                             (S(I+1)-S(I))
   AI=FI/2.
   BI=I/2
   IF(AI-BI)22,21,22
21 FJ1=FJ1+4.*SSIN*FCOS*FDIV
   FJ2=FJ2+4.*SSIN*FSIN*FDIV
   GO TO 200
22 FJ1=FJ1+2.*SSIN*FCOS*FDIV
   FJ2=FJ2+2.*SSIN*FSIN*FDIV
00 CONTINUE
   FJ1=H*FJ1/3.
   FJ2=H*FJ2/3.
   FJ=FJ1**2+FJ2**2
   IF(MA)24,23,24
23 IF(MB)28,27,28
27 MB=1
   FJJ=FJ
   GO TO 7
28 MA=1
   FFI=SORTF(FJJ+FJ)
   PRINT 70,EFI
   GO TO 4
24 IF(MB)26,25,26
26 FJJ=FJ
   MB=0
   GO TO 11
25 ETH=SQRTF(FJJ+FJ)
   PRINT 60,ETH
00 CONTINUE
40 FORMAT(4F10.0)
50 FORMAT(F10.0,I10,F10.0)
60 FORMAT(4HETH=,E14.8)
70 FORMAT(4HEFI=,E14.8)
   STOP
   END
```

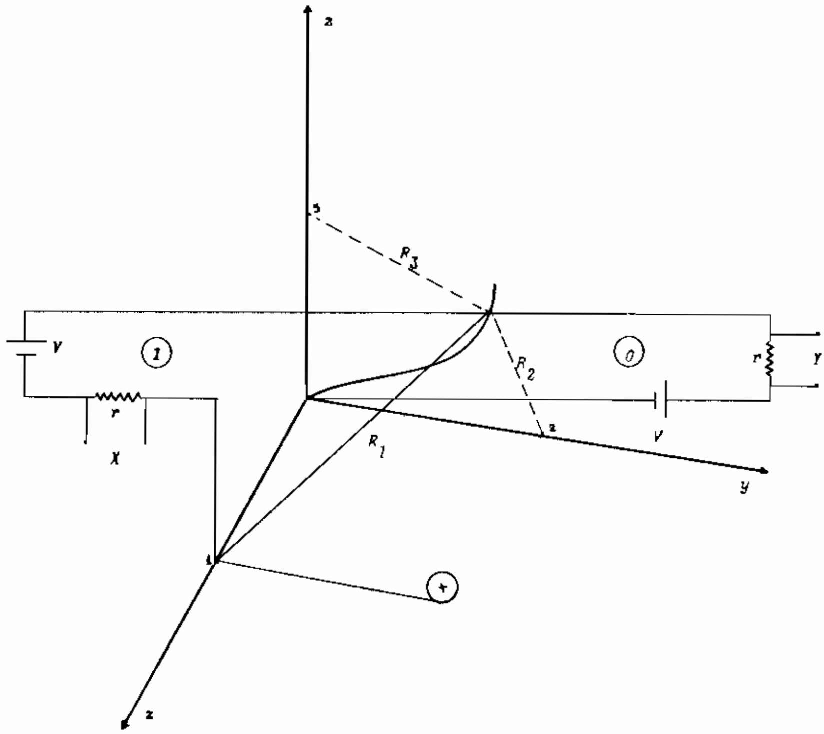
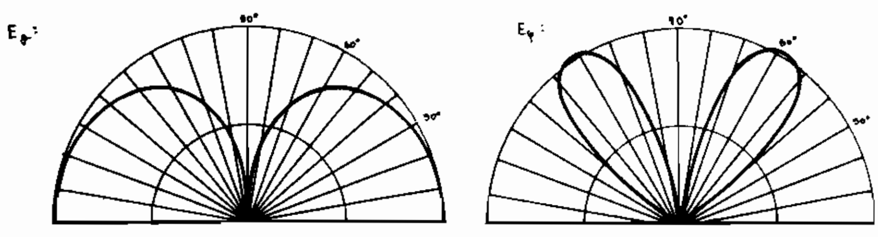


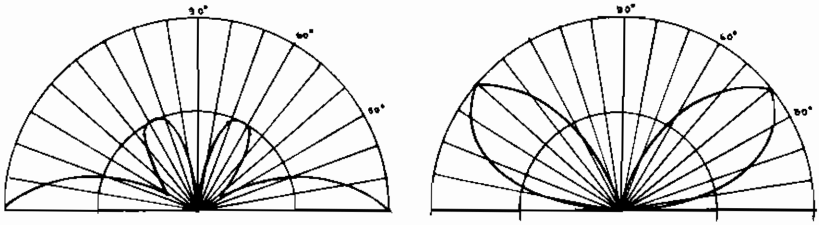
Fig. 2

5. RESULTS

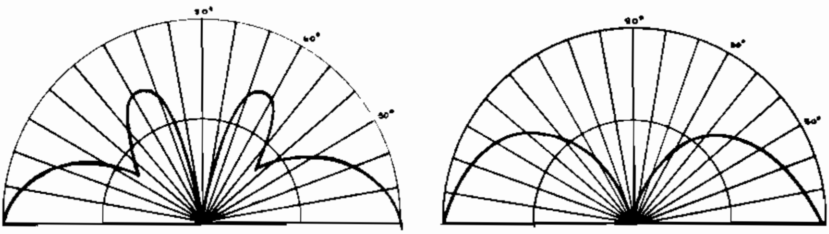
The antenna patterns have been evaluated for many cases of antenna of Known and used shapes as well as of antennas of various shapes. The results are the following :



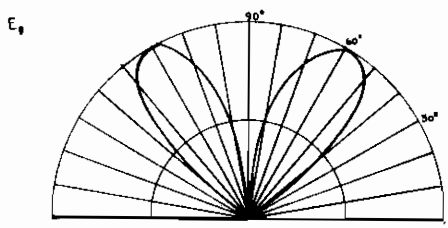
1. 2 dipoles with $L = \lambda$ and $d/L = 0,5$



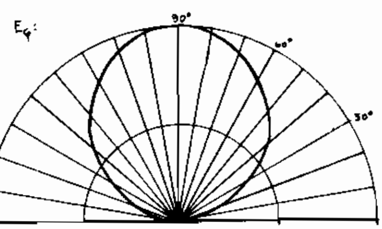
2. Antenna «V» with $L = \lambda$ and $a = 60^0$



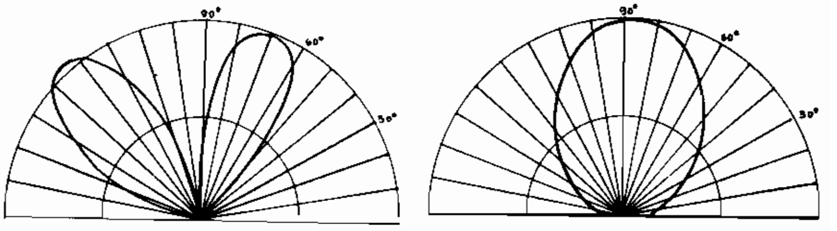
3. Antenna «Γ» with $H + L = 1,5$ and $H/L = 0,5$



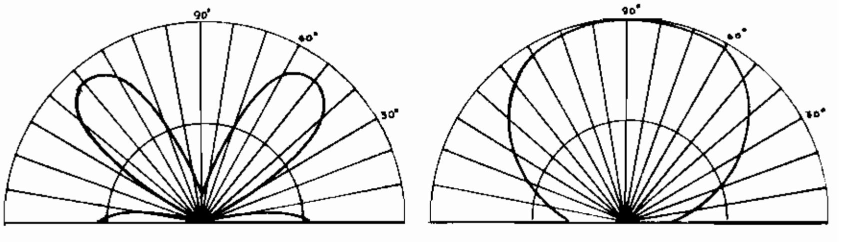
4. Vertical dipole $\lambda/2$



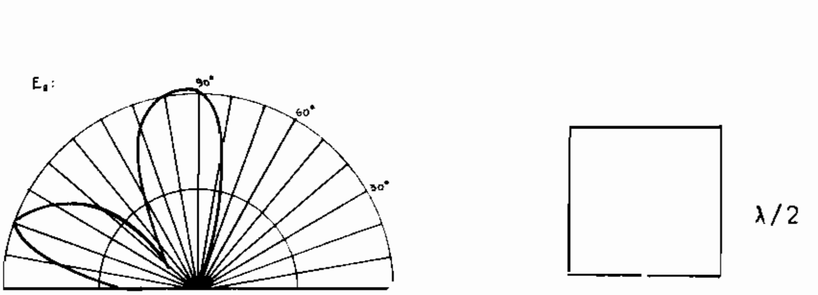
5. Horizontal dipole



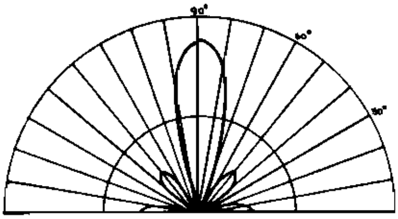
6. Dipole λ/2 with a = 10°



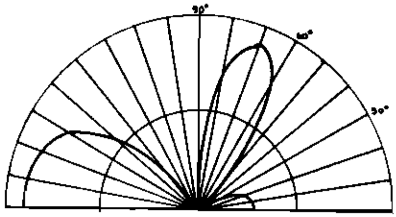
7. Dipole λ/2 with a = 30°



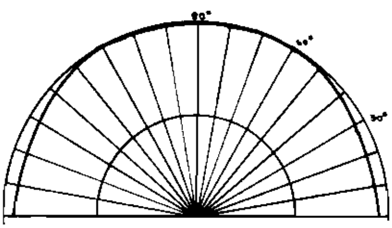
8. Loop antenna



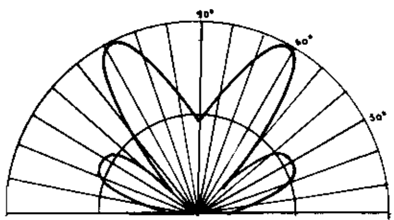
9. 3 dipoles with $d/\lambda = 0{,}375$, $\delta = 0^0$

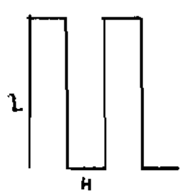


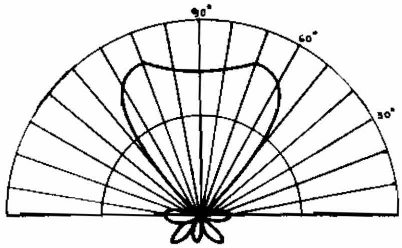
10. 3 dipoles with $d/\lambda = 0{,}375$, $\delta = 90^0$



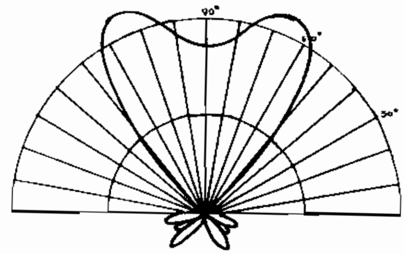
11. $H + L = 4{,}75\lambda$ *and* $H/L = 0{,}25$



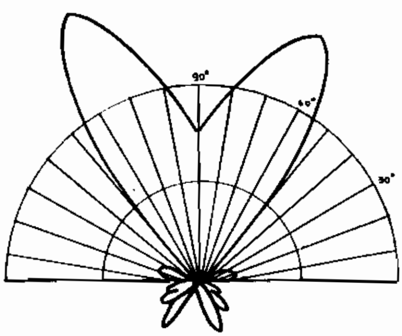




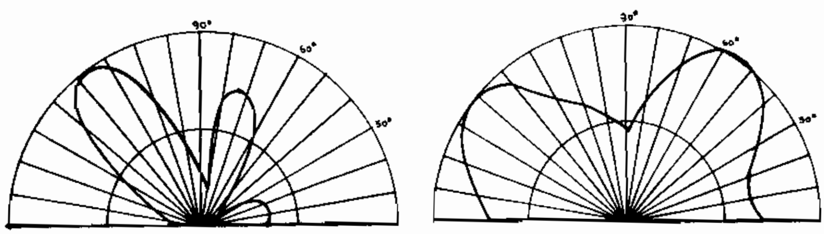
12. Helical antenna with $h = 3$



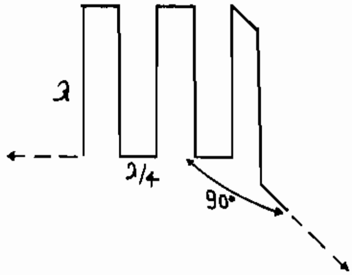
13. Helical antenna with $h = 4$



14. Helical antenna with $h = 5$



15. $H + L = 8{,}625\lambda$ and $H/L = 0{,}25$



6. CONCLUSION

The evaluation of antenna patterns is generally possible, approximately, in any case. That is effected by the evaluation of A_θ and A_φ of the vector potential. The use of the above described method is obvious, given that we can construct antennas of various complex shapes which are to perform with the desired directivity and gain.

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ΠΕΡΙΛΗΨΙΣ

ΔΙΑΓΡΑΜΜΑΤΑ ΑΚΤΙΝΟΒΟΛΙΑΣ ΚΕΡΑΙΩΝ ΟΙΟΥΔΗΠΟΤΕ ΣΧΗΜΑΤΟΣ

Ὑπὸ

ΙΩΑΝΝΟΥ ΣΑΧΑΛΟΥ

('Εργαστήριον Φυσικῆς 'Εκτάκτου Αὐτοτελοῦς Ἕδρας Πανεπιστημίου Θεσσαλονίκης)

Μελετᾶται ὁ τρόπος εὑρέσεως διαγραμμάτων ἀκτινοβολίας κεραιῶν οἱουδήποτε σχήματος, διὰ προσεγγιστικοῦ ὑπολογισμοῦ τῶν ἐντάσεων τοῦ ἠλεκτρομαγνητικοῦ πεδίου $\vec{E}$ καὶ $\vec{H}$.

Ὁ ὑπολογισμὸς πραγματοποιεῖται δι' ἀριθμητικῆς ὁλοκληρώσεως τῶν συνιστωσῶν A_φ καὶ A_θ τοῦ διανυσματικοῦ δυναμικοῦ.

Προτείνεται ὁ ὑπολογισμὸς τῶν συντεταγμένων ἑκάστου σημείου τῆς κεραίας συναρτήσει τοῦ μήκους L αὐτῆς, διὰ φωτογραφικῆς καὶ ἠλεκτρικῆς μεθόδου.