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A REVIEW OF CURRENT THEORIES ON THE HALL EFFECT IN FERROMAGNETICS

by

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Summary: *A review is given for the up today proposed theories for the ferromagnetic Hall effect.*

1.1. In 1879, during one of his investigations on the nature of the forces exerted on a conductor carrying a current in a magnetic field, E. H. Hall | 1 | observed that a voltage is developed across the specimen in the direction perpedicular to both the current and the magnetic field, in the way shown in fig. 1.

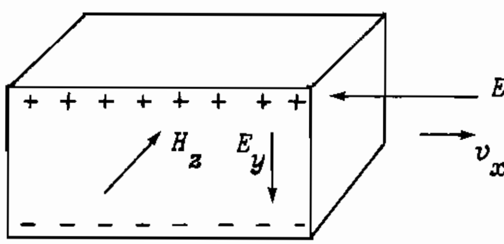


Fig. 1: Schematic representation of the Hall effect.

This voltage is called Hall voltage and is produced by the accumulation of the charges on a face of the specimen until the electric field, thus resulting, is large enough to counter-balance the force exerted by the magnetic field.

The effect was attributed to the action of the Lorentz force on the moving electrons.

If we consider a rectangular sample as in fig. 1 the Lorentz force on a charge carrier is

$$\vec{F} = e|\vec{E} + \frac{1}{c}\vec{v} \times \vec{H}| \quad (1)$$

The HALL electric field in the y direction results from the condition:

$$F_y = 0 = e|E_y - \frac{1}{c} v_x H|$$

which becomes

$$E_y = E_H = v_x H|c = j_x H|Nec$$

given that

$$j_x = Nev_x$$

where j_x is the current density and N the carrier concentration.
The quantity:

$$R_H = E_H|j_x H = 1|Nec$$

is called the HALL coefficient. In current bibliography one finds the following quantities:

HALL resistivity ρ_H, defined as the ratio of the HALL electric field per longitudinal current density i.e.

$$\rho_H = E_H|j_x = R_H H$$

HALL angle θ, defined by its tangent:

$$\tan\theta = R_H \sigma H = \mu_H H$$

where σ is the conductivity of the material,

$$\mu_H = R_H \sigma = R_H|\rho$$

is the HALL mobility and ρ is the electrical resistivity.

1.2. THE HALL EFFECT IN MAGNETIC MATERIALS

The Hall effect is metals which show appreciable magnetization exhibits quite unexpected features from the ones appearring in other metals. It would be reasonable to be expected that due to the LORENTZ force the HALL effect should vary linearly with the magnetic induction. The typical behaviour of the HALL effect in a ferromagnetic sample is shown in fig. (2).

A rapid linear increase in HALL voltage is observed by increasing B, followed by a linear section having a relatively smaller slope. It is evident that in this case, the HALL effect could not be the simple result of the action of the LORENTZ force on the conduction electrons. It was for this reason that the effect was called «anomalous». It is now known that the effect is not restricted to ferromagnetic materials only, but it could appear in any material in which localized magnetic moments are present, as in the case of paramagnetic or aniferromagnetic materials

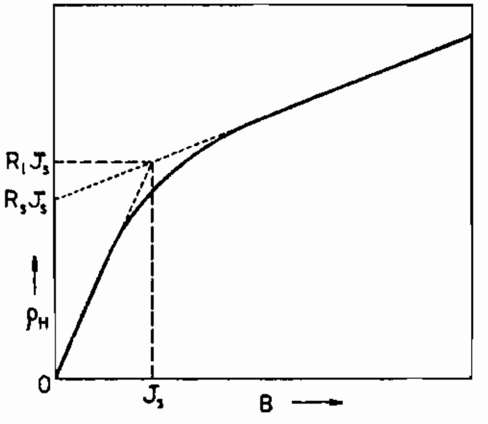


Fig. 2: Hall pesistivity ρ_H vs magnetic induction B in a ferromagnetic material.

A. W. SMITH and R. SEARS | 2 | and E. PUGH | 3 | - working independently - tried to explain the anomalous behaviour of the HALL effect considering two contributions to it: the one beeing the result of tbe action of the LORENTZ force, the second beeing a strongly temperature dependent contribution added to the first. They proposed a semi-empirical formula to explain the curve of fig. (2)

$$\rho_H = R_0 \mu_0 H + R_1 J_s \quad \text{(Giorgi system)}$$
$$\rho_H = R_0 \mu_0 H + R_1 4\pi J_s \quad \text{(C.G.S. system)} \qquad (2)$$

where μ_0 i the magnetic permeability of vacuum.

$\vec{H}$ is the intensity of the magnetic field.

$\vec{J}_s$ is the spontaneous magnetization of the sample.

The first term in the right member of equation (2) is attributed to the action of the LORENTZ force and is characterized by the constant R_0, which is called «normal» or «usual» beeing indetical with the one of the HALL effect in non magnetic metals.

The second term of the right member of eq. (2) is met only in the case of magnetic metals. It could be also present in a ferromagnetic domain even if an external magnetic induction is not applied.

It is evidently a «spontaneous» contribution to the HALL resistivity and it is characterized by the constant R_1 which is called «anomalous» or «spontateous» HALL constant.

R_1 was found to be a strongly temperature dependent quantity, attaining its maximum value just below the CURIE temperature of the materials.

R_0 exhibits also a temperature dependence though it is found to be much weaker.

In a ferromagnetic domain in the absence of an external applied field the following relations hold |4|

$$\left.\begin{array}{l} \vec{H} = 0 \\ \vec{J} = J_S \end{array}\right| \quad (3)$$

In this case in the ferromagnetic domain, we have a spontaneous HALL resistivity equal to $R_1 J_S$, the HALL effect beeing macroscopically and for a large number of domains, zero.

When an external field is applied the randomly oriented magnetic moments tend to align parallel to each other and to the apllied field with increasing induction, and then the HALL effect is macroscopically observable.

When the applied induction is not large enough to saturate the sample, the sample has large magnetic permeability, the product $\mu_0\vec{H}$ is negligible, the magnetic induction B is practically equal to J ($\vec{B} \simeq J$) the first linear section of the curve of fig. (2) having slope equal to R_1.

When the external field become large enough to saturate the sample, since J does not change, the second linear portion of the curve has a R_0 slope. The two linear portions of the curve intersect at a point having an abscissa equal to spontaneous magnetization J_S, and an ordinate equal to $R_1 J_S$.

In current bibliography there are many differences in the definitions of the various coefficients, e.g. application of the empirical equation (2) implies that

$$\left.\begin{array}{l} \vec{B} = \mu_0\vec{H} + \vec{J}_s \\ \vec{B}_e = \mu_0\vec{H}_e \end{array}\right| \quad (4)$$

and that also

$$\mu_0\vec{H} = \vec{B}_e - N\vec{J} \quad (5)$$

where $\vec{B}$ is the magnetic induction.

$\vec{H}$ is the intensity of magnetic field.

J_s is the intensity of magnetization inside the sample.

$\vec{B}_e$ the magnetic induction	(Both supposed to be uniform at a large distance from the sample).
$\vec{H}_e$ the intensity of the magnetic field	

where N is the demagnetizing factor which is equal to 1 for a rectangular thin sample on which an external induction B is applied normally. Smit writes equation (2) in the following form

$$\rho_H = R_0 B + R_s J_s \tag{6}$$

where $$R_s = R_1 - R_0$$

E. Pugh and S. Foner | 6 | write the equation (2)

$$\rho_H = R_0(\mu_0 H + aJ_s) = R_0 B_{eff} \tag{7}$$

where $$a = R_1 | R_0$$

The coefficient a is called «field parameter» and it is considered that the material is behaving as if an «effective» induction B was acting upon the conduction electrons instead of B.

The field parameter a is usually much larger than unity and in some cases is negative, which means that the two linear portions of the Hall curve have opposite slopes.

2.1. Phaenomenological theory of the Hall effect

In 1939, Meixner | 7 | presented a theory combining, in a very general way, the various transverse thermomagnetic and galvanomagnetic effects in an anisotropic conductor, and the corresponding currents giving rise to them.

According to this theory

$$\left. \begin{aligned} E_i &= \rho_{ik} j_k + \varepsilon_{ik} G_k + \frac{1}{e}\frac{\partial \xi}{\partial x_i} \\ w_i &= - \pi_{ik} j_k + \lambda_{ik} G_k + \frac{\xi}{e} j_i \end{aligned} \right| \tag{8}$$

where

E_i is the electric field
j_i is the current's density
$G_i = - \partial T | \partial x_i$ is the negative temperature gradient
w_i is the heat current density
ρ_{ik} is the electrical resistivity tensor
λ_{ik} is the thermal conductivity tensor
ε_{ik} is the absolute thermoelectric power tensor
π_{ik} is the Pertier tensor
ξ is the chemical potential or Fermi energy

The current and field vectors are represented by their components in a Cartesian coordinate system (x_1, x_2, x_3).

Callen | 8 | modified the above equations writing:

$$\left. \begin{aligned} E_i^* &= E_i - \frac{1}{e} \cdot \frac{\partial \xi}{\partial x_i} \\ w_i^* &= w_i - \frac{\xi}{e} j_i \end{aligned} \right| \qquad (9)$$

where $-e$ is the electronic charge

Thus equations (8) become:

$$\left. \begin{aligned} E_i^* &= \rho_{ik} j_k + \varepsilon_{ik} G_k \\ w_i^* &= - \pi_{ik} j_k + \lambda_{ik} G_k \end{aligned} \right| \qquad (10)$$

The coefficients appearing in equations (10) depend on the magnetic induction and obey the Onsager reciprocal relations:

$$\begin{aligned} \rho_{ik}(\vec{B}) &= \rho_{ki}(-\vec{B}) \\ \lambda_{ik}(\vec{B}) &= \lambda_{ki}(-\vec{B}) \end{aligned} \qquad (11)$$

In the case of an isotropic medium, with the magnetic induction directed along the x_3 axis we get:

$$\begin{aligned} \rho_{ii}(\vec{B}) &= \rho_{ii}(-\vec{B}) \qquad & \rho_{11} &= \rho_{22} \\ \rho_{ij}(\vec{B}) &= -\rho_{ij}(-\vec{B}) \qquad & \rho_{12} &= -\rho_{21} \\ & & \rho_{13} &= \rho_{32} = 0 \end{aligned} \qquad (12)$$

i.e. the diagonal elements of the transport tensors are even functions of magnetic induction, the off-diagonal elements beeing odd.

Thus equations (10) become (taking into account only the galvanomagnetic effects)

$$\left.\begin{aligned} E_1^* &= \rho_{11}j_1 + \rho_{12}j_2 + \varepsilon_{11}G_1 + \varepsilon_{12}G_2 \\ E_2^* &= \rho_{21}j_1 + \rho_{11}j_2 + \varepsilon_{21}G_1 + \varepsilon_{11}G_2 \\ E_3^* &= \rho_{33}j_3 + \varepsilon_{33}G_3 \end{aligned}\right| \qquad (13)$$

In the case of an isotropic medium with completely isothermal conditions, i.e. in the absence of thermal gradients ($G_i = - \partial T / \partial x_i = 0$), equations (13) take a more simple form:

$$\left.\begin{aligned} E_1 &= \rho_{11}j_1 - \rho_{21}j_2 \\ E_2 &= \rho_{21}j_1 + \rho_{11}j_2 \\ E_3 &= \rho_{33}j_3 \end{aligned}\right| \qquad (14)$$

or according to JAN's notation | 4 |

$$\left.\begin{aligned} E_1 &= \rho_{\perp}j_1 - \rho_H j_2 \\ E_2 &= \rho_H j_1 + \rho_{||}j_2 \\ E_3 &= \rho_{||}j_3 \end{aligned}\right| \qquad (15)$$

where $\rho_{\perp} = \rho_{11} = E_1 | j_1$ is the resistivity in a transverse field
$\rho_{||} = \rho_{33} = E_3 | j_3$ is the resistivity in a longitudinal field
$\rho_H = \rho_{21} = E_2 | j_1 = R_H B$ is the isothermal HALL resistivity

The two quanttiies E_i^* and E_i are equal under the assumption of isothermal and homogeneous conditions.

In many texts, the current densities are expressed as functions of the components of the electric field given that the tensors of resistivity and conductivity obey the relation:

$$\rho_{ij}\sigma_{jk} = \delta_{ik} \qquad (16) \qquad \delta_{ik} \begin{cases} \rightarrow 0 & i \neq k \\ \rightarrow 1 & i = k \end{cases}$$

Hence equations (14) become

$$
\left.\begin{aligned}
j_1 &= \sigma_{11}E_1^* - \sigma_{21}E_2^* \\
j_2 &= \sigma_{21}E_1^* + \sigma_{11}E_2^* \\
j_3 &= \sigma_{33}E_3^*
\end{aligned}\right| \qquad (17)
$$

which implies that the components of the resistivity and conductivity tensors are connected with the relations

$$
\sigma_{11} = \frac{\rho_{11}}{\rho_{11}\rho_{22} - \rho_{12}\rho_{21}} \qquad G_{12} = \frac{-\rho_{12}}{\rho_{11}\rho_{22} - \rho_{12}\rho_{11}} \qquad (18)
$$

or $$G_{11} = \frac{\rho_{11}}{\rho_{11}^2 + \rho_{12}^2} \qquad G_{12} = \frac{-\rho_{12}}{\rho_{11}^2 + \rho_{12}^2}$$

Provided that the symmetry relations (16) hold.

In the case of a sample of rectangular geometry fig. 3 with the current parallel to Ox_1 direction, equations

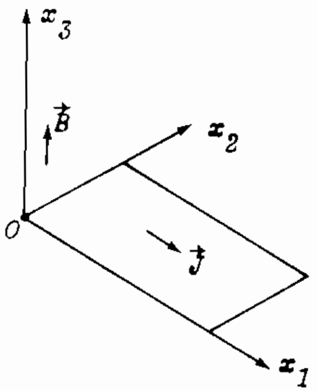


Fig. 3.

(15) simplify to

$$
\begin{aligned}
E_1 &= \rho_1 j \\
E_2 &= \rho_H j
\end{aligned}
$$

since $j_1 = j$ and $j_2 = j_3 = 0$

The potential difference V_{AB} is equal to:

$$V_A - V_B = E_2 b = \rho_H j b \qquad (19)$$

where $b = AB$ is the distance between A and B.

Thus the HALL effect formula results from very general considerations. Now it is easy to define the HALL angle as:

$$\tan\theta = E_2 | E_1 = \rho_H | \rho = RB | \rho = \mu B \qquad (20)$$

where $\mu = R_H | \rho$ is the HALL mobility.

Many authors use the quantities «HALL conductivity» defined as

$$\gamma = \frac{\rho_H}{\rho_1^2 + \rho_H^2} \simeq \frac{\rho_H}{\rho_1^2} \quad (\rho_H << \rho_1) \qquad (22)$$

assuming it to be a measure of the HALL effect | 4 |.

2.2 SEMICLASSICAL THEORIES OF THE HALL EFFECT.

It is easy to explain the HALL effect according to the semiclassical theory of the motion of the electron in a magnetic field.

An electron with Fermi energy E_f, moving with velocity v in a magnetic field is subject to the action of the Lorenz force

$$\vec{E} = \frac{e}{c} \vec{v} \times \vec{H} \qquad (22)$$

where e is the electron charge
c the velocity of light
$\vec{H}$ the intensity of the magnetic field

The equation of motion of the electron in momentum space is evidently

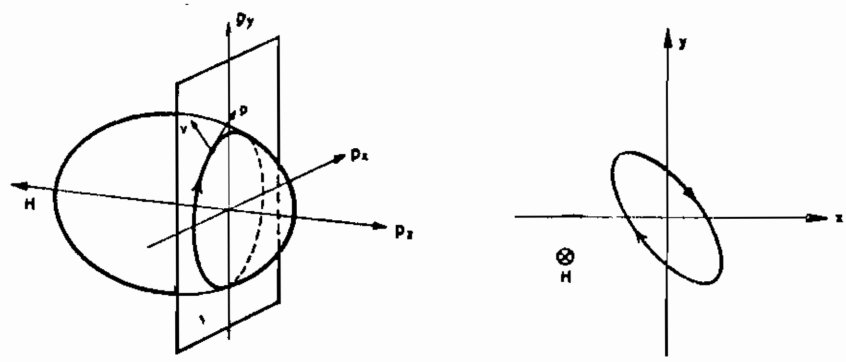
$$\vec{F} = \frac{d\vec{p}}{dt} \qquad (23)$$

where $\vec{p}$ is the crystal momentum.

It is evident from equation (22) and (23) that the representative point which represents the electron in the momentum space p, under the influence of the magnetic field is perpendicular to the direction of vector $\vec{H}$, the motion taking place on the Fermi surface.

The point describes a cyclotron orbit |9| which is defined by the intersection of the Fermi surface and the plane normal to $\vec{H}$, Figs. 4,5.

If we assume that the electron is not subject to collisions then it describes a complete orbit defined by equation:



Ffig. 4: Showing a cyclotron orbit in momentum space (a) and in real space (b) when only $\vec{H}$ is present.

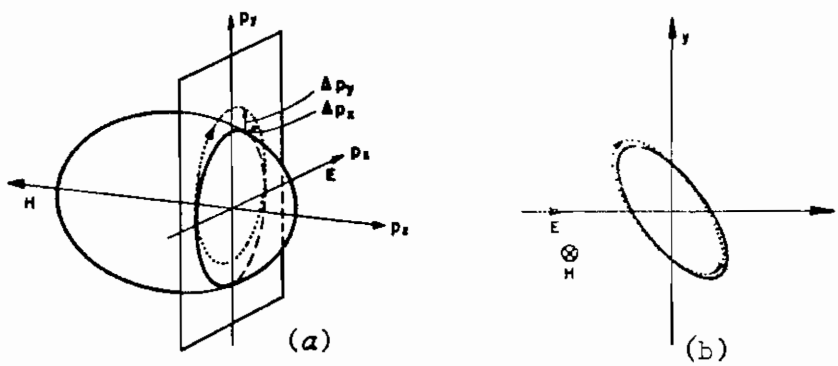


Fig. 5.

$$\frac{2\pi}{\omega_c} = \frac{eh}{\rho_H} \int_S \frac{dk}{v_\perp} \tag{24}$$

where $v_\perp$ is the component of $\vec{v}$ in the plane normal to $\vec{H}$ at $\vec{k}$.

ω_c is the cyclotron frequency, which is given by

$$\omega_c = \frac{2\pi}{\tau_c} = \frac{eH}{m^*c} \tag{25}$$

where $m^* = \frac{h^2}{2\pi}\left(\frac{\delta A}{\delta \varepsilon}\right)$ is the cyclotron mass, defined for the particular orbit which encloses the area A in p space and τ_c is the time taken by the electron to complete one orbit [10].

The equation of motion in real space results substituting

$$\vec{v} = \frac{d\vec{r}}{dt} \tag{26}$$

in equations (22) and (23).

Thus obtaining

$$\frac{d\vec{p}}{dt} = \frac{e}{c}\frac{d\vec{r}}{dt} \times \vec{H} \qquad (1.3) \qquad (27)$$

It is obvious that when the motion of the electron is taking place in a direction parallel to $\vec{H}$, its motion is not affected by the external magnetic field. If an external electric field is added then the energy of the electron is disturbed and its representative point in p space no longer moves on a surface of constant energy.

If we consider an external electric field acting along the x axis, normal to the magnetic field which acts along the z axis, then after some computation* we get the equation of motion

$$\frac{d\varepsilon}{dt} = ev_x E_x = \frac{cE_x}{H}\frac{dp_y}{dt} \tag{28}$$

where ε is the electron's energy.

The orbit of the representative point in p space is displaced from the Fermi surface (on which $\varepsilon = \varepsilon_F$) by an amount

$$\Delta\varepsilon = \varepsilon - \varepsilon_F = \frac{cE_x}{H} p_y + C \tag{29}$$

If Δp_x and Δp_y are the displacements of a point on the perimeter of the section of the Fermi surface in the xy plane we get

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*
$$F_x = eE_x$$

$$F_y = \frac{ev_x H}{c}$$

$$\frac{dp_x}{dt} = eE_x \qquad \frac{dp_y}{dt} = \frac{ev_x H}{c} \rightarrow v_x = \frac{c}{eH}\frac{dp_y}{dt}$$

$$\frac{d\varepsilon}{dt} = F_x v_x = eE_x v_x = eE_x \frac{c}{eH}\frac{dp_y}{dt} = \frac{cE_x}{H}\frac{dp_y}{dt}$$

$$\left.\begin{aligned}\Delta p_y &= \frac{\Delta\varepsilon}{v_y}\\ \Delta p_x &= \frac{\Delta\varepsilon}{v_x}\end{aligned}\right| \qquad (30)$$

A displacement of the Fermi surface in p space results in producing a net electric current. It is easily found that the current densities along the x and y axes are correspondingly:

$$\left.\begin{aligned}\Delta j_y &= \frac{2e}{h^3}\,\Delta p_z \int_S v_y \Delta p_y \Delta p_x = \frac{2e}{h^3}\,\Delta p_z\,\frac{cE_x}{H}\,A(z)\\ \Delta j_x &= \frac{2e}{h^3}\,\Delta p_z \int_S \frac{eE_x}{B}\,p_y d_y = 0\end{aligned}\right| \qquad (31)$$

where Δp_z is the thickness of the intersection of the Fermi surface and xy plane, A(z) being its area.

The Hall effect is thus resulting from the classical theory of the combined action of an electric and magnetic fields.

M. KOHLER |11| used the free electron theory to compute the HALL coefficient for which he obtained the very known expression

$$R_H = \frac{1}{ne} \qquad (32)$$

where n is the number of the conduction electrons per unit volume, R_H being negative, since e is negative.

The application of this formula is quite doubtful |12|. In the case of an one band model it is applicable only for spherical energy surfaces and for relaxation times depending on energy alone.

This simple theory was replaced by a more elaborate theory introduced by SONDHEIMER and WILSON |12|, which includes conduction in two bands and has dominated for a long time.

According to this theory there are two overlapping bands, the s and d bands partially filled. It is assumed that s-d transitions can be neglected. Furthermore, for the sake of simpicity, it is assumed that the behaviour of the electrons in one band is not influenced by the electrons of the other band and that quantum effects of the orbits - orbit's quantization - due to the application of a magnetic field, can be also neglected.

The energy levels in the two bands are assumed to be proportional to the square of the wave vector. Thus for the s band we get:

$$E = \frac{h^2|\vec{k}|^2}{2m_s} \tag{33}$$

where $\vec{k}$ is the wave vector $\vec{k}(k_1, k_2, k_3)$ and m_s the effective mass of the electrons in the s band.

Similarly for the d band we have

$$E = A - \frac{h^2|\vec{k}|^2}{2m_d} \tag{34}$$

where A is the overlapping energy of the two bands.

Sondheimer and Wilson solved the Boltzmann equation for this model using classical transport theory and obtained for the Hall coefficient:

$$R = -\frac{1}{e}\,\frac{\frac{\sigma_s^2}{n_s} - \frac{\sigma_d^2}{n_d}}{(\sigma_s + \sigma_d)^2} = -\frac{1}{e}\left|\frac{2}{n_s}\left(\frac{\sigma_s}{\sigma}\right)^2 - \frac{2}{n_d}\left(\frac{\sigma_d}{\sigma}\right)^2\right| \tag{35}$$

where $\sigma = \sigma_s + \sigma_d$

σ_s and σ_d the conductivities in s and d bands respectively

n_s and n_d the number of carriers in s and d bands respectively

E. M. Pugh |13| remarked that the experimental results for the ordinary Hall coefficient in ferromagnetics, which coincides with the one given in classical theory, could not be explained either with the simple free electron model or with the Sondheimer - Wilson model from which differed by a factor of two. On the other hand no provision was made in this latter model for the ferromagnetic character of the metals Fe, Co, Ni.

Pough extended the Sondheimer - Wilson model assuming that both 3d and 4s bands are divided into two sub-bands in which the spin of the electrons are aligned either parallel or antiparallel to the magnetic field.

Thus the Sondheimer - Wilson formula for the Hall coefficient transforms to

$$R = -\frac{1}{Ne}\,\frac{1}{n_{sp}}\left(\frac{\sigma_{sp}}{\sigma}\right)^2 + \frac{1}{n_{sd}}\left(\frac{\sigma_{sa}}{\sigma}\right)^2 - \frac{1}{n_{dp}}\left(\frac{\sigma_{dp}}{\sigma}\right)^2 - \frac{1}{n_{da}}\left(\frac{\sigma_{da}}{\sigma}\right)^2 \tag{36}$$

The indices p and a divide the 3d and 4s bands into two bands in which the spins of the electrons are aligned either parallel or antiparallel to the magnetic field. Only above the CURIE temperature of the elements the two sub-bands are indentical and the PUGH model is equivalent to the SONDHEIMER - WILSON model.

This model proved to he very successful in explaining numerous experimental results of ferromagnetic metals and alloys.

3.1 QUANTUM MECHANICAL THEORIES OF THE ANOMALOUS HALL EFFECT

Till 1954 it had not been possible to explain the anomalous behaviour of the Hall effect in ferromagnetics by classical conduction theories.

In that year R. KARPLUS and J. LUTTINGER |14| proposed a new model based on safe quantum mechanical reasoning and their formalism became henceforth the background of all later theories.

It is for this reason that this theory will be given here is some extent. In this model a «gas» of magnetic electrons, which correspond to the vacancies of d band, is moving through the periodic potential of the non magnetic ions.

In order that the spontaneous magnetization is taken into account it is assnmed that the carriers with spin up electrons are more numerous than those with spin down electrons.

The temperature dependent difference in occupation spin states is responsible for the spontaneous magnetization in ferromagnetic materials. In this model the itinerant carriers are thought to be responsible for the electric and magnetic properties of the material. The carriers moving under the influence of the external electric field through the periodic potential of the ions, suffer spin orbit type coupling between their spin and their angular momentum. Since the number of the carriers having spin of different directions is unequal, this fact is capable of producing transverse current which according to K and L has the right order of magnitude, in such a way that one could think it, as being responsible for the anomalous Hall effect. The quantum mechanichal formalism is based on the fact that as a result of the spin orbit interaction, the stationary states of the system acquire a left - right assymetry and when an external electric field is added, a current results which is perpendicular to both the field and to the mean direction of spin i.e. to magnetization.

The Hamiltonian of the system is considered as a sum of three terms

$$\mathbf{H}_T = \mathbf{H}_o + \mathbf{H}_{so} + \mathbf{H}^{li} \tag{37}$$

where $\mathbf{H}_o = p^2/2m + v(\vec{r})$ is the Hamiltonian of an electron in the crystal potential energy $v(\vec{r})$ and

$$\mathbf{H}_{so} = \frac{|\vec{\sigma} \times \vec{\nabla} v(\vec{r})| \cdot \vec{p}}{4m^2c^2} \qquad (\hbar = 1)$$

where $\mathbf{H}_{so}$ is the spin orbit interaction * and σ the Pauli spin operator.

The usual representation of the Pauli spin operator σ is:

$$\vec{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \vec{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \vec{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and $\mathbf{H}'' = -e\vec{E}.\vec{r}$ is the Hamiltonian of the electrons in the external electric field $\vec{E}$.

To compute the effect of the spin orbit coupling on the transverse conductivity, the wave functions Φ_l corresponding to stationary so lutions of the Bloch type of equation:

$$\mathbf{H}\Phi_l = (\mathbf{H}_o + \mathbf{H}_{so})\Phi_l = \varepsilon_l \Phi_l \tag{38}$$

are introduced (v(r) periodic and p translationally invariant).

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* Since the magnitude of the motional magnetic field of an electron is

$$\vec{B} = -\frac{\vec{p} \times \vec{E}}{mc}$$

Where $\vec{p}$ is the momentum of the electron, $\vec{E}$ is the electric field strength at the electron and $\vec{B}$ is the resultant magnetic field seen by the moving electron.

The interaction gives rise to a new term in the HAMILTONIAN:

$$\mathbf{H}_{so} = \frac{1}{4m^2c^2} \vec{\sigma}.(\vec{\nabla} v \times \vec{p})$$

It is easily found that |15 |

$$\mathbf{H}_{so} = \frac{e}{mcr^3} \vec{M}.\vec{L}$$

where $\vec{M}$ is the localized magnetic moment, taken at the origin of the coordinates and L is the angular momentum of the charge carriers about the origin.

The functions Φ_l are orthonormal in the volume of the crystal Ω.

$$(\Phi_l,\Phi_{l'}) = \int_\Omega \Phi_l^* \Phi_{l'}(\vec{r}) d^3 x = \delta_{ll'}, \qquad (39)$$

The classical computation's techniques using the solution of the Boltzmann equation give |16| for the average velocity of an electron in a crystal subject to an electric field:

$$< v_a > = - eE_b \tau \sum_l \rho_o'(\varepsilon_l) \frac{\partial \varepsilon_l}{\partial k_\beta} V_a(l) \qquad (40)$$

where v(l) and ε_l are the diagonal elements of the velocity and Hamiltonian operators, including spin - orbit coupling in* the l state. It is shown that this expression could never contain a Hall effect and thus classical computation fails to explain the effect. It is therefore necessary to obtain an extention for the interaction of H^{ii} with the electric field.

The computation of the off - diagonal elements of H^{ii} gives:

$$(n\vec{k}|H''|n'\vec{k}') = \int_\Omega e^{-i\vec{k}\cdot\vec{r}} \omega^*_{nk} H'' e^{i\vec{k}'\cdot\vec{r}} \omega_{n'k'} d^3 x =$$

$$= -eE_b(n\vec{k}|x_b|n'\vec{k}') = - eE_b\{i\delta_{nn'} \frac{\partial}{\partial k_b} \delta_{kk'} + i\delta_{kk'} J^{nn'}_b(\vec{k}) \qquad (41)$$

where
$$J^{nn'}_b(\vec{k}) = \int_\Omega \omega^*_{nk}(\vec{r}) \frac{\partial}{\partial k_b} \omega_{n'k}(\vec{r}) d^3x$$

$$J^{nn}_b(\vec{k}) = J_b(l) \neq 0 \qquad (42)$$

and
$$\Phi_l = e^{i\vec{k}\cdot\vec{r}} \omega_{nk}(\vec{r})$$

given that in the general case we have:

$$(nk|x_a|n'k') = \int_\Omega e^{-i\vec{k}\cdot\vec{r}} \omega^*_{nk} x_a e^{i\vec{k}'\cdot\vec{r}} \omega_{n'k'} d^3 x = \int_\Omega e^{-i\vec{k}\cdot\vec{r}} \omega^*_{nk} \frac{1}{i} \frac{\partial e^{i\vec{k}'\cdot\vec{r}}}{\partial k'_a} \cdot$$

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* The current's dependence on the spin orbit coupling comes through the velocity of equation (40).

$$\cdot\, \omega_{n'k'}\, d^3x = \int_\Omega e^{i\vec{k}\cdot\vec{r}}\, \omega^*_{nk} \frac{1}{i} \left| \frac{\partial}{\partial k'_a} \left(e^{i\vec{k}'\cdot\vec{r}}\, \omega_{n'k'}\right) - e^{i\vec{k}'\cdot\vec{r}} \frac{\partial \omega_{n'k'}}{\partial k'_a} \right| d^3x =$$

$$= -i \frac{\partial}{\partial k'_a} (\delta_{nn'}\delta_{kk'}) + i \int_\Omega e^{(i\vec{k}'-\vec{k})\cdot\vec{r}}\, \omega^*_{nk} \frac{\partial \omega_{n'k'}}{\partial k'_a} d^3x \qquad (43)$$

Taking into consideration that ω_{nk} are orthonormal the above equation (41) simply becomes:

$$(n\vec{k}\,|x_a|n'k') = i\delta_{nn'} \frac{\partial}{\partial k_a} \delta_{kk'} + iJ_a^{nn'}(\vec{k}) \qquad (44)$$

K and L proved that the second term in equation (44) is entirely responsible for the anomalous Hall effect. They introduced time dependent methods in evaluating the average velocity due to the periodic Hamiltonian * including a regular periodic perturbation with matrix elements

$$-ie\delta_{kk'}E_b J_b^{nn}(\vec{k}') \qquad (45)$$

The computation of the average velocity is effected using the known |18| relations

$$< v_\alpha > = \mathrm{Tr}\{\rho_o v_\alpha\} \text{ and } \mathrm{Tr}\{\rho_o\} = 1 \qquad (46)$$

The average velocity is then given by the formula

$$< v_\alpha > = -ieE_b \sum_l \rho'_o(\varepsilon_l) v_b(l) J_\alpha(l) \qquad (47)$$

in which the current $J\alpha(l)$ is included.

It is also proved that the current is perpendicular to both the electric field and magnetization and that it vanishes in the absence of a spin orbit coupling.

Their final formula for the average velocity in vector notation is:

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* Periodic Hamiltonian defined as

$$\mathbf{H}_p = \mathbf{H}_o + \mathbf{H}_{sp} + \mathbf{H}_i^{\cdot\cdot} + \mathbf{H}_1^{\cdot\cdot}$$

$\mathbf{H}_i^{\cdot\cdot}$ is a singular operator responsible for usual conductivity effects.
$\mathbf{H}_1^{\cdot\cdot}$ perturbing Hamiltonian with matrix elements described above. The velocity tensor is defined as

$$v_a = i\ |\mathbf{H},\ x_a\ |$$

with [ ] meaning as usual the commutator $[\mathbf{A}, \mathbf{B}] = \mathbf{AB} - \mathbf{BA}$

$$< \vec{v} > = -\frac{e}{m\Delta^2} \sum_{l(d)} \rho_o'(\varepsilon_l) \left| \vec{E} \cdot \vec{v}(l) \right| \vec{F}(l) \qquad (48)$$

where Δ^2 is some mean square energy separation between different d bands for the same $\vec{k}$ at the top of their Fermi surfaces.

Replacing in the above expression $\vec{F}(l)$ — $\vec{F}(l)$ being the spin orbit force - with

$$\vec{F}(l) = \frac{e}{c} \left| \vec{v}(l) \times \vec{H} \right| \qquad (49)$$

and

$$\mathbf{H} = \frac{H_{so}\vec{M}}{M_s}$$

we finally get:

$$< \vec{v} > = -\frac{e^2}{\Delta^2 mc} H_{so}\vec{E} \cdot \left| \sum_{l(d)} \vec{v}(l)\rho_o'(\varepsilon_l)\vec{v}(l) \right| \times \frac{\vec{M}}{M_s} \qquad (50)$$

which-for a cubic crystal transforms to

$$\vec{v} = \frac{e^2}{\Delta^2 mc} H_{so} \frac{\vec{M} \times \vec{E}}{M_s} \sum_{l(d)} \rho_o'(\varepsilon_l) v_x^2(l) \qquad (51)$$

Thus the resultant transverse current is

$$J_y = N_d e \bar{v}_y = r M_z E_x \qquad (52)$$

where N_d is the total number of magnetic electrons contributing to the current *.

Since the Hall voltage is proportional to current density

$$E_y = -\rho J_y \qquad (53)$$

the Hall resistivity finally becomes:

$$\rho_H = \frac{E_y}{j_x} = \rho r M_z E_x / j_x = -\rho^2 r M_z \qquad (54)$$

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* Their assumptions to their own opinion anyway, were quite «crude» leading to the oversimplification of the situation and to the critisism of SMIT |17, 18 |.

Thus the anomalous Hall coefficient $R_s = -\rho^2 r M_z$* depends on the temperature, varying as the square of the resistivity.

R_s is resulting by integration of the physical quantities on a band, taking averages as

$$R_s = \frac{2e^2}{m\Delta^2} H_{so}\delta < \frac{m}{m^*} > d_{av}\rho^2 \qquad (55)$$

where m^* is the effective mass

δ is the number of the non complete bands.

K and L using the experimental results of JAN |19| and JAN and GIJMAN |20| as an application of their theory, concluded that a law of the form

$$R_s \simeq A\rho^n \text{ with } n \simeq 2 \qquad (56)$$

does hold.

In a subsequent paper SMIT |21| remarked that K and L did not estimate rightly the consequences of the spin orbit interaction and that its action leads to a modification of the Bloch functions but not to the scattering in a perfectly periodic lattice **. Such a scattering could be caused only by impurities or lattice imperfections.

To his opinion such a scattering combined with spin orbit interaction could cause a Hall field, being assymetric.

He also remarked that the off - diagonal elements of the current density could not contribute to the Hall effect as K and L assumed in their theory.

In a succesion of papers LUTTINGER and SMIT |22| argued on the above matters and it seems that this has not be settled. Both of the above theories (Smit's theory of lattice imperfections and impurities and K and L's of spin orbit interaction) have the same disadvantage***: they are not applicable in the case of the rare earth elements, for which it is quite well established from experimental measurements - specific

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* r is given by the detailed formula $r = -\frac{e^3}{mc\Delta^2} H_{so} \frac{N_d}{M_s} \sum_{l(d)} \rho_o(\varepsilon_l) v x^2(l)$

** The charge carrier beeing scattered asymmetrically with respect to a plane defined by the carriers incoming direction and the direction of its spin.

*** SMIT's theory suffers from an additional drawback: the equation obtained for the anomalous HALL angle changes sign according to whether the perturbing potential of the impurity is attractive or repulsive. This prediction is disproved by his own experiments with impurities in Ni. The HAAL effect remained negative although the impurities atomic numbers were lower or higher than Ni's (Al or Si for the first case, Sn or W for the second).

heat and magnetic susceptibility measurements - that they consist of tri - valent ions and localized 4f magnetic electrons with almost free conduction electrons.

STRACHAN and MURRAY |23| argued that K and L's results concerning the qualitative behaviour of the extraordinary HALL coefficient was not the product of a detailed theoretical analysis but a direct consequence of its definition.

STRACHAN and MURRAY calculated the anomalous HALL coefficient taking into account not only the spin orbit coupling but defining in detail the wave functions of the electrons as well, and regarding the final states of the electrons after their collision with the lattice as a superposition of BLOCH functions with arbitrary phases.

They calculated the BLOCH functions for the s and d bands for Ni and gave the formula

$$R_s = -3{,}25 \times 10^{-14} \frac{(\rho \times 10^6)^2}{M_s(T)} \left\{1 + 0{,}089 \left(\frac{T}{100}\right)^2\right\} z^2\theta \qquad (57)$$

where ρ the resistivity of Ni.

$M_s(T)$ the saturation magnetization of Ni.

T the temperature in absolute grades.

θ a constant ≤ 1.

z energy difference between s and d bands.

IRKHIN and SAVROV |24| obtained a similar expression for R_s as K and L considering the scattering by phonons which becomes important as temperature is raised from absolute zero. Their theoretical treatment is based on second quantization methods.

Their result for the average velocity of the carriers is in close agreement with the one of K and L, but it differs substantially when the scattering is effected by impurities |22|.

Their calculation of the anomalous HALL coefficient is based on the calculation of the σ_{xy} element of the conductivity tensor, starting from the expression of the average velocity, which is:

$$< v_\beta > = ieE_\alpha \sum_l \frac{\partial \rho_l^o}{\partial \varepsilon_l} v_l^a \Big|_l^b$$

they calculated $J_n^\beta(\vec{k})$ and gave in the first approximation of pertubation theory

$$J_n^b(\vec{k}) = 2 \sum_{n' \neq (n)} \frac{I_{nn'}^{\beta}(\vec{k}) H_{n'n}^{so}(\vec{k})}{\varepsilon_n^o(\vec{k}) - \varepsilon_{n'}^o(\vec{k})} \tag{59}$$

using the symmetry relations
introduced by K and L

$$\left.\begin{array}{l} I_{nn'}^b = - I_{n'n}^b = - \dfrac{p_{nn'}^{\beta}}{m\omega_{nn'}(\vec{k})} \\ H_{n'n}^{so} = - H_{nn'}^{so} \end{array}\right| \tag{60}$$

where m the mass and p the momentum of the electron and $\omega_{nn'} = \varepsilon_n^o - \varepsilon_{n'}^{(o)}$ they obtained

$$J_n^{\beta}(\vec{k}) = \frac{1}{m} \sum_{n'} \frac{H_{nn'}^{so} p_{n'n}^{\beta} - p_{n'n}^{\beta} H_{n'n}^{so}}{\omega_{nn'}^2} \tag{61}$$

substituting (61) into (58) we get

$$\sigma_{\beta\alpha} = \frac{en<v_{\beta}>}{F_{\alpha}} = \frac{e^2 n}{m\Delta^2} \sum_l \frac{\partial \rho_l^o}{\partial \varepsilon_l} v_l^a i \left| H^{so}, \vec{p} \right|_l^{\beta} \tag{62}$$

where n is the number of current carriers in unit volume and calculating*

* The calculation is effected replacing σ by relative magnetization M/M_z in (62a) which then becomes

$$|H^{so}, \vec{p}|^{\beta} = \frac{i}{4m^2c^2M_s} \left(\left| \vec{p} \times \vec{M} \right|^{\gamma} \frac{\partial}{\partial r_{\gamma}} \right) \left| \frac{\partial v}{\partial r_{\beta}} \right. \tag{63a}$$

The diagonal bloch matrix elements are

$$\left| H^{so}, \vec{p} \right|_l^{\beta} = - \frac{i}{4m^2c^2M_s} |\vec{k} \times \vec{M}|^{\gamma} \int u^*_{nk}(r) \frac{\partial^2 v}{\partial r_{\gamma} \partial r_{\beta}} u_{nk}(\vec{r}) d\vec{r}_o \tag{64}$$

substituting equation (65) and taking into consideration Poisson's equation we have

$$\sigma_{\beta\alpha} = - \frac{\pi}{3} \frac{e^4 n}{m^3 c^2 \Delta^2 M_s} \sum_l v_l \frac{\partial \rho_l^o}{\partial \varepsilon_l} v_l^a | \vec{k} \times \vec{M} |^{\beta} \tag{65}$$

where $$v_e = \int u^*_{nk}(\vec{r}) \rho(\vec{r}) u_{nk}(\vec{r}) d\vec{r}_o \tag{66}$$

$\rho(\vec{r})$ is the density of the charge responsible for the potential v. Finally setting $M = M_z$ and integrating over $\vec{k}$ averaging over all bands n we find equation (63).

$$\mathbf{H}_{so} = \frac{1}{4m^2c^2} \vec{\nabla} v \left| \vec{p} \times \vec{\sigma} \right| \tag{62a}$$

finally (62) becomes

$$\sigma_{yx} = \frac{\pi}{3} \frac{e^4 n < \vec{v} >}{n^2c^2\Delta^2} \frac{M_z}{M_s} \delta < \frac{1}{m^*} > \tag{63}$$

Thus the anomalous HALL coefficient is

$$R_s = - \rho^2 \sigma_{yx} \frac{1}{4\pi M_z} = - \frac{1}{12} \frac{e^4 n\vec{v}}{m^2c^2\Delta^2} \frac{\delta}{M_s} < \frac{1}{m^*} > \rho^2 \tag{67}$$

IRKHIN and SHAVROV replacing the corresponding physical quantities in (67) with their numerical values

$$\Delta \simeq 10^{-12}\,\text{erg} \qquad n \sim 10^{22}\,\text{cm}^{-3} \qquad m^* \simeq 10\,\text{m}$$
$$M_s \sim 10^3\,\text{Oe} \qquad \rho(300\,^\circ\text{K}) \simeq 7.10^{-6}\Omega\cdot\text{cm}$$

estimated R_s to be

$$R_s \simeq 10^{-11} \frac{\text{V.cm}}{\text{A} \cdot \text{G}}$$

which is in close agreement with the experimental results, under the assumption that $\vec{v} = 10^{27}\,\text{cm}^{-3}$, which means that there is a localization of the electron density in small regions of linear dimensions 10^{-9} cm. Furthermore a comparison of the R^1_s for scattering by impurities to the R_s^{ph} for scattering by phonons, results to the conclusion that even at high temperature the contribution of impurity scattering in the total, is very small, and that the total HAöL coefficient

$$R_s^T = - \rho_T^2\, \sigma_{yx}^T / 4\pi M_z \simeq R_s \tag{68}$$

remains proportional to ρ^2.

Finally IRKHIN and SHAVROV concluded that the experimentally observed deviations from the square law might be due to magnetic inhomogeneities which can lead to the appearance of a linear term in the dependence of $R_s(\rho)$. In a subsequent paper |25| IRKHIN and ABEL'KII* discussed the spontaneous HALL effect in ferromagnetic as arising

* The interaction mechanism introduced by IRKHIN and ABEL skii is the mixed type of s-orbit/d-spin interaction.

This interaction is the one between the force which experiences a moving from the scattering of conduction electrons by the spin inhomogeneities taking into acount the intrinsic spin orbit interactions of the magnetic electrons as well as the interactions between the orbital momenta of the conduction electrons and the spins of the magnetic electrons.

They obtained a solution for the Kinetic equation for the scattering, using second quantization methods, the method giving a temperature dependence of the HALL coefficient $R_s(T) \sim \rho_{mag}(T)$ ($\rho_{mag}(T)$ is the magnetic part of the electric resistivity).
Especially for $s = 1/2$ they found

$$R_s(T) \simeq M_s^2(0) - M_s^2(T)$$

M_s beeing the spontaneous magnetization. It is a doubtful procedure anyway the one which estimates the ρ_{mag} using the method proposed by Weiss and Marotta |26| i.e. by extrapolating the temperature dependence curve of the phonon part of the resistivity ρ_{ph} from the interval $T > T_c$ to a region of lower temperatures.

GUREVICH and YASSIEVICH |27| considered the case of scattering by phonons and impurities simultaneously and obtained the same temperature dependence of the anomalous HALL effect as before i.e. $R_s \sim \rho^2$ which they found to be valid for high and low temperatures as well. They also examined the temperature dependence of the ratio of the ferromagnetic HALL constant to the ordinary one, due to the temperature dependence of the relaxation time and magnetization. It was for the first time that the possibility of a sign change in the ferromagnetic Hall coefficient with changing temperature was discussed.

KONDORSKII |28| used spin orbit interaction and scattering by impurities and phonons simoultaneously and was able to obtain a formula which includes a linear term in ρ i.e.

$$R_s = a\rho + b\rho^2 \qquad (69)$$

The spin orbit interaction used here refers to spin orbit interaction between the conduction electrons and the orbits of the localized electrons (electrons bound on ions).

In a paper presented in 1969, KONDORSKII |29| suggested that all the above theories using itinerant electrons consider that each electron has the same magnetic moment M_z along the direction of the average magnetization. Thus, those theories are not taking into account the fact

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conduction electron by the magnetic field of a localized magnetic moment on an ion.

that the exchange interaction which is responsible for the properties of ferromagnetism splits in energy the distributions corresponding to the two spin directions.

In ferromagnetic state the itinerant carriers occupy states on the Fermi surface consisting of different sheets for up - spin and for down - spin orientations. When the splitting is comparable with the width of the energy gap in the non ferromagnetic state distinction must be made between the electron and the hole like characteristics for each of the spin sheets of the Fermi surface.

KONDORSKII writes for the anomalous HALL coefficient

$$R_s = \frac{c}{I_s \sigma^2} \sum_n J_n^y \tag{70}$$

where c is a constant

I_s the spontaneous magnetization

σ the electrical conductivity

J_n^y is the contribution of the nth spin zone, for which KONDORSKII obtained the following expression

$$J_n^y = cM_n \int_{S_n} \left| \frac{qv_x k_x}{|\mathrm{Grad} E|} \right| \begin{matrix} dS_n \\ E = E_F \end{matrix} \tag{71}$$

the intergal is taken on the Fermi surface S_n of the n - th spin zone M_n is the average z component of the magnetic moment of an electron in the n - th band.

q dimensionless parameter of order of unity

ε is the energy of an electron with wave vector k and ε_F the Fermi energy. The sign of each contribution to the anomalous Hall effect depends upon the signs of M_n and the integral over S_n. KONDORSKII assumes that the anomalous effect is a result of the scattering from phonons or thermal disorder of the spin system in connection with a spin orbit interaction of the itinerant electrons. His theory is in agreement with experimental results in Ni.

KONDO |30| proposed a new model in which the charge carriers (which he calls s electrons) are equally distributed between states of opposite spin. The ions then have a nonvanishing total spin which accounts for the magnetic properties of the material. This is due to the spin of the d electrons which are localized on the ions. The spins of the magnetic ions are disordered by temperature and the charge carriers are scattered by the non periodic potential which is thus generated and which acts on their spins. Periodicity is destroyed by the disorder of the ionic magnetic moments rather, than by the introduction of impurities. KONDO introduces the mechanism known as s - d* spin - spin interaction, which accounts principaly for the resistivity of magnetic materials in terms of scattering of s electrons by d electrons. The s - d interaction provides no skew** scattering by itself, even though KONDO shows that it is anisotropic if the orbital ground state of the electron is degenerate.

On the other hand this anisotropy disappears when the groundstate is nodegenerate.

It was for this reason that KONDO introduced a new interaction mechanism the intrinsic spin orbit interaction of d electrons within the magnetic ions *** KONDO thought that the skew scattering is caused by an anisotropy of the s - d interaction. The function of the spin - orhit interaction on the electrons within the ions is to permitt an odd power of the s - d interaction proper to a degenerate ground state to appear in the transition probabilities.

KONDO's theory provides the correct temperature dependence of the anomalous HALL effect in metals.

KONDO himself made the following remark:

If the intrinsic spin orbit interaction is quenched then in this model there is no skew scattering and consequently, no anomalous HALL effect results. KONDO |31| therefore suggested that the intrinsic spin - orbit interaction is dominated by the s - d mixing interaction****.

The mixed interaction gives rise to skew scattering of the s electrons

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* This interaction is taking place between the spin of the conduction electron and the spin angular momentum of the incomplete d shell.

** Skew symmetric in this model is thought to be a scattering producing a wave function of the scattering charge carrier which is not invariant with respect to mirroring along a plane defined by the incoming direction of the s electron and the direction of the spin of the scattering ion.

*** This interaction is taking place between the spiu of the electrons and the apparent magnetic field-produced when we consider the coordinate system to move with the valence electrons, since there is au apparent orbital motion of the nucleus about the electron. It is defined for a specific orbit in an ion and it is clearly zero for the s part of the wave function.

**** This is a covalent mixing between ionic orbitals (incomplete shells) and conduction electrons. The coupling is always antiferromagnetic (J is negative) since a conduction electron interchanged with a d-orbital vacancy, must have a spin opposed to the resident d electron in the state. Beeing basically a resonance effect the s and d states must have similar energies.

and hence an anomalous HALL contribution.

KAGAN and MAKSIMOV |32| computed the anomalous HALL effect introducing spin - orbit interaction between non polarized conduction electrons and the spin system of a ferromagnetic metal. The scattering process of this model is thought to be the one, of the conduction electrons, by fluctuations of the spin system and by impurities. They employed the density matrix method taking into account non elastic scattering processes.

They found that a

$$R_S \sim T^4$$

law for low temperatures hold, and a $R_S \sim T^2$ law for higher temperatures.

RHYNE |33| found a $R_S \sim T^2$ law for Gd in the temperature range $4.2 \leq \theta \geq 80$ K°, which seems to be in direct contradiction with the above theory.

RHYNE suggested that the neglect of GAGAN and MASKIMOV of an appreciable anisotropy in the energy gap in the wave spectrum may be an important factor in their calculation.

IRKHIN et al |34| found that an $R_S \sim T^3$ law holds for low temperature region.

Leribaux |35| examined the case that the electron transport is limited by electron phonon interactions. His theoretical treatment is based on KUBO's formalism of current correlation functions. He proved that to the first order in the magnetization, the off diagonal conductivity is of order zero in the electron phonon interaction and to this order is equivalent to K and L's results. Leribaux calculated the anomalous HALL coefficient for monocrystalline bcc Iron, in the low temperature regime, and found his results to be correct in sign but differing by a factor of 1/3 from the experimental results given by DHEER |36| for iron whiskers.

Finally MARANZANA |37| introduced the following theory: The ions possess a nonvanishing magnetic moment due to localized d electrons which account for the magnetic properties of solids. The charge carriers (s electrons) are equally distributed between states with spin up and spin down and they are scattered by the disorder induced by temperature in the system of magnetic ions. In addition to s - d interaction, which is responsible for the resistivity of the material, another interaction mechanism is added a d spin * s orbit interaction.

MARANZANA's theory was capable of explaining, the temperature dependence of the HALL effect, the sign of the anomalous HALL coefficient but failed to explain the anomalies in the paramagnetic region of antiferromagnetics. It is quite evident from the dispersion of the various theories, the various theoretical treatment and the variety of their conclusions that the HALL effect in ferromagnetics is not fully understood and that a rather more elaborate or effective theory is needed to cover all the drawbacks and defficiencies of current theories.

* MARANZANA proved that a term is added to the HAMILTONIAN of the electron of the type

$$\mathbf{H} = -\frac{e}{ML}\,\frac{\vec{M}\cdot\vec{L}}{r^3}$$

where M is a magnetic moment placed at the origin of the coordinate system and L the orbital angular momentum of the charge carrier.

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ΠΕΡΙΛΗΨΙΣ

ΕΠΙΣΚΟΠΗΣΙΣ ΝΕΩΤΕΡΩΝ ΘΕΩΡΙΩΝ ΕΠΙ ΤΟΥ ΦΑΙΝΟΜΕΝΟΥ HALL ΕΙΣ ΣΙΔΗΡΟΜΑΓΝΗΤΙΚΑ ΥΛΙΚΑ

Ὑπὸ

ΙΩΑΝΝΟΥ Α. ΤΣΟΥΚΑΛΑ καὶ Ε. ΠΑΠΑΔΗΜΗΤΡΑΚΗ - ΧΛΙΧΛΙΑ

(Ἐργαστήριον Γ' Ἕδρας Φυσικῆς)

Ἡ παροῦσα ἐργασία ἀποτελεῖ ἔκθεσιν τῶν μέχρι σήμερον ἰσχυουσῶν θεωριῶν περὶ τοῦ φαινομένου Hall εἰς τὰ σιδηρομαγνητικὰ μέταλλα.