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ON THE SYNTHESIS OF NONUNIFORM PLANAR ANTENNA ARRAYS

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Abstract : *A synthesis of a nonuniform planar array is obtained by the method of orthonormalization of the base where the array factor f(φ) is referred. Some applications are carried out.*

I. Theoretical Synthesis

The problem of synthesizing a planar array can be solved like the one of linear arrays |1|, |2|.

The planar factor of an array consisting of N discrete elements is given by

$$f(\varphi) = \sum_{i=1}^{N} A_i \exp \mid jkr_i \cos(\varphi - \varphi_i) \mid \qquad (1)$$

where $k = 2\pi/\lambda$ and (r_i, φ_i) are the planar co-ordinates of the ith element One can find the amplitudes A_i by orthonormalizing the base referring to $f(\varphi)$. Thus, the infinite set

$$\{\Phi_i(\varphi) = \exp \mid jkr_i \cos(\varphi - \varphi_i) \mid\}$$

will, on applying Schmidt's method, give the new set

$$\{\Psi_i(\varphi) = \sum_{j=1}^{i} C_j^{(i)} \exp \mid jkr_i \cos(\varphi - \varphi_i) \mid\} \qquad (2)$$

the coefficients $C_j^{(i)}$ of (2) will be determined with the aid of the following relations:

$$\Psi_1(\varphi) = \frac{\Phi_1(\varphi)}{\mid <\Phi_1, \Phi_2> \mid^{1/2}}$$

$$\Psi_n(\varphi) = \frac{\Phi_n(\varphi) - \sum_{j=1}^{n-1} < \Phi_1, \Psi_j, > \Psi_j(\varphi)}{| < A_n, A_n > |^{1/2}} \quad (3b)$$

where A_n is the numerator of (3b).

Taking into account that

$$< \Phi_n, \Psi i > = \int_o^{2\pi} \Phi_n(\varphi)\Psi^*_j(\varphi)d\varphi = 2\pi \sum_{i=1}^{j} C^{*(j)}_i J_o(ke_{ni})$$

(J_o is the zeroth Bessel function)
where

$$e_{ni} = \frac{r_n \sin\Phi_n - r_i \sin\Phi_i}{\sin\Phi_{ni}}$$

and

$$\text{cost}\Phi_{ni} = \frac{r_n \cos\Phi_n - r_i \cos\Phi_i}{r_n \sin\Phi_n - r_i \sin\Phi_i}$$

considering (1) and (2) and solving (3a) and (3b), we get

$$C^{(n)}_n = \frac{1}{D_n}$$

$$C^{(n)}_k = -\frac{2\pi}{D_n} \sum_{j=1}^{n-1} C^{(j)}_k \left(\sum_{i=1}^{j} C^{(j)}_i J_o(ke_{ni}) \right)$$

$$D_n = \left| 2\pi - 4\pi^2 \sum_{j=1}^{n-1} \left(\sum_{l=1}^{j} C^{(j)}_i J_o(ke_{ni}) \right)^2 \right|^{1/2} \quad (4)$$

$$B_j = \int_o^{2\pi} f(\varphi)\Psi^*_j(\varphi)\, d\varphi$$

and

$$A_i = \sum_{j=i}^{N} B_j C^{(j)}_i$$

11. Maximum Gain

To have a maximum gain in a certain direction φ_o, it is necessary for the radiation pattern to be an impluse function in this direction. Given that the array plane can be considered as the Oxy plane of an orthogonal system *Oxyz*, there follows that the maximum will appear in a direction $\theta = \pi/2$, $\varphi = \varphi_o$.

The radiation pattern is

$$F(\varphi,\theta) = g(\varphi,\theta)\Sigma A_i e^{jkr_i\cos(\varphi - \varphi_i)} \tag{5}$$

On the array plane,

$$F(\varphi,\pi/2) = g(\varphi,\pi/2)\ \Sigma A_i \exp\{jkr_i\cos(\varphi - \varphi_i)\} \tag{6}$$

In agreement with the above discussion, it is necessary that

$$\left.\begin{array}{l} g(\varphi_o, \pi/2) = 1 \quad \text{and} \\ F(\varphi, \pi/2) = \delta(\varphi - \varphi_o) \end{array}\right| \tag{7}$$

Then

$$B_j = \int_o^{2\pi} f(\varphi)\Psi_j^*(\varphi)d\varphi = \int_o^{2\pi} \frac{F(\varphi,\pi/2)}{g(\varphi,\pi/2)} \Sigma \Psi_i^*(\varphi)d\varphi$$

$$= \int_o^{2\pi} \frac{\delta(\varphi - \varphi_o)}{g(\varphi,\pi/2)} \Psi_i^*(\varphi)d\varphi = \Psi_i^*(\varphi_o) \tag{8}$$

The gain will be

i) for isotropic elements $(g(\varphi,\theta) = 1)$

$$G = \frac{|F(\varphi_o, \pi/2)|^2}{1/4\pi \int_o^{\pi} \int_o^{2\pi} |F(\varphi,\theta)|^2 \sin\theta d\varphi d\theta} = \frac{|f(\varphi_o)|^2}{1/4\pi \int_o^{\pi} \int_o^{2\pi} |f(\varphi)|^2 \sin\theta d\varphi d\theta} =$$

$$= 2\pi \frac{\Sigma B_i B_i^{*2}}{\Sigma B_i B_i^*} = 2\pi \Sigma B_i B_i^* \tag{9}$$

ii) For parallel dipoles, normal to the array plane,

$$(g(\varphi,\theta) = \sqrt{1 - \sin^2\theta\cos^2\varphi}),$$

$$G = \frac{|F(\varphi_o, \pi/2)|^2}{\int_0^{\pi} \int_0^{2\pi} |F(\varphi,\theta)|^3 \sin\theta d\varphi d\theta} =$$

$$= 2\pi \frac{|\Sigma B_i B_i^*|^2}{\Sigma B_i B_i^* - 2/3 \; \Sigma\Sigma A_i A_j \{J_o(ke_{ij}) - \cos\varphi_{ji} J_1(ke_{ij})\}} \tag{10}$$

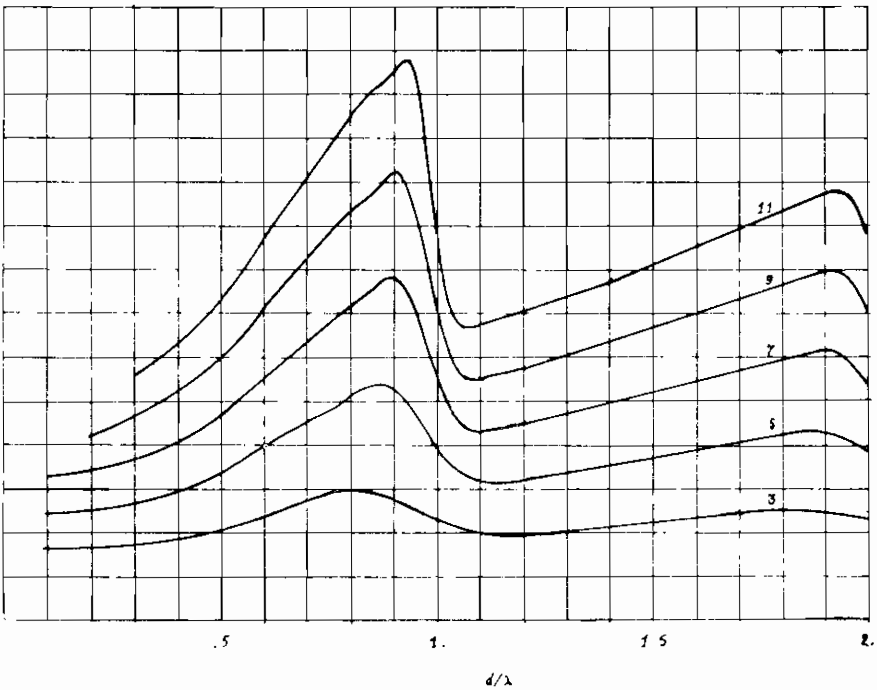

Fig. 1
Optimum gain of axial arrays of crossed dipoles
($N = 3,5,7,9,11$)

Figs (1) and (2) represent the gain for two cases: The first regards a uniform circular array of isotropic elements, and the second a uniform linear array of dipoles; for both cases, one cannot fail to remark the coincidence with the nomographs given by Cheng |4|, Tai |5| respectively.

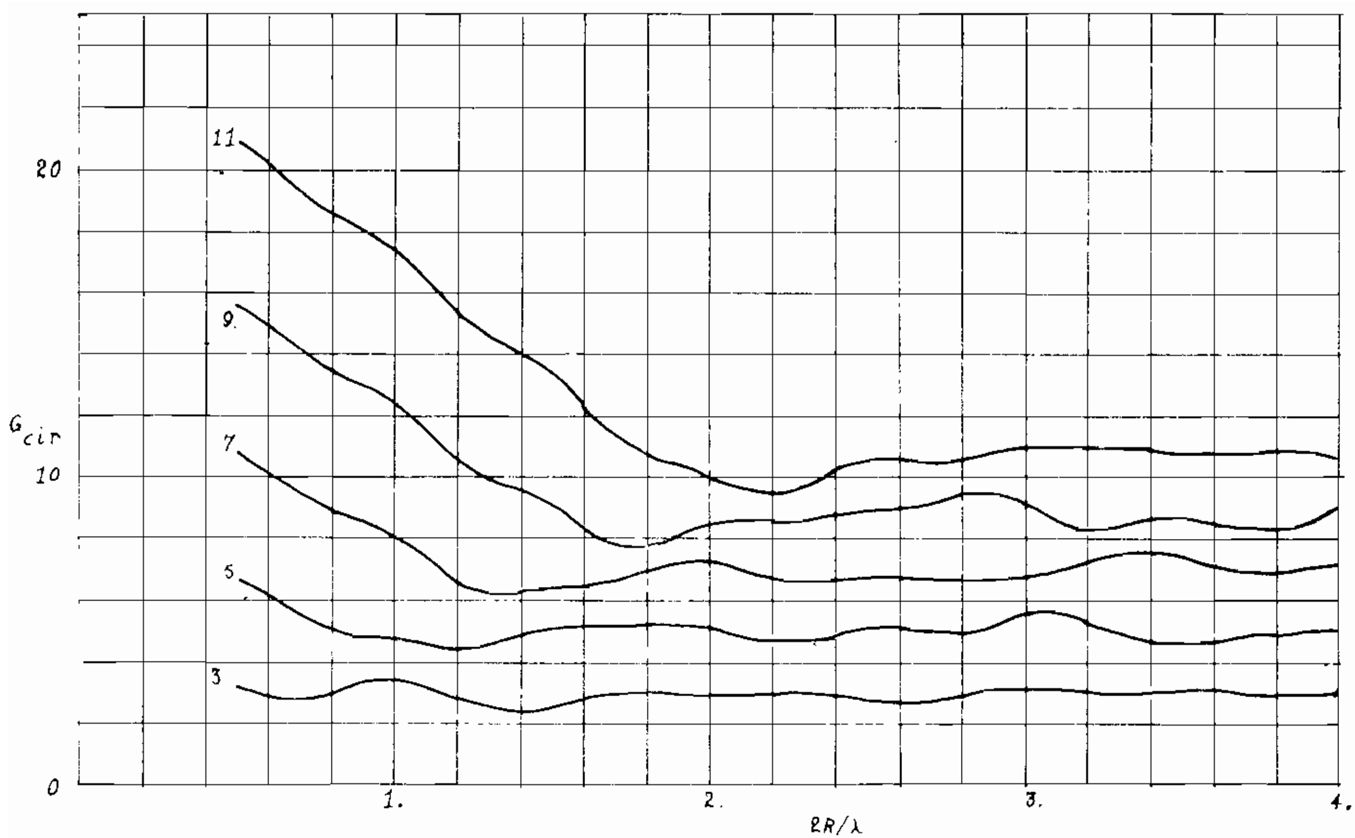

Fig. 2 Optimum gain of circular arrays of isotropic elements ($N = 3, 5, 7, 9, 11$)

III. A Directive Radiation Pattern

A directive radiation pattern can be expressed as

$$f(\varphi) = \exp\{p\cos(\varphi - a)\}\cos\{q\sin(\varphi - a)\} \tag{11}$$

Maximum occurs at $\varphi = a$, while the first null when

$$q\sin(\varphi_o - a) = \pi/2 \Rightarrow \varphi = \frac{\pi}{2\sin(\varphi_o - a)} \tag{12}$$

A secondary maximum can be defined by zeroing the derivative of $f(\varphi)$, that is

$$P\sin(\varphi_1 - a)\cos|q\sin(\varphi_1 - a)| + q\cos(\varphi_1 - a)\sin|q\sin(\varphi_1 - a)| = 0$$

The side lobe level of the radiation pattern is

$$M = \frac{F(a)}{F(\varphi\ )} = \frac{\exp\{P|1 - \cos(\varphi_1 - a)|\}}{\cos|q\sin(\varphi_1 - a)|} \tag{14}$$

For $a = 0$, one gets

$$\left.\begin{aligned} q &= \frac{\pi}{2\sin\varphi_o} \\ P &= -\frac{\pi}{2\sin\varphi_o} \cdot \frac{\tan\left|\dfrac{\pi\sin\varphi_1}{2\sin\varphi_o}\right|}{\tan\varphi_1} \\ M &= \frac{\exp\{P(1 - \cos\varphi_1)\}}{\cos(q\sin\varphi_1)} \end{aligned}\right| \tag{15}$$

With the aid of (15) we can get P, q and M versus φ_1, φ_o, as shown in figs (3a,b,c).

From (11) we can get the values of B_j as follows:

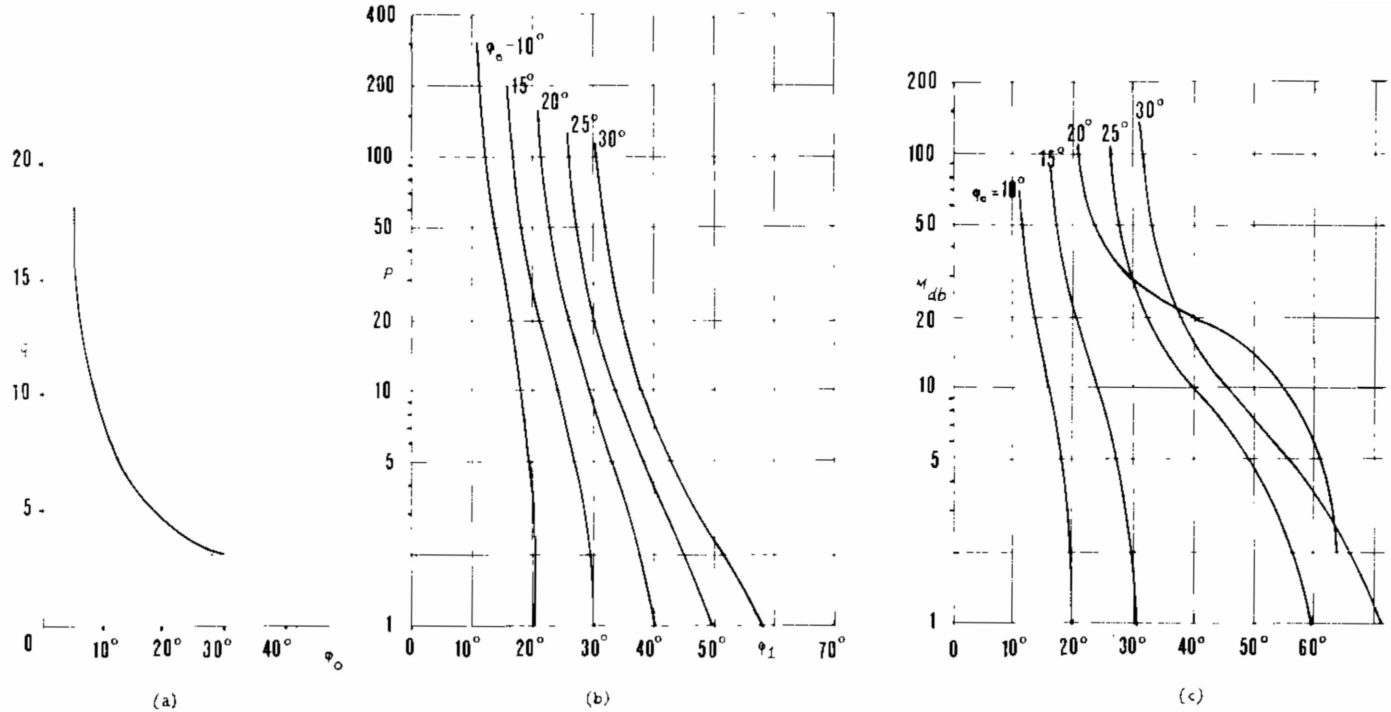

Fig. 3 *g*, *P* and M_{db} *versus* φ_o, φ_1

$$B_i = \pi \sum_{i=1}^{j} C_i^{*(j)} \mid I_o (\sqrt{b_i - jC_i}) + I_o(\sqrt{d_i - jf_i}) \mid \qquad (16)$$

where

$$b_i = P^2 - g_i^2 \quad , \quad C_i = -2Pg_i\cos(\Phi_i - \Psi_i)$$
$$d_i = P^2 - h_i^2 \quad \quad f_i = -2Ph_i\cos(\Phi_i - Z_i)$$

$$g_i = \frac{q\cos a + kr_i\sin\Phi_i}{\sin\Psi_i} \quad , \quad h_i = \frac{q\cos a - kr_i\sin\Phi_i}{\sin Z_i}$$

$$\text{cost}\ \Psi_i = \frac{q\sin a + kr_i\cos\Phi_i}{q\cos a + kr_i\sin\Phi_i} \quad , \quad \text{cost}\ Z_i = \frac{q\sin a - kr_i\cos\Phi_i}{q\cos a - kr_i\sin\Phi_i}$$

IV. Conclusion

As shown by the preceding discussion, it is possible to synthesize non-uniform planar arrays by the orthonormalization method. The maximum gain can be easily found on replacing the radiation pattern by the δ — Dirac function.

By choosing a directive radiation pattern it is possible to synthesize an array having pre-defined i) angle of maximum, ii)width between the first nulls and iii) Side lobe level.

All these apply equally well to linear arrays, too.

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ΠΕΡΙΛΗΨΙΣ

ΠΕΡΙ ΤΗΣ ΣΥΝΘΕΣΕΩΣ ΜΗ ΟΜΟΓΕΝΩΝ ΕΠΙΠΕΔΩΝ ΣΤΟΙΧΕΙΟΚΕΡΑΙΩΝ

Ὑπὸ
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