Sci. Annals, Fac. Phys. & Mathem., Univ. Thessaloniki, **15**, 235, (1975)

ON THE GENERAL ANTENNA ARRAY SYNTHESIS

by
J. SAHALOS - K. MELIDIS - H. PAPADIMITRAKI-CHLICHLIA
(Dept. of Physics - University of Thessaloniki - Thessaloniki - Greece.
(Received 24.10.75)

Abstract: *An attempt is made to synthesize an antenna array by use of the othogonal method. The array factor is expressed as a power series of e, resulting in the determination of the amplitudes and phases of the elements' feedings with the help of simple formulas. Two applications are carried out.*

1. Introduction

The problem of synthesizing general nonuniform arrays has been studied |1| recently by orthonormalization of the base in which the array factor, $f(\varphi,\theta)$, is referred.

The application of this method in definite problems presents some difficulties arising from the inappropriate expression of $f(\varphi,\theta)$. In special, though, cases of planar or linear arrays the problem can be considerably simplified, and the synthesis has been done by use of other methods.

2. Formulation

In the case of a general nonuniform array of N discrete identical elements (Fig. 1) the radiation pattern is given by:

$$F(\varphi,\theta) = g(\varphi,\theta) \sum_{i=1}^{N} A_i \exp\{jkr_i|\sin\theta\sin\theta_i\cos(\varphi - \varphi_i) + \cos\theta\cos\theta_i|\} \qquad (1)$$

where the A_i 's represent complex amplitudes, $k = 2\pi/\lambda$ and $g(\varphi,\theta)$ is the radiation pattern of one element.

The array factor

$$f(\varphi,\theta) = \frac{F(\varphi,\theta)}{g(\varphi,\theta)}$$

has a base of the form

$$\{\Phi_i = \exp\{jkr_i[\sin\theta\sin\theta_i\cos(\varphi - \varphi_i) + \cos\theta\cos\theta_i]\}\} \quad (2)$$

It would be tiresome, if not tedious, to repeat here the procedures of orthonormalizing this base and arriving at the formulas that give the complex amplitudes. One can refer to [1] — [3] for the intermediate steps. It is sufficient to recall that

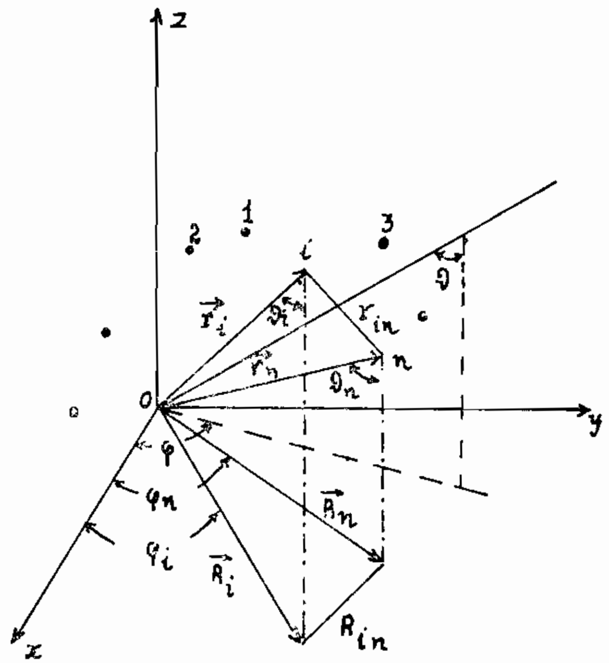


Fig. 1. Geometry of the general nonuniform array discussed

$$A_i = \sum_{j=1}^{N} B_j C_i^{(j)} \quad (3)$$

where

$$C_k^{(n)} = -\frac{4\pi}{D_n} \sum_{j=1}^{n-1} \left| C_k^{(j)} \left(\sum_{i=1}^{j} C_i^{(j)} S_{ni}\right) \right|$$

$$C_n^{(n)} = \frac{1}{D_n}$$

$$D_n = \left| 4\pi - 4^2\pi^2 \sum_{j=1}^{n-1} (\sum_{i=1}^{j} C_i^{(j)} S_{ni})^2 \right|^{\frac{1}{2}}$$

$$B_j = \int_o^{\pi} \int_o^{2\pi} f(\varphi,\theta)\Psi^*_j(\varphi,\theta)\sin\theta d\varphi d\theta \qquad (4)$$

$$\Psi_j(\varphi,\theta) = \sum_{i=1}^{j} C_i^{(j)}\Phi_i(\varphi,\theta)$$

and $$S_{ni} = \frac{\sin kr_{ni}}{kr_{ni}}$$

As we can see, B_j is the starting point of the synthesis. But a suitable representation of the array factor as a series is a way of solving the problem numerically. Now $f(\varphi,\theta)$ is a vector of the vector space that has the set $\{\Phi_i(\varphi,\theta)\}$ as a base. The property of $f(\varphi,\theta)$ is that its absolute value changes in general. Its phase in the vector space $\{\Phi_i\}$ is a vector. Thus $f(\varphi,\theta)$ can be expressed as

$$f(\varphi,\theta) = \Phi(\varphi,\theta) \, . \, \exp|jS(\varphi,\theta)| \qquad (5)$$

The unit vector $\exp|jS(\varphi,\theta)|$ can be expressed as a function of the Φ_i 's (since the set $\{\Phi_i\}$ fills the space), by a linear relation of the form

$$\exp|jS(\varphi,\theta)| = \sum_{i=1}^{N} \lambda_i\Phi_i(\varphi,\theta) \qquad (6)$$

Now the Φ_i 's are of the form $\exp|jS'(\varphi,\theta)|$, so we can accept that $S(\varphi,\theta)$ has a form similar to that of $S'(\varphi,\theta)$, i.e.,

$$S(\varphi,\theta) = kr_o \cdot |\sin\theta \, . \, \sin\theta_o\cos(\varphi - \varphi_o) + \cos\theta\cos\theta_o| \qquad (7a)$$

Moreover, $|f(\varphi,\theta)|$ must represent some properties—loci and values of maxima, beamwidths etc. Given that periodic functions of the form $e^{s(\varphi,\theta)}$ can define a locus of maximum and a beamwidth, a sum of the form

$$|f(\varphi,\theta)| = \sum_{m=1}^{M} T_m e^{S_m(\varphi,\theta)} = \sum_{m=1}^{M} T_m\, e^{P_m \cos(\varphi - \varphi_m)\sin\theta + q_m\cos\theta}$$

could represent the argument of the array factor.
We thus, by use of (4), (5), (7a,b) get, for B_l, (see Appendix)

$$B_l = 4\pi \sum_{i=1}^{l} \sum_{m=1}^{M} C_i^{*(l)} T_m \frac{\sin Z_{im}}{Z_{im}} \tag{8}$$

where Z_{im} is a known coefficient.
Formula (8) gives, in a simple form, the B_l's, which, together with the $C_i^{(l)}$'s can generate the complex amplitudes.

3. Remarks on the array factor.

When the argument of the array factor is expressed in the form

$$|f(\varphi,\theta)| = \sum_{m=1}^{M} T_m \exp|P_m \cos(\varphi - \varphi_m)\sin\theta + q_m\cos\theta| \tag{9}$$

one can write

$$|f| = \sum_{m=1}^{M} T_m \exp|R_m \cos a_m| \tag{10}$$

where

$$R_m = \sqrt{P^2_m + q^2_m}$$
$$\cos a_m = \cos(\varphi - \varphi_m)\sin\theta \,.\, \sin\theta_m + \cos\theta\cos\theta_m$$

$$\sin\theta_m = \frac{P_m}{\sqrt{q^2_m + P^2_m}} \quad \text{and} \quad \cos\theta_m = \frac{q_m}{\sqrt{q^2_m + P^2_m}}$$

Local maxima appear when $\cos a_m = 1$, or

$$\cos(\varphi - \varphi_m)\sin\theta \,.\, \sin\theta_m + \cos\theta\cos\theta_m = 1 \qquad (11)$$

(11) implies

$$\varphi = \varphi_m \qquad \text{and} \qquad \theta = \theta_m$$

leading to the conclusion that the series (10) will have as many terms as the maxima sought.

Another useful information are the half power angles. Now, if the $\exp(R_m)$'s are set equal to X_m, one can have, from (10), 2M equations of an equal number of unknowns, as follows:

$$T_1X_1 + T_2X_2^{\cos a^1_2} + \dots\dots\dots\dots + T_mX_m^{\cos a^1_m} = k_1$$

$$T_1X_1^{\cos a^2_1} + T_2X_2 + \dots\dots\dots\dots + T_mX_m^{\cos a^2_m} = k_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$T_1X_1^{\cos a^m_1} + T_2X_2^{\cos a^m_2} + \dots\dots\dots\dots + T_mX_m = k_m$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$T_1X_1^{\cos b^1_1} + T_2X_2^{\cos b^1_2} + \dots\dots\dots\dots + T_mX_m^{\cos b^1_m} = 2^{-\frac{1}{2}}\, k_1$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \qquad (12)$$

$$T_1X_1^{\cos b^m_1} + T_2X_2^{\cos b^m_2} + \dots\dots\dots\dots + T_mX_m^{\cos b^m_m} = 2^{-\frac{1}{2}}\, k_m$$

where a^i_j are the angles where the maxima occur, b^i_j are the half-power angles, and k_i the values of f at the maxima.

The solution of (12) will give the values of T_i and R_i that are necessary to determine $f(\varphi,\theta)$. The phase of the array factor must be of the form (7a).

4. Application in the case of a single-lobe radiation pattern.

In such a case one has, according to the previons discussion,

$$\left.\begin{array}{l} T_1X_1 = k_1 \\ T_1X_1^{\cos b^1_1} = 2^{-\frac{1}{2}}\, k_1 \end{array}\right| \qquad (13)$$

Dividing eqs. (13) by parts we get

$$X_1 = 2^{\frac{1}{2(1 - \cos b^1_1)}}$$

Then

$$R_1 = \frac{\ln 2}{2(1 - \cos b^1_1)} \quad \text{and} \quad T_1 = \frac{k_1}{R_1}$$

In the special case of uniform arrays the phase can be considered as a constant, independent of φ and θ.

Numerial application was carried out for two array forms, to get a single main lobe with a beamwidth of 20°.

For a linear array we can produce only a planar pattern with the above property, because of the axial symmetry of the array.

Putting the elements on the x-axis for a uniform linear array with twelve $\lambda/4$ - spaced elements we have a max at $\varphi = 0$, $\theta = 90$ (Fig. 2). For a circular uniform array of 12 elements, having a radius of 0.6λ and a max at $\varphi = 90$, $\theta = 90$ (Fig. 3), we put the elements on the xy plane (Fig. 4).

The amplitudes are

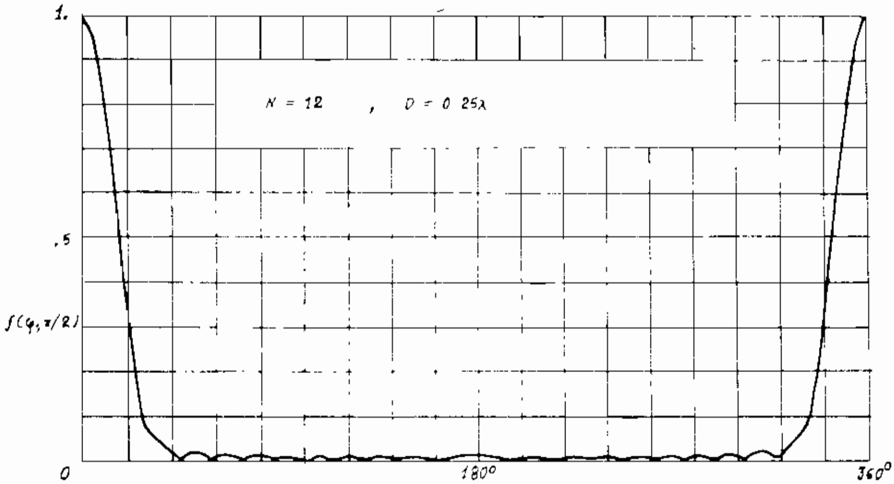


Fig. 2. Planar radiation pattern for the linear array

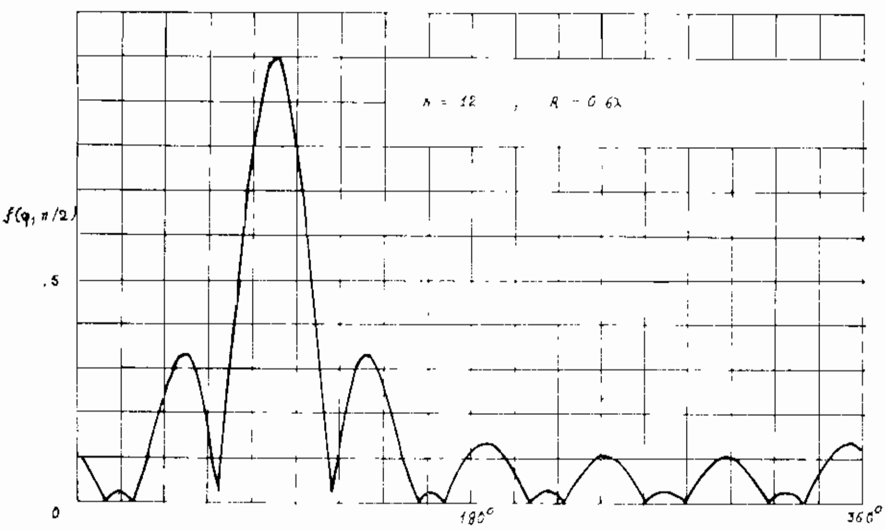
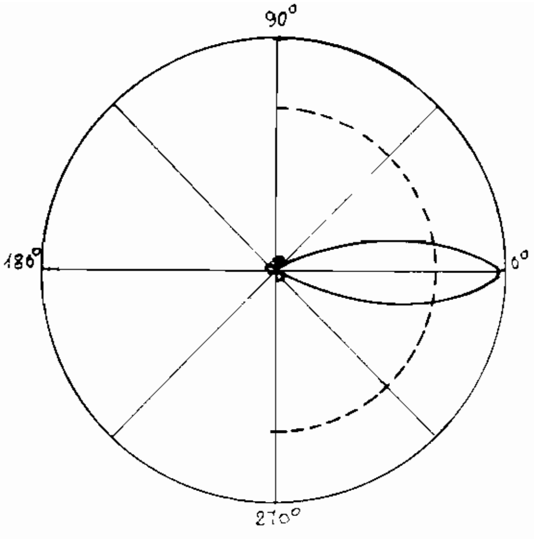
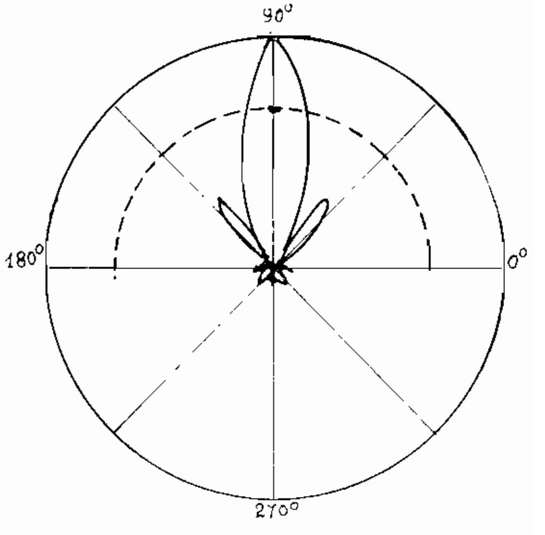


Fig. 3. Planar radiation pattern for the circular array





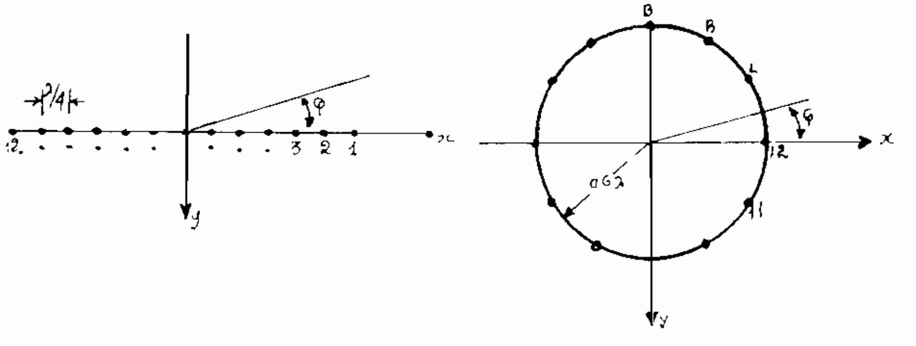


Fig. 4. Space geometry of the linear and circular arrays discussed

LINEAR ARRAY			*CRCULAR ARRAY*		
	Amplitude	*Phase, deg.*			
1	.008	127.9	1,5	.970	—168.4
2	.060	—35.1	2,4	.924	18.9
3	.213	154.7	3	.907	—158.6
4	.483	—16.6	6,12	1.00	0.0
5	.795	171.8	7,11	.97	168.4
6	1.0	0.0	8,10	.924	—18.6
7	.984	—171.7	9	.907	158.9
8	.76	16.7			
9	.454	—154.5			
10	.202	35.0			
11	.061	—134.5			
12	.01	58.1			

Appendix

Recalling that |4|

$$\int_0^{2\pi} \exp(a\cos x + b\sin a)\cos(\lambda\cos x + \mu\sin x)dx = \pi|I_o(\sqrt{C + jD}) + I_o(\sqrt{C - jD})| \quad (14)$$

and

$$\int_0^{2\pi} \exp(a\cos x + b\sin x)\sin(\lambda\cos x + \mu\sin x)dx = -i\pi|I_o(\sqrt{C + jD}) - I_o(\sqrt{C - jD})| \quad (15)$$

we have

$$B_1 = \int_0^{\pi}\int_0^{2\pi} \left| \sum_{m=1}^{M} T_m \exp|P_m\cos(\varphi - \varphi_m)\sin\theta + q_m\cos\theta| \right| \sum_{i=1}^{l} C^{*}{}_i{}^{(l)} \exp |$$

$$| -jk(Z_{io}\cos\theta)| \left| \exp(-jkR_{io}\,.\,\cos(\varphi - \varphi_{io})\sin\theta) \right| \sin\theta d\varphi d\theta$$

where $Z_{io} = r_i \sin\theta_i - r_o \cos\theta_o$, $\varphi_{io} = \varphi_i - \varphi_o$, $R_{io} = (R^2_i + R^2_o - 2R_iR_o \cos\varphi_{io})^{1/2}$, $R_i = r_i \sin\theta_i$ and $R_o = r_o \sin\theta_o$

From (14), (15) the integral

$$M = \int_o^{2\pi} \exp|P_m \cos(\varphi - \varphi_m)\sin\theta| . \exp| - jkR_{io}\cos(\varphi - \varphi_{io})\sin\theta| . d\varphi$$

becomes

$$M = 2\pi I_o(\sqrt{C - jD})$$

where

$$C = \sin^2\theta|P^2_m - k^2R^2_{io}|$$

$$D = - 2\sin^2\theta P_m kR_{io}\cos(\varphi_m - \varphi_{io})$$

and

$$\sqrt{C - jD} = \sin\theta|P^2_m - k^2R^2_{io} + 2jP_m kR_{io}\cos(\varphi_m - \varphi_{io})|^{1/2} =$$
$$= ja_{im}\sin\theta$$

(a_{im} is a complex number)

Now M takes on the form

$$M = 2\pi J_o(a_{im}\sin\theta) \tag{17}$$

(J_o is the zeroth Bessel function)

and (16) becomes

$$B_l = 2\pi \int_o^{\pi} \sum_{i=1}^{I} \sum_{m=1}^{M} T_m C_i^{*(l)} \exp . (q_m - jkZ_{jo})\cos\theta | J_o(a_{im}\sin\theta) .$$
$$. \sin\theta d\theta \tag{18}$$

Placing

$$q_m - jkZ_{io} = jb_{im}$$

we get

$$B_1 = 4\pi \sum_{i=1}^{I} \sum_{m=1}^{M} C_i^{*(1)} T_m \sqrt{\pi/2}\, (a_{im}^2 + b_{im}^2)^{-1/4}\, J_{1/2}\, |\, (a_{im}^{2} + b_{im}^2)^{\frac{1}{2}}\, | \quad (19)$$

or, substituting

$$Z_{im}^2 = a_{im}^2 + b_{im}^2 \qquad ,$$

$$B_1 = 4\pi \sum_{i=1}^{I} \sum_{m=1}^{M} C_i^{*(1)} T_m \frac{\sin Z_{im}}{Z_{im}}$$

REFERENCES

1. SAHALOS, J.: "A solution of General Nonuniformly spaced Antenna Array". Proc, of IEEE, 1974, 62, pp. 1292-1294.
2. UNZ, H.: "Nonuniformly spaced arrays. The orthogonal method". Proc. of IEEE 1966, 54, pp. 53-54.
3. SAHALOS, J.: "The orthogonal method of nonuniformly spaced arrays". Proc. of IEEE, 1974, 62, pp. 281-282.
4. GRADSHTEYN, I and RYZHIK, I.: "Table of integrals Series and products". Academic Press, 1965, p. 488.

ΠΕΡΙΛΗΨΙΣ

ΠΕΡΙ ΤΗΣ ΣΥΝΘΕΣΕΩΣ ΓΕΝΙΚΩΝ ΣΤΟΙΧΕΙΟΚΕΡΑΙΩΝ

Ὑπὸ
Ι. ΣΑΧΑΛΟΥ, Κ. ΜΕΛΙΔΗ καὶ Ε. ΠΑΠΑΔΗΜΗΤΡΑΚΗ - ΧΛΙΧΛΙΑ
(Γ' Ἕδρα Φυσικῆς Πανεπιστημίου Θεσσαλονίκης)

Γίνεται μία προσπάθεια συνθέσεως στοιχειοκεραίας διὰ χρήσεως τῆς μεθόδου ὀρθογωνοποιήσεως. Ὁ παράγων σειρᾶς ἐκφράζεται ὑπὸ μορφὴν δυναμοσειρᾶς τοῦ e. Τοῦτο ἔχει ὡς ἀποτέλεσμα τὸν προσδιορισμὸν τῶν πλατῶν καὶ τῶν φάσεων τῆς τροφοδοσίας τῶν στοιχείων τῇ βοηθείᾳ ἁπλῶν τύπων. Ἀναφέρονται δύο ἐφαρμογαί.