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A GENERALIZATION OF WASSCHER'S METHOD FOR MEASURING THE GALVANOMAGNETIC COEFFICIENTS OF ANISOTROPIC MATERIAL

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Abstract: *Wasscher's modification of Van der Pauw's method for measuring galvanomagnetic coefficients is generalized in order to become applicaple in materials of unknown charaeteristics. The possible effect of spacing anomalies on the spatial distribution is also investigated.*

1. INTRODUCTION

The method proposed by Van der Pauw for measuring galvanomagnetic coefficients [1] is an elegant and sensitive method when applied to isotropic materials. In the case of anisotropic materials the method, although applicalbe in its bacic configuration needs readjustment to yield correct results. Wasscher [2] did just that, he extended the treatment of the data in cases where structural anisotropies determine the values of the measured quantities. Since Wasscher developed his method for applying it in a specific case, e.i to measure the anisotropic galvanomagnetic coefficients of MnTe, he did not generalize it to cover all possible experimental encounters.

The high concentration of carriers in the case of degenerate semiconductors, e.g. GeTe or SnTe, combined with a slight anisotropy of the low temperature modification, presented a case where the general principles of Wasscher's method were applicable, but required refinement to yield reliable and meaningfull results. The aim of this paper is to present the way that Wasscher's method was applied in the case of GeTe [3] which we believe is quite general and can be used in the case of unknown materials.

2. GENERAL PRINCIPLES

Suppose that we are dealing with a material whose resistivities ρ_i lie along three principal, mutually orthogonal axes, x_i. A flat circular sample of this material, of thickness d and radius r, with its surface plane normal to the principal axis x_3 would contain the principal resistivities ρ_1 and ρ_2 in its surface plane. This will be electrically equivalent, as proved by Wasscher, to an anisotropic sample with the following characteristics (fig. 1)

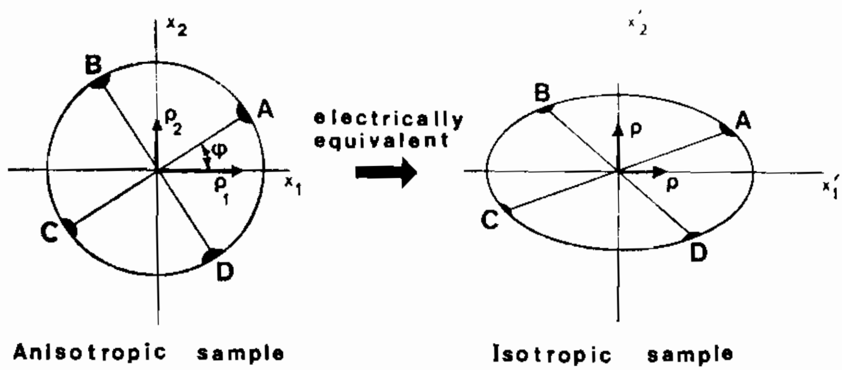

Fig. 1 An anisotropic circular sample is equivalent to an isotropic elliptical one. The case is for $\varrho_1 > \varrho_2$.

a. thickness $$d' = d\left(\frac{\rho_3}{\rho}\right)^{1/2} \qquad (1)$$

b. semi-axes $$a = r\left(\frac{\rho_1}{\rho}\right)^{1/2}$$

$$b = r\left(\frac{\rho_2}{\rho}\right)^{1/2} \qquad (2)$$

c. specific resistance $$\rho = (\rho_1\rho_2\rho_3)^{1/3} \qquad (3)$$

To perform measurements two pairs of contacts AC and BD are formed at the circumference of the sample at the opposite sides of two mutual perpendicular diameters (fig. 1), forming an angle with the principal directions x_1 and x_2. With this arrangemend two resistances, as defined by Van der Pauw, can be measured

$$R_1 = \frac{V_D - V_C}{I_{AB}}$$

$$R_2 = \frac{V_A - V_D}{I_{BC}} \qquad (4)$$

These resistances may be used to determine the specific resistance from the relation

$$\exp\left(-\pi R_1 \frac{d'}{\rho}\right) + \exp\left(-\pi R_2 \frac{d'}{\rho}\right) = 1 \qquad (5)$$

which leads to the expression

$$\rho = \frac{\pi d'}{2\ln 2}(R_1 + R_2)\, f\left(\frac{R_1}{R_2}\right) \qquad (6)$$

The parameter $f(R_1/R_2)$ is a correcting factor derermined experimentally, which Van der Pauw presented graphically using as coordinates f vs R_1/R_2. A usefull parametric relation to express Van der Pauw's graphical results for $f(R_1/R_2)$ is given by Wasscher (2)

$$f\left[\left(\frac{R_1}{R_2}\right)_{max}\right] = \frac{\ln\left(\frac{1}{4}\right)}{\ln\left(\frac{1}{2}+x\right) + \ln\left(\frac{1}{2}-x\right)} \quad \frac{R_1}{R_2} = \frac{\ln\left(\frac{1}{2}-x\right)}{\ln\left(\frac{1}{2}+x\right)} \qquad (7)$$

with $-\frac{1}{2} < x < \frac{1}{2}$. Now for an anisotropic sample, by substituting ρ and d' from (3) and (1) in equation (6) the following relation is obtained

$$(\rho_1 \rho_2)^{1/2} = \frac{\pi d}{2\ln 2}\,(R_1 + R_2)\, f\left(\frac{R_1}{R_2}\right) \qquad (8)$$

If the equivalent ellipse representing the resistance anisotropy of the circular sample is mapped conformally into a çircle of unit radius, the expressions for the measured values R_1 and R_2 transform to the following

$$R_1 = \frac{(\rho_1 \rho_2)^{1/2}}{\pi d} \ln \left[\frac{2}{1 - k sn(2u)} \right]$$

$$R_2 = \frac{(\rho_1 \rho_2)^{1/2}}{\pi d} \ln \left[\frac{2}{1 + k sn(2u)} \right] \qquad (9)$$

where $u = 2K(k)\frac{\varphi}{\pi}$ and $sn(2u)$ is a complete elliptical integral of the first kind; k is the parametric argument. The values of R_1 and R_2, as determinded by the relations (9), obey the fundamental equation 5.

In the case that $\varphi = 45^0$ the relations (9) are reduced to the following

$$(R_1)_{max} = \frac{(\rho_1 \rho_2)^{1/2}}{\pi d} \ln \left[\frac{2}{1 - k} \right]$$

$$(R_2)_{min} = \frac{(\rho_1 \rho_2)^{1/2}}{\pi d} \ln \left[\frac{2}{1 + k} \right] \qquad (10)$$

Dividing by terms we obtain the relation

$$\left(\frac{R_1}{R_2} \right)_{max} = \frac{\ln \left[\frac{1}{2} (1 - k) \right]}{\ln \left[\frac{1}{2} (1 + k) \right]} \qquad (11)$$

from which the value of the parameter k is determined. Furthermore by adding the relations (10) and substituting the result $[(R_1)_{max} + (R_2)_{min}]$ into equation (8) one obtains the following relation for the correcting factor

$$f\left[\left(\frac{R_1}{R_2}\right)_{max}\right] = \frac{\ln\left(\frac{1}{4}\right)}{\ln\left[\frac{1}{2}(1-k)\right] + \ln\left[\frac{1}{2}(1+k)\right]} \tag{12}$$

from which it is evident that k lies within limits $-1 < k < 1$.

The dependence of the ratio of resistances R_1/R_2, measured at arbitrary angles, to the angle φ was investigated by Wasscher graphically by plotting $\ln(R_1/R_2) / \ln(R_1/R_2)_{max}$ vs φ for three values of (ρ_1/ρ_2). He also constructed a diagram plotting the ratio $(R_1/R_2)_{max}$ vs (ρ_1/ρ_2) for various geometries of the sample. Thus the treatment of the data is inherently dependent upon two diagrams.

3. EXTENSION OF THE TREATMENT

From the above it is evident that Wasscher's method needs further refinement to be applicable in a general case.

The first limitation is that it requires the knowledge of the angle φ. This is possible in materials for which the principle axes are known. In unknown materials the reference frame may be chosen arbitrarity, so that $\varphi = \theta + \omega$, where θ is the experimental angle of rotation of the sample in respect to the set of contacts and ω is the phase angle. To obtain ω we notice that

$$\text{for} \quad \theta = -\omega, \left(\frac{\pi}{2} - \omega\right) \text{ or } (\pi - \omega) \qquad \ln\left(\frac{R_1}{R_2}\right) = 0$$

$$\text{for} \quad \theta = \frac{\pi}{4} - \omega \qquad \ln\left(\frac{R_1}{R_2}\right) = \max \tag{13}$$

$$\text{for} \quad \theta = \frac{3\pi}{4} - \omega \qquad \ln\left(\frac{R_1}{R_2}\right) = \min$$

from which a set of two orthogonal directions is obtained, the relative maximum and minimum. Supposing that the above process is constructed on two samples with surface planes perpendicular to each other, it results in a pair of two directions of relative maximun resistances. Now if we draw on each sample a plane containing the direction of its maximum and perpendicular to the surface, on the crossing of these two planes will lie the direction of the absolute maximum of the resistivity ellipsoidal. Reminding that $\rho_1 > \rho_2 > \rho_3$ the last defined direction is parallel to ρ_1. So at last, the resulted critical directions on a sample perpendicular to the determined ρ_1, will give the other principal directions of ρ_2 and ρ_3.

The second point that needs further investigation is the dependence of the ratio (R_1/R_2) on φ and (ρ_1/ρ_2), which has been presented graphically by Wasscher. This may be done by investigating separetely the case for which the anisotropy is small and when it is rather large.

Thus by using values of $(R_1/R_2)_{max} = \gamma < 1.4$ the relation of R_1/R_2 vs φ, as obtained by using equations (9), leads to an elliptical spatial distribution of R. The values of the resistances are given by the following relations

$$R_1^2 = \frac{a^2b^2}{a^2\sin^2\theta + b^2\cos^2\theta}$$
$$R_2^2 = \frac{a^2b^2}{a^2\cos^2\theta + b^2\sin^2\theta} \qquad (14)$$

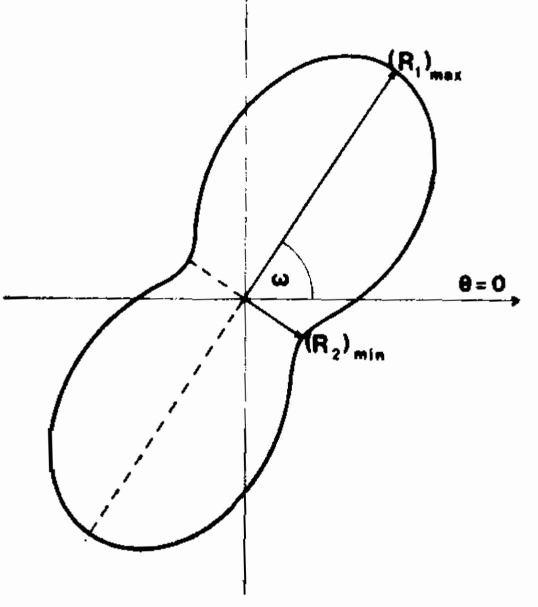

Fig. 2 Polar distribution of resistance R_1, R_2 as function of the experimental-arbitrary-angle θ in respect to the set of contacts.

where a and b are the semi-axis of the ellipse. By defining the ratio R_1/R_2, at an arbitrary angle θ, as w, it is easily proved that

$$\ln w = \ln\left[\frac{\gamma^4 + \tan^2(\varphi - 45)}{1 + \gamma^4 \tan^2(\varphi - 45)}\right] \quad (15)$$

For values of $\gamma > 1.4$ the simple elliptical form is not sufficient to describe the dependence of w on φ. In order to generalize the method we used the more general expressions (9). Using values of sn(2u) for various k as given in tables [4], a parametric relation could be constructed. Thus for values $\gamma < 69$ we found that we could express the following approximate relation

$$\log w = [\sin 2\varphi - 0.02[\,|\sin 4\varphi| + 0.145\,|\sin 8\varphi|\,]\log\gamma]\log\gamma \quad (16)$$

In fig. 3 the plotts of logw vs sn(2u), as obtained from the relations (9), and logw vs φ, as obtained from (16), are given side by side for comparison. From this diagram it is evident that the apporximation (16) is satisfactory (<0.1 %).

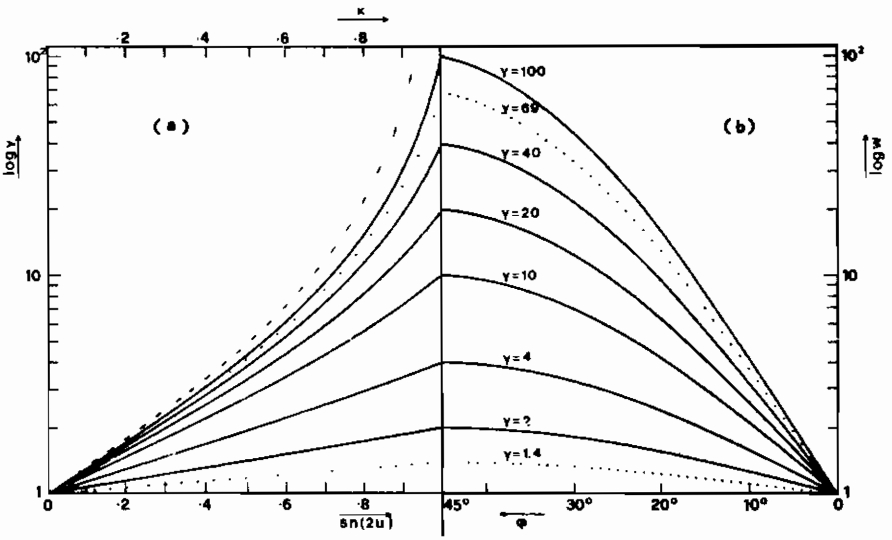

Fig. 3. Diagram showing the dependence of the functions logγ and logw upon the sn(2u) and the angle φ for various values of the anisotropy parameter γ.

By using the approximate expression (16) the values of the resistances are given by the following relations

$$R_1 = R_{max} \frac{\ln(1+\gamma^{y/\sqrt{2}})}{\ln(1+\gamma^{1/\sqrt{2}})}$$

$$R_2 = R_{min} \frac{\ln(1+\gamma^{-y/\sqrt{2}})}{\ln(1+\gamma^{-1/\sqrt{2}})} \tag{17}$$

with $y = \ln w/\ln\gamma$. Polar diagrams, using a normalized value of b=1, is given in figure 4. From these diagrams the various expected forms of the spatial distribution of R are evident.

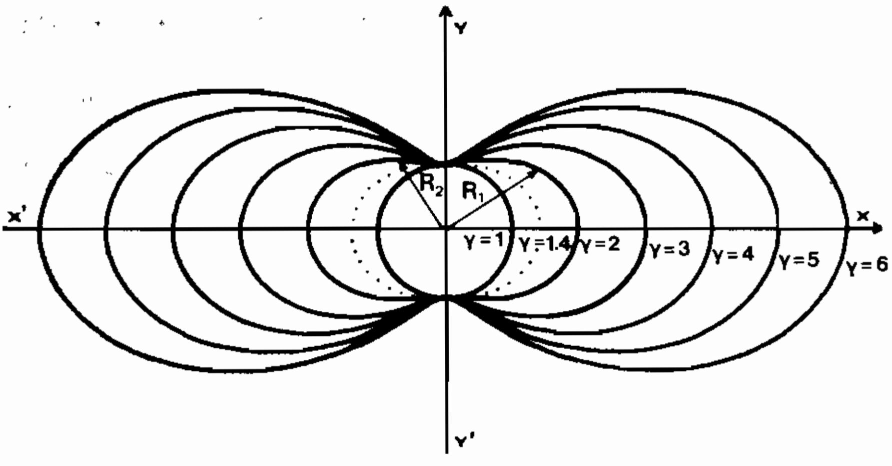

Fig. 4. Set of the polar dispersion of the resistance R, for different values of the anisotropic parameter γ, refered to a common arbitrary value of R_{min}.

In addition the dependence of on the ratio of the principle resistances $\lambda = \rho_1/\rho_2$ (which again is given by Wasscher in a graphical form) was investigated. It was found that a parametric equation which expressed the results of Wasscher's plott for a circular sample was of the form

$$\lambda = \exp\left(m \frac{\gamma^c - 1}{\gamma^c + 1}\right) \tag{18}$$

with c = 0.2142 and m = 3.2633.

Equations (18) and (8) form a set of simultaneous equations, the solution of which leads to the values ρ_1 and ρ_2. Thus the following relations are obtained upon substitution of (18) in (8)

$$\rho_1 = \frac{\pi d}{\ln 2} R_{max} \left(\frac{\gamma+1}{2\gamma}\right) f(\gamma) \exp\left(\frac{m}{2} \frac{\gamma^{\circ}-1}{\gamma^{\circ}+1}\right)$$

$$\rho_2 = \frac{\pi d}{\ln 2} R_{min} \left(\frac{\gamma+1}{2}\right) f(\gamma) \exp\left(\frac{m}{2} \frac{1-\gamma^{\circ}}{1+\gamma^{\circ}}\right) \quad (19)$$

To simplify the expression of the experimental results, the specific conductivity is expressed by the relation

$$\sigma_l = S_l \Phi_l(\gamma) \quad (20)$$

where

$$S_l = \frac{\ln 2}{\pi d} \quad \frac{I_l}{V_l} \quad (21)$$

and

$$\Phi_1(\gamma) = \frac{2\gamma}{f(\gamma)(\gamma+1)} \exp\left(-\frac{m}{2}\frac{\gamma^{\circ}-1}{\gamma^{\circ}+1}\right) \quad (22)$$

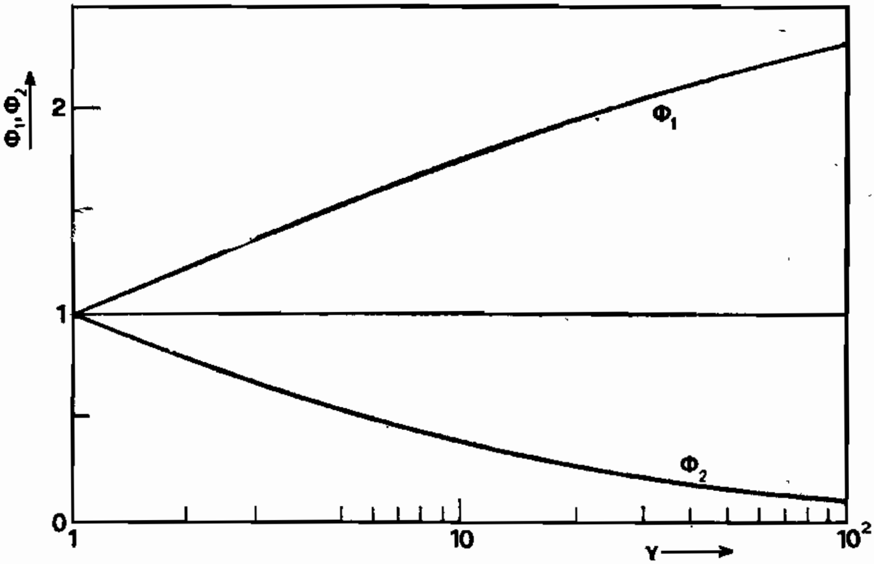

Fig. 5. Diagram showing the dependence of the «anisotropy terms» Φ_1 and Φ_2 upon the variation of the anisotropy parameter γ.

$$\Phi_2(\gamma) = \frac{\gamma}{f(\gamma)(\gamma+1)} \exp\left(-\frac{m}{2}\,\frac{1-\gamma^c}{1+\gamma^c}\right) \qquad (23)$$

The parameter $\Phi_1(\gamma)$ is the expression of the anisotropy, the «anisotropy term», since for γ=1 it is equal to 1. In fig. 5 the dependence of Φ_1 and Φ_2 upon γ is plotted.

4. INFLUENCE OF SPACING ANOMALIES

The method proposed by Van der Pauw, and also Wasscher's treatment of the data, requires homogeneous samples. In cases that secondary effects, as for instance spacing anomalies prevail, these may influence the spatial distribution of resistances. In cases that the walls of the spacing anomalies follow a relatively regular pattern, a wavy character in the spatial distribution is expected. This is actually an effect observed in GeTe [3]. Now if we suppose that there is a change in the specific resistance caused by the secondary effects, this change is manifested as

$$\frac{d\lambda}{\lambda} = 2mc\,\frac{\gamma^c}{(1+\gamma^c)^2}\,\frac{d\gamma}{\gamma} \qquad (24)$$

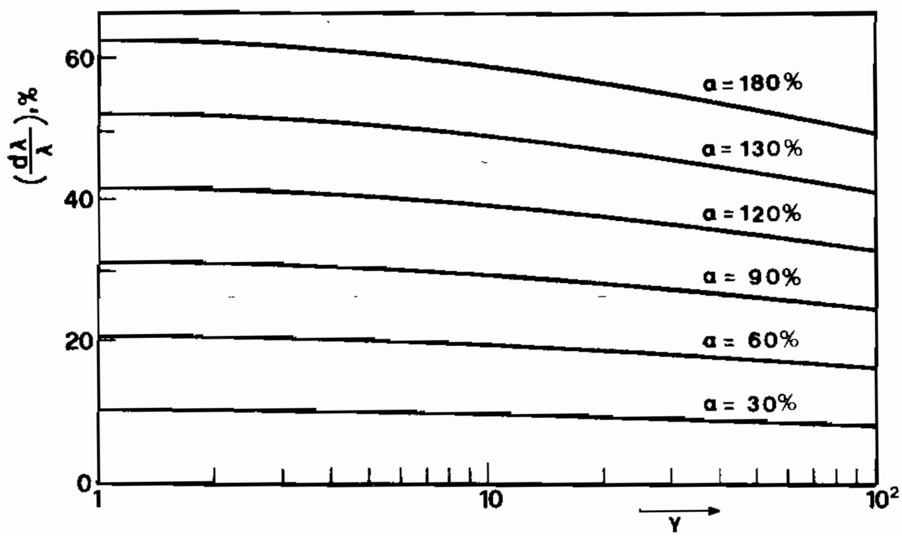

Fig. 6. Dependence of the method sensitivity, upon the fluctuations of the anisotropy ratio λ as a function of γ.

For $\gamma \gg 1$ we may approximate the expression (24) by the following

$$\frac{d\lambda}{\lambda} = 2mc\gamma^{-\circ} \frac{d\gamma}{\gamma} \qquad (25)$$

By defining a modulation parameter $a = d\gamma/\gamma$ in percentage manner, we may construct diagrams of $d\lambda/\lambda\%$ vs γ for various values of a (fig. 6). The sensitivity of the method is evident, since small changes in λ influence the measured values of γ in an exponential manner.

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ΠΕΡΙΛΗΨΗ

ΓΕΝΙΚΕΥΣΗ ΤΗΣ ΜΕΘΟΔΟΥ WASSCHER ΓΙΑ ΤΗ ΜΕΤΡΗΣΗ ΓΑΛΒΑΝΟΜΑΓΝΗΤΙΚΩΝ ΣΤΑΘΕΡΩΝ ΑΝΙΣΟΤΡΟΠΩΝ ΥΛΙΚΩΝ

Ὑπὸ

Ο. ΒΑΛΑΣΣΙΑΔΗ καὶ Ν. Α. ΟΙΚΟΝΟΜΟΥ

('Εργαστήριο Β' Ἕδρας Φυσικῆς Πανεπιστημίου Θεσσαλονίκης)

Ἡ μεθοδολογία Wasscher, ποὺ εἶναι τροποποίηση τῆς μεθόδου τοῦ Van der Pauw, εἶναι κατάλληλη γιὰ μετρήσεις γαλβανομαγνητικῶν σταθερῶν σὲ ἀνισότροπα ὑλικὰ γνωστῶν χαρακτηριστικῶν. Ἡ μέθοδος αὐτὴ ἐπεκτείνεται καὶ παρουσιάζεται μὲ γενικότερη μορφή, ποὺ μπορεῖ νὰ ἐφαρμοσθῇ σὲ περιπτώσεις ὑλικῶν ἄγνωστης ἠλεκτρικῆς συμπεριφορᾶς. Ἐπίσης ἀναλύεται ἡ δυνατότητα μελέτης τῆς ἐπίδρασης ἐκτεταμένων ἀνωμαλιῶν δομῆς στὴν πολικὴ κατανομὴ τῆς ἠλεκτρικῆς συμπεριφορᾶς.