Fractals: USE in physical geography and geology
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SUMMARY

Fractals can be defined as irregular continuous curves without derivatives. The fractal or Hausdorff dimension is not a necessary integer parameter and characterizes the form of a curve.

Fractal curves are located in many geographic objects such as coastlines, Karst regions, drainage basins and so on.

The distribution of the areas of sixteen Greek islands, the areas of Greek lakes and Greek plains display a Pareto distribution and a fractional behaviour as well. Fractional dimensions for the recent faulted zones of Sifnos and Andros are $D=1.10$ and $D=1.02$ respectively.

Lengths of rivers such as selinous have values close to $D=1.10$.

Generally fractal dimensions may have low values when the faults measured are recent.

The values of $D$ depend on the geomorphologic stage of a region.
Low values are observed in the period of the original uplift and the panplain. On the other hand, the relief indicates elevate values in the intermediate stages.

The value of fractional dimension of the Landscapes is proportional to Entropy, which means that maximum Entropy reflects maximum disorder or randomness in nature and consequently Gaussian distribution of different variables. So we expect normal distribution for landscape variables on high D values.
ΠΕΡΙΛΗΨΗ
Συν Fractals ορίζονται συνεχείς μη "ομαλές" καμπύλες, μη παραχωρήσεις, Η fractal dimension (F.D) είναι μία παράμετρος που μπορεί να λάβει και μη ακέραιες τιμές και χαρακτηρίζει την μορφή μιας καμπύλης.

Πολλές γεωμορφοφές όπως ακτογραμμές, καροτ, υδρογραφικές λεκάνες κλπ. μπορούν να χαρακτηρισθούν σεν Fractal μορφές. Η κατανομή του εμβαδού των 16 Ελληνικών νησιών των Ελληνικών λιμνών και πεδίων είναι μία Pareto κατανομή και επομένως "συμπεριφέρονται" σεν Fractals. Μετρήσεις της F.D. για τις ρηχωτωμένες πρόσφατα ακτές της Σέληνου και Ανδρού, έδωσαν τιμές D = 1.10 and 1.02 αντίστοιχα.

Τα μήκη ποταμών όπως o Σέληνου έχουν D = 1.10 περί που. Γενικά ναμένουμε χαμπές τιμές της F.D. όταν τα υπό μελέτη ρηχματα είναι πρόσφατα.

Οι τιμές της F.D. εξαρτώνται από το στάδιο αξέλιξης μιας περιοχής. Είναι στο στάδιο ανύψωσης της περιοχής και στο στάδιο του πανεπίπεδου οι τιμές της F.D. θα είναι μικρές, αντίθετα στο ενδεικτικό στάδιο μειώσεως. Η τιμή της F.D. του αναγλύφου είναι ανάλογη της Εντροπίας, μεγάλη εντροπία σημαίνει μεγάλη "ατάξια" ή "τυχαιότητα" στη φύση και κατά συνέπεια οι μεταβλητές του αναγλύφου θα σκολοφθούν κατανομή κατά θεωρία. Είναι για υψηλές τιμές της F.D. αναμένουμε κανονική κατανομή των μεταβλητών των γεωμορφών.

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INTRODUCTION

In 1967 Mandelbrot introduced "fractal" geometry, a new kind of geometry which is entirely different from the broadly accepted Euclidean geometry. The fractal curves can be defined as irregular continuous curves without derivatives.

Most natural objects contain this kind of irregularity and this fact becomes clearer when we move our observation to different scales for example from small hills to great mountains.

This is the main characteristic of the fractal concept and it is called self-similarity.

A very well know geometrical example of self-similarity is the "Koch" curve (Figure 1).

![Figure 1: The "Koch" curve.](image)

This curve called "Koch" island" was first studied by the mathematician Helge von Koch in 1904. It is constructed by adding in the middle of each of the three sides of the triangle, another smaller triangle whose side length is equal to one third side length of the initial triangle.

The second important point in fractal theory is the
fractal dimension, which is a number declaring the
degree of curve irregularity and it is possible to
take non-integer value as well. The formula to obtain
the value of F.D. is $D = -\frac{\text{LOG}N}{\text{LOG}r}$ (N) where $N$ is the
number of decomposed parts of a figure and $r$ (N) is
the similarity ratio. In the case of the figure 1 we have

The shape of fractal curves as it happens in "Koch"
curves seems too geometrical to share characteristics
in common with natural features, so that
the concept of randomness must be necessarily introduced
to provide more realistic curves. This is the model
called fractional Brownian motion (Mandelbrot 1982)
and the resulting fractal curves display statistically
self-similarity.

Observing natural objects we easily realise that
many of them represent fractional curves and so it is
useful to study them with this new geometry.

Fractal’s use in physical geography and geology

Coastlines can be regarded as the most typical fractal
curves. Richardson (1961) stated an empirical law
saying that the length of a coastline depends on the
various yardstick lengths.

The value of $D$ for a coastline can be calculated by
means of a log-log graph plot, the number of segments
of length and versus various sampling intervals. This
slope of the line represents the value of $D$.

It is clear that the value of $D$ reflects the shape
of a coastline and for $D=1$ a straight line arises. If $D$ is increased, the shape of a coast becomes more irregular (Figure 2).

![Figure 2: coastlines with $D=1.16$](image)

Observing the coast of Aegean islands the fractal dimension varies. For this reason faulted coasts like the Northeastern zone of Sifnos and the Southwestern part of Andros have $D=1.10$ and $D=1.02$ (Figure 2A).

![Figure 2A: Fractal dimension of the Northeastern part of Sifnos. $D=1.105$.](image)

but the same islands have coast with much more irregularity and the value of their $D$ is $1.99$ and $1.122$ respectively.
It is worth noting that fractal shape "islands" (Figure 3) can be produced from Mandelbrot's simple formula

\[ X_{n+1} = F(X_n) = X_n^2 + c \]

for \( c = -0.12375 + 0.54508i \)

Figure 3:
Fractal shape "islands"

In a broad sense many natural features such as areas of lakes, islands and stretches of rivers display an hyperbolic size distribution (Korcak Law). This kind of distribution is the typical consequence of fractal landscapes.

The distribution of the areas of Greek islands, the areas of Greek plains and the lengths of rivers can be studied (Figures 4, 5, 6).
Figure 4: A plot between log(rank) and log(area) of 16 Greek islands.

Figure 5: A plot between log(rank) and log(length) of the 17 Greek rivers.
Fig. 6: A plot between log(rank) and log (Area) of 9 Greek Plains

Observing the figures 4, 5 and 6 we confirm the fractal nature of the above geographic objects.

Fractal "fragmentation" also shows the length of caves with $D \approx 1.4$ approximately (curl 1986, Lawry 1987). Drainage system elements indicate a fractional behaviour such as the areas of drainage basins and stretches of streams. An instance of this is the fractal dimension of the length of Selinous drainage basin which has been calculated and gives value $D$.
Self similar characteristics indicate also different tectonic elements such as fractures (Chiles 1988), faults related to earthquakes, folds or world-wide crust deformation.

The study of fractures on the island of Mykonos shows a fractal dimension $D = 2.23$ (Figure 8).

![Fractal dimension of the fractures of Mykonos](image)

Figure 8: Fractal dimension of the fractures of Mykonos $D = 2.23$

The fractal dimension of a fault indicates its age and new faults have values close to one. Finally the soil covered topography displays a fractal geometry. The fractal dimension of this surface in temperate regions varies between 2.0 and 2.3 (Culling and Datko 1987).

It is obvious that the values of $D$ vary because of the stage of development of a region in the Davisian period.
Thus during the first uplift and in the stage of plain the fractal surface will have values close to 2, while in the intermediate stage the value $D$ will be greater.
CONCLUSIONS

The existence of fractal behaviour has been proved for several geographic features in Greece, such as for areas of islands, plains and lakes and for the lengths of rivers.

The fractal dimension is "a handy tool" to obtain greater knowledge of the form of a curve. Low fractal dimension has been calculated for the faulted zone of Sifnos and Andros and the values of D have been calculated as 1.10 and 1.02 respectively. On the other hand non-recent faulted zones present higher D values.

Finally, the fractal dimensions of traces of fractures of Mykonos and the drainage basins of selinous river have been measured to be D=2.23 and D=2.33.

It becomes clear that the landscape value of D is proportional to entropy and as the erosion proceeds the D diminishes.

On the other hand maximum entropy results in maximum disorder which means randomness in nature and Gaussian distribution of different landscape variables.

BIBLIOGRAPHY


of the soil-covered landscape.

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